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29 Steel Design Guide

Vertical Bracing Connections—Analysis and Design

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Preface

This Design Guide provides guidance for the design of braced frame bracing connections based on structural principles and adhering to the 2010 AISC *Specification for Structural Steel Buildings* and the 14th Edition AISC *Steel Construction Manual*. The content expands on the discussion provided in Part 13 of the *Steel Construction Manual*. The design examples are intended to provide a complete design of the selected bracing connection types, including all limit state checks. Both load and resistance factor design and allowable stress design methods are employed in the design examples.

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Chapter 1
Introduction

1.1 OBJECTIVE AND SCOPE

This Design Guide illustrates a method for the design of braced frame bracing connections based on structural principles, and presents the design basis and complete design examples illustrating the design of:

1. All orthogonal and nonorthogonal connections involving a brace, a beam and a column (corner type)
2. Connections involving a beam or column and one or two braces, such as chevron or K-bracing, and eccentric braces (center type)
3. Connections of braces to columns at column base plates (base type)
4. Both nonseismic and seismic situations are covered

1.2 DESIGN PHILOSOPHY

All structural design, except for that which is based directly on physical testing, is based either explicitly or implicitly on the principle known as the lower bound theorem of limit analysis. This theorem is important because it allows structural engineers to be confident that 1) their assumptions about the internal force field will not over-predict the strength of an indeterminate structure, and 2) different methodologies for determining an admissible force field, while they may vary significantly in their predictions of the available strength, are nonetheless all valid. This theorem, which was first proven in the form given in the following in the 1950s (Baker et al., 1956), states that:

Given: An admissible internal force field (i.e., a distribution of internal forces in equilibrium with the applied load)

Given: Satisfaction of all applicable limit states

Then: The external load in equilibrium with the internal force field is less than, or at most equal to, the connection capacity.

The lower bound theorem is applicable to ductile limit states, and most connection limit states have some ductility. For instance, bolts in shear undergo significant shear deformation, on the order of ⅜ in. for a ¾-in.-diameter bolt, before fracture. Limit states such as block shear and net shear can accommodate significant distortion of the material before fracture. Plate or column buckling, while generally conceived as a nonductile limit state, is in a sense a ductile limit state; when a plate or column buckles, it does not become incapable of supporting any load, but rather will continue to support the buckling load as long as any excess load can be distributed to other components of the structural system. This phenomenon can be observed in the laboratory when a displacement-type testing machine is used. If a force-type machine is used, the load will increase continuously, and kinking and complete collapse will occur.

Actually all structural design relies on the validity of the lower bound theorem. For instance, if a building is modeled by a frame analysis computer program, a certain distribution of column loads will result. This distribution is dependent on thousands of assumptions. Shear connections are assumed not to carry any moment at all, and moment connections are assumed to maintain the angle between members. Neither assumption is true. Therefore, the column design loads at the footings will sometimes be drastically different from the actual loads, if these loads were measured. Some columns will be designed for loads smaller than the true load, and some will be designed for larger loads. Because of the lower bound theorem, this is not a concern.

Ductility can also be provided to an otherwise nonductile system by support flexibility. For instance, transversely loaded fillet welds are known to have limited ductility. If a plate is fillet welded near the center of a column or beam web and subjected to a load transverse to the web, the flexibility of the web under transverse load will tend to mitigate the low ductility of the fillet weld and will allow redistribution to occur. This same effect can be achieved with transversely loaded fillet welds to rigid supports by using a fillet weld larger than that required for the given loads. The larger fillet weld allows the given applied loads to redistribute within the length of the weld without local fracture.

The term "admissible force field" perhaps needs some further explanation. Bracing connections are inherently statically indeterminate. Therefore, there will be many possible force distributions within the connection. All of those force distributions that satisfy equilibrium are said to be "admissible" or "statically admissible." There are theoretically an infinite number of possible admissible force fields for any statically indeterminate structure. There will also be an infinite number of internal force fields that do not satisfy equilibrium; these are said to be "inadmissible." If such a force field is used, the lower bound theorem is not valid and any design obtained with this inadmissible force field cannot be said to be safe; i.e., the failure load may be less than the applied load. When an admissible force field is used, the calculated failure load will be less than, or at most equal to, the load at which failure occurs; therefore, a safe design is achieved.

Chapter 2
Common Bracing Systems

Figure 2-1 shows some common concentric bracing configurations. The sketches of the member are meant to show the member orientation (all elements are W-shapes), which is often missing on computer generated drawings. These sketches are not meant to show work point locations. In a concentrically braced frame, the gravity axes of all members at any one joint (such as Detail A of Figure 2-1) meet at a common point, called the work point (W.P.). Figure 2-2 shows the connection at Detail A of Figure 2-1(a). As with trusses, all joints in concentrically braced frames are assumed to be pinned.

Nonconcentrically braced frames are similar to concentrically braced frames; however, the work point is not located at the common gravity axis point. Figure 2-3 illustrates a nonconcentric bracing arrangement. The work point location results in a couple at the joint. This couple must be considered in the design of the system's connections and main members in order to ensure that the internal force system is admissible. In many cases, this couple will be small and will not affect member size.

Eccentrically braced frames have a different appearance than concentrically braced frames. The braces intersect the beams at points quite distinct from the usual joint working point, as shown in Figure 2-4. Eccentrically braced frames can be used to provide better access through the braced bay for doors, windows or equipment access. They are also used in seismic design because they can provide significant inelastic deformation capacity primarily through shear or flexural yielding in the links.

2.1 ANALYSIS CONSIDERATIONS

Braced frames can be analyzed as simple trusses with all joints pinned. In most cases, secondary forces (also called distortional or rotational forces) due to joint rigidity can be ignored. The AASHTO Bridge Code (AASHTO, 2012) Section 6.14.2.3, for example, states that these forces can be ignored if member lengths are greater than 10 times their cross-sectional dimension in the plane of distortion. In regions of high seismicity, braced frames can be designed as pinned, but corner connections (those involving a column, beam and brace) may have to explicitly include consideration of distortional forces in the design of the connections.

In concentrically braced frames, all members are assumed to be subjected only to axial forces due to lateral loads. This greatly simplifies the structural analysis because, in many cases, the frame will be statically determinate.

In nonconcentrically braced frames, it is common practice to analyze the frame as concentric and then, when the work point location is moved, to superimpose the resulting member moments with the originally calculated axial member forces.

Eccentrically braced frames are usually analyzed with the brace work point displaced from the beam-column work point when the braces do not intersect the beam at a common point (Figure 2-4). The beam-column work point is usually considered to be concentric as shown in Figure 2-3, but if nonconcentric beam-column joints are used as shown in Figure 2-4, the small resulting member moment can be superimposed on the original eccentric analysis results.

2.2 CHEVRON BRACED FRAMES (CENTER TYPE)

Typical chevron bracing configurations are shown in Figure 2-5. These arrangements are used extensively in commercial buildings because the story height is typically half of the bay width. The chevron arrangement will, in this case, provide a brace slope close to a 12 on 12 bevel.

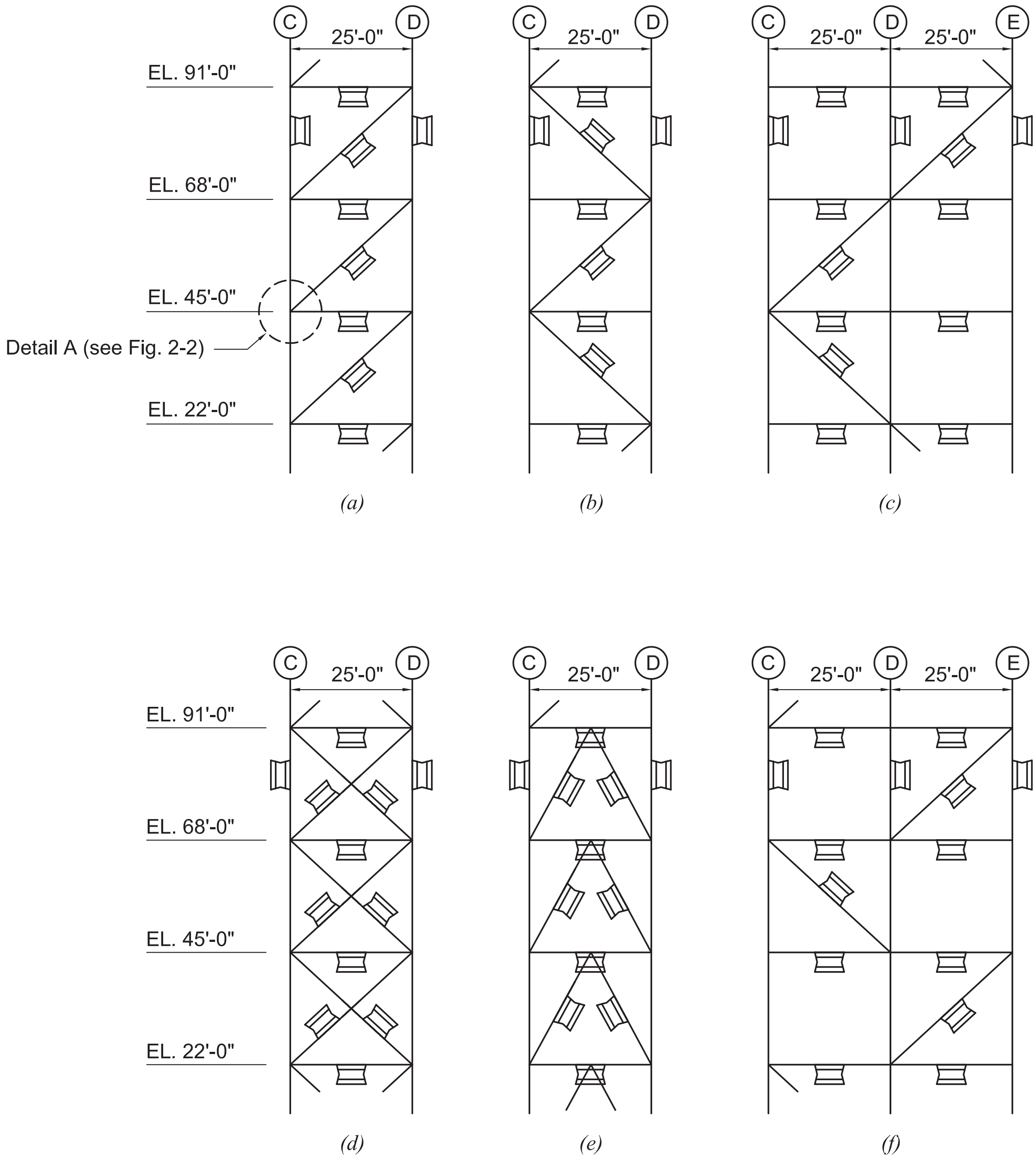


Fig. 2-1. Various vertical bracing arrangements.

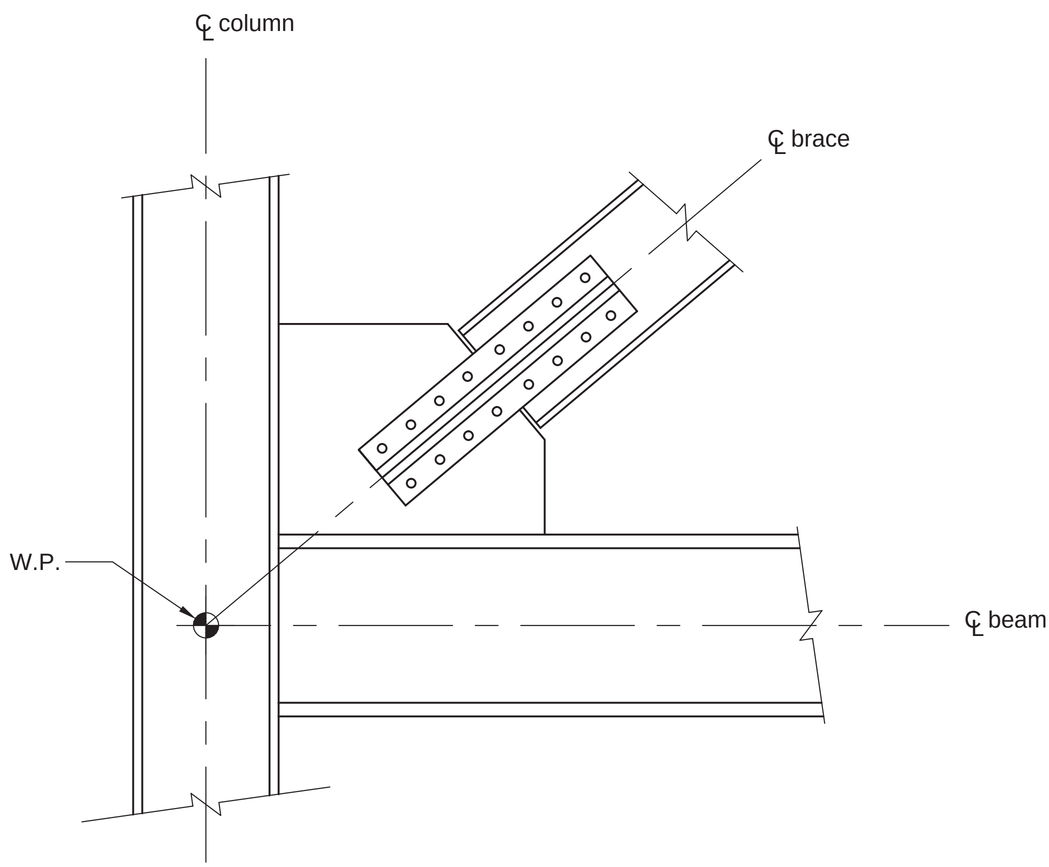


Fig. 2-2. Detail A—typical concentric gusset connection.

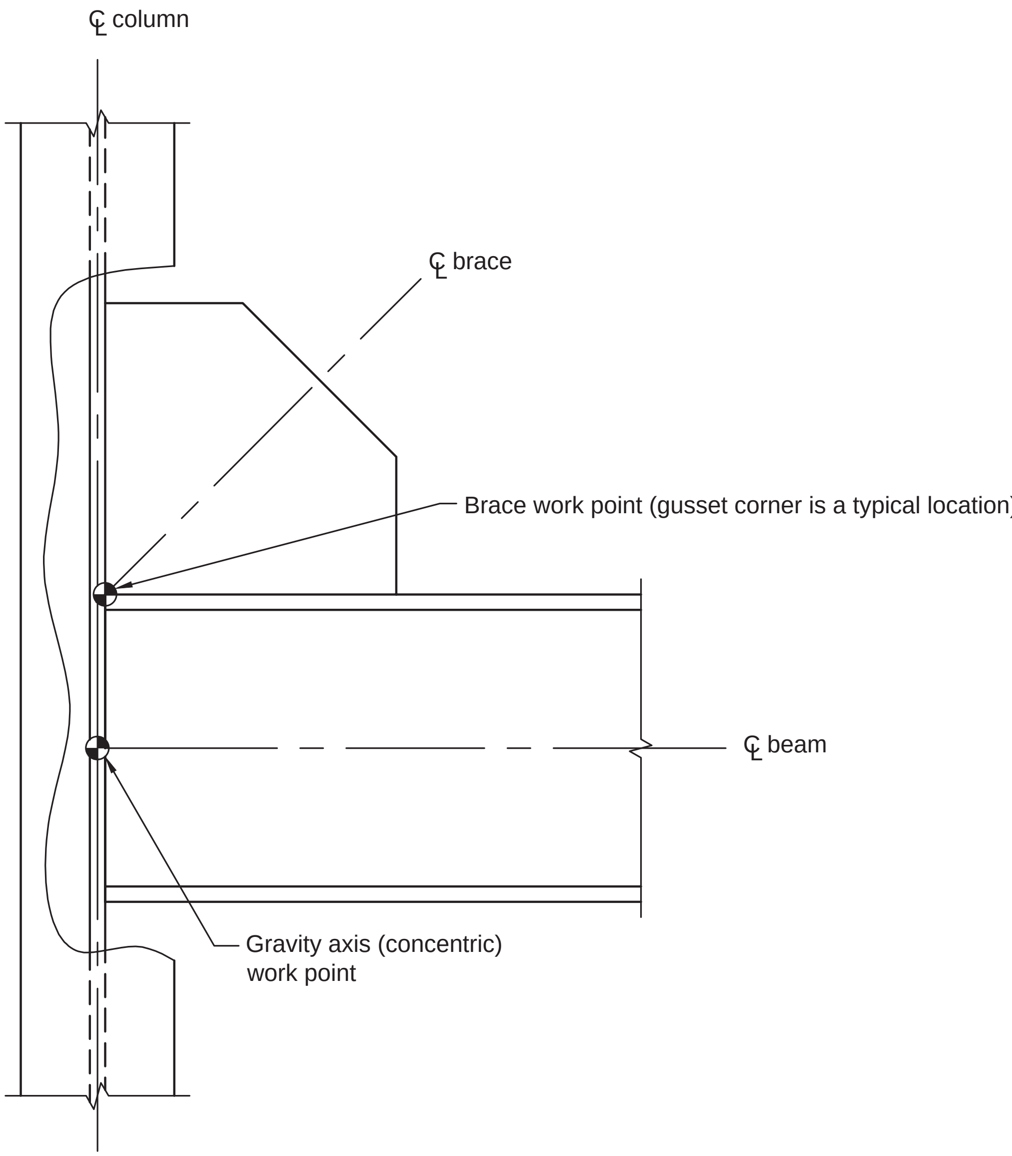


Fig. 2-3. Nonconcentric brace work point.

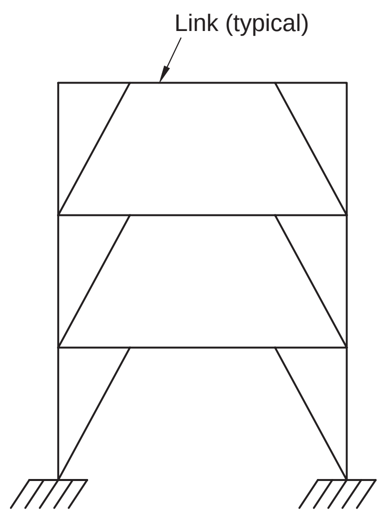


Fig. 2-4. Typical eccentrically braced frame.

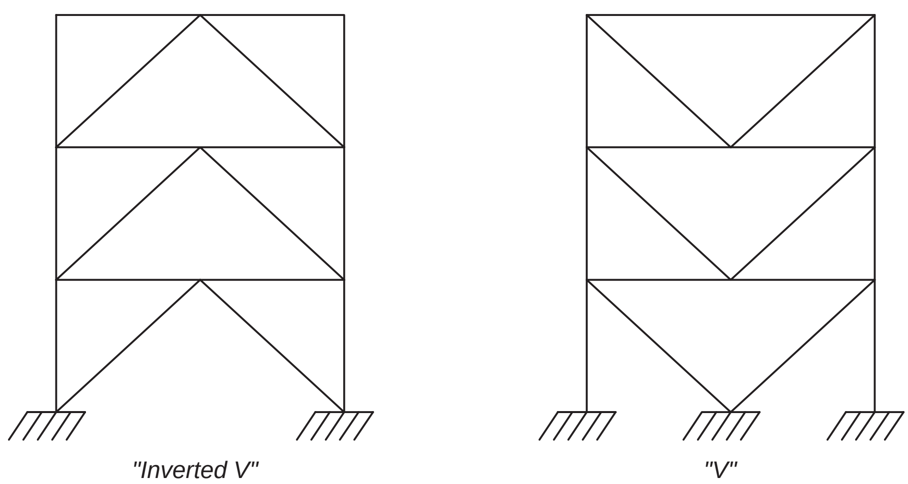


Fig. 2-5. Chevron braced frames.

Chapter 3
Brace-to-Gusset Connection Arrangements

Every bracing connection at the beam/column/brace (corner) consists of at least four connections, and sometimes five, as shown in Figure 3-1. These connections are labeled as follows:

(A) Brace to gusset

(B) Gusset to beam

(C) Gusset to column

(D) Beam to column

(E) Collector, or drag, beam to column

The collector beam-to-column connection occurs when there is a drag or transfer force at the column, as would clearly exist in the bracing arrangement shown in Figures 2-1(c) and 2-1(f), but can also exist in any of the configurations shown in Figure 2-1. Regardless of whether the gusset is a corner or center type, the brace-to-gusset connection is designed in the same way and should constitute the first phase of the connection design. Once the size and location of the brace-to-gusset connection is known, the size of the gusset and the remaining connections can be determined.

3.1 SMALL WIDE-FLANGE BRACES

Figures 3-1 and 3-2 show wide-flange braces, with the web to view in elevation. Small wide-flange braces with this orientation are typically connected to the gussets by WTs or double angles back-to-back on the near and far side of the gusset. Alternatively, single angles on each side of the brace could be employed. If the brace is subjected to compression as well as tension, plates should not be used in place of the WTs or angles. Figure 3-3 shows a wide-flange brace, flange to view in elevation. This brace is connected to the gusset with four angles, two on the near side and two on the far side of the gusset.

3.2 LARGE WIDE-FLANGE BRACES

When larger wide-flange sections are used as braces with the web to view in elevation, they can be attached to the gusset at both the flanges and at the web as shown in Figure 3-4. Plates can be used to attach the web, and “claw” angles can be used to attach the flanges. The outstanding angle legs provide for stability. Since about 80% of the force will be transferred through the flanges when a W14 column section is used, the web plates can often be eliminated. This is especially true for tension/compression braces, because the buckling strength of the brace will usually be less than 80% of the tensile strength. A very compact flange connection can be made to wide-flange braces, web to view in elevation, as shown in Figure 3-5. This arrangement resembles a truss tension-chord splice.

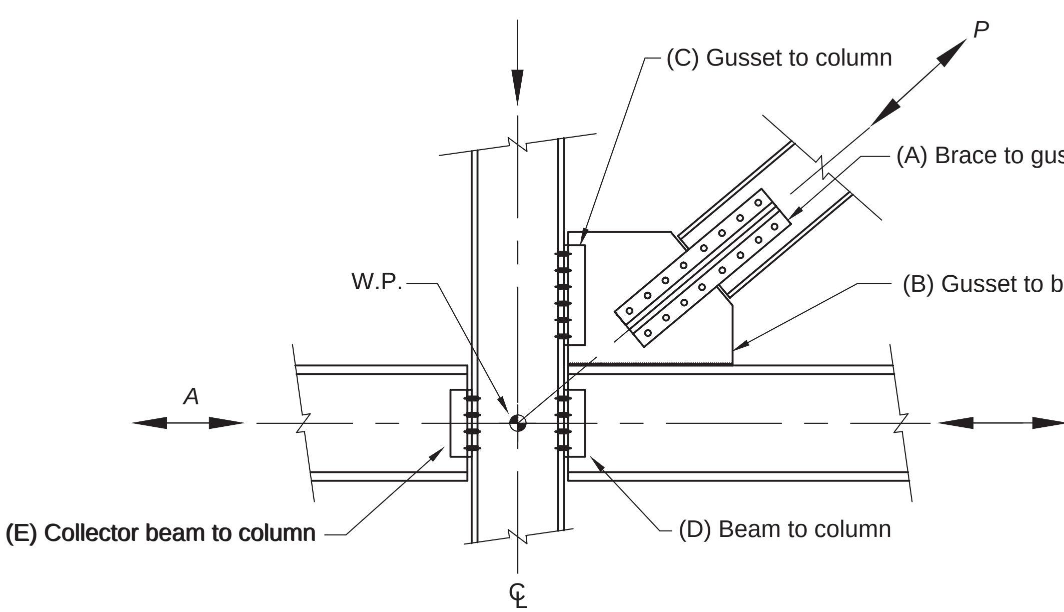


Fig. 3-1. Concentric (corner gusset) bracing connection.

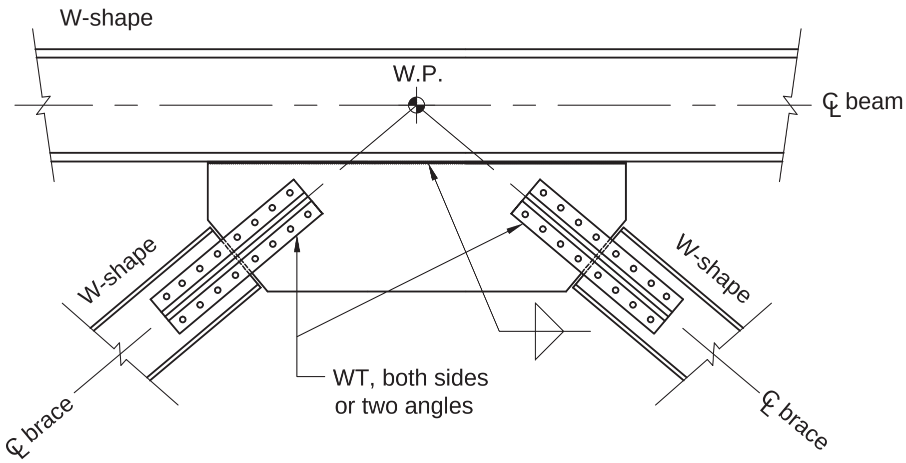


Fig. 3-2. Typical chevron brace connection configuration (center gusset).

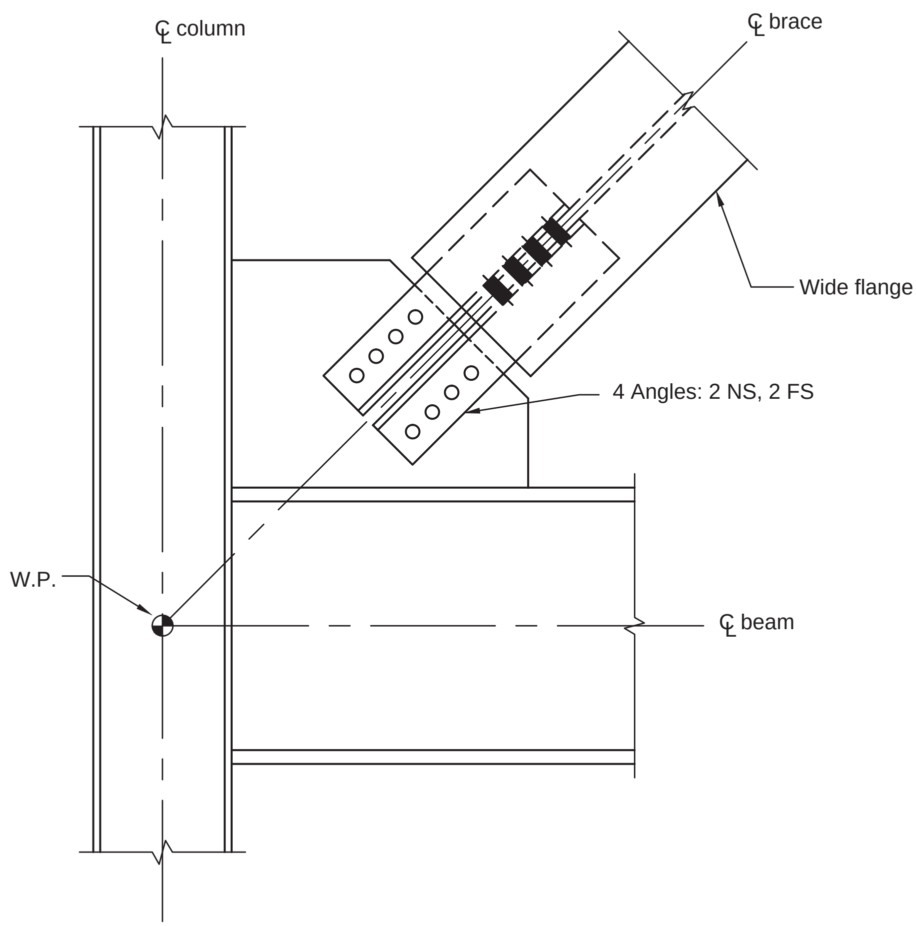


Fig. 3-3. Wide-flange brace (flange to view) with four angles connecting to gusset.

When wide-flange braces are flange to view in elevation, the flanges can be very effectively connected to the gusset as shown in Figure 3-6. The plates that connect to the gusset are "bird mouthed" and welded to the gusset.

3.3 ANGLE AND WT-BRACES

Figure 3-7 shows a double-angle brace. The angles will be stitched together as shown. This is an effective brace for tension and compression, but is often not desirable for use in external locations or in industrial applications because it is difficult to maintain (i.e., keep painted). The double angles should be galvanized if this configuration is used in external applications.

Single-angle braces and WT-braces (Figure 3-8) are excellent for maintenance purposes and are probably the most economical bracing members for tension-only bracing. Because of member eccentricity, they are not desirable for compression bracing.

3.4 CHANNEL BRACES

Channel braces can be either back-to-back with stitches (Figure 3-9) or toe-to-toe with stitches (Figure 3-10). The back-to-back arrangement has little out-of-plane buckling strength. The toe-to-toe arrangement is similar to an HSS brace. These types of bracing arrangements are rarely used and are generally expensive compared to other bracing options.

3.5 FLAT BAR BRACES

Flat bars can be used as tension-only braces. As shown in Figure 3-11, the bars can be lapped onto the gusset and field bolted. When an available flat bar size can be used, there is very little fabrication cost involved, but these are very slender for erection and may be subject to objectionable vibration due to building movement and equipment frequency input.

3.6 HSS BRACES

An HSS brace connection is shown in Figure 3-12. HSS sections are very commonly used as braces. As shown in Figure 3-12, the HSS is slotted and field welded to the gusset. The slot must be made long enough to allow for erection. HSS sections are more expensive than wide-flange sections, but they are a more efficient section for buckling, and may be a lighter alternative to a wide-flange section of equivalent strength. They are extensively used for seismic design because the tensile and compressive strengths of the members are closer than they are for wide-flange sections.

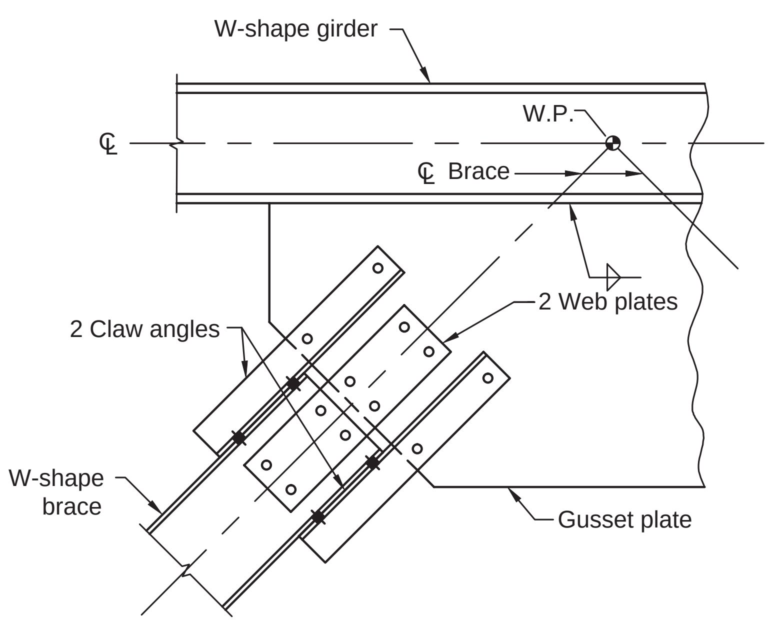


Fig. 3-4. Wide-flange brace (web to view) with "claw" angles and web plate.

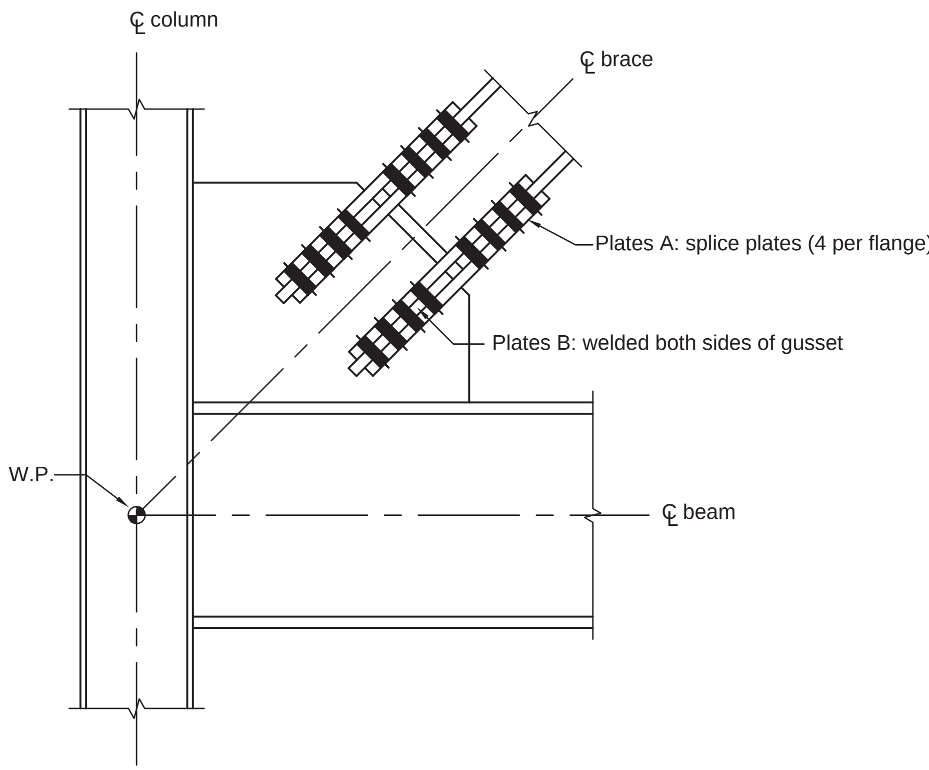


Fig. 3-5. Compact splice-type connection.

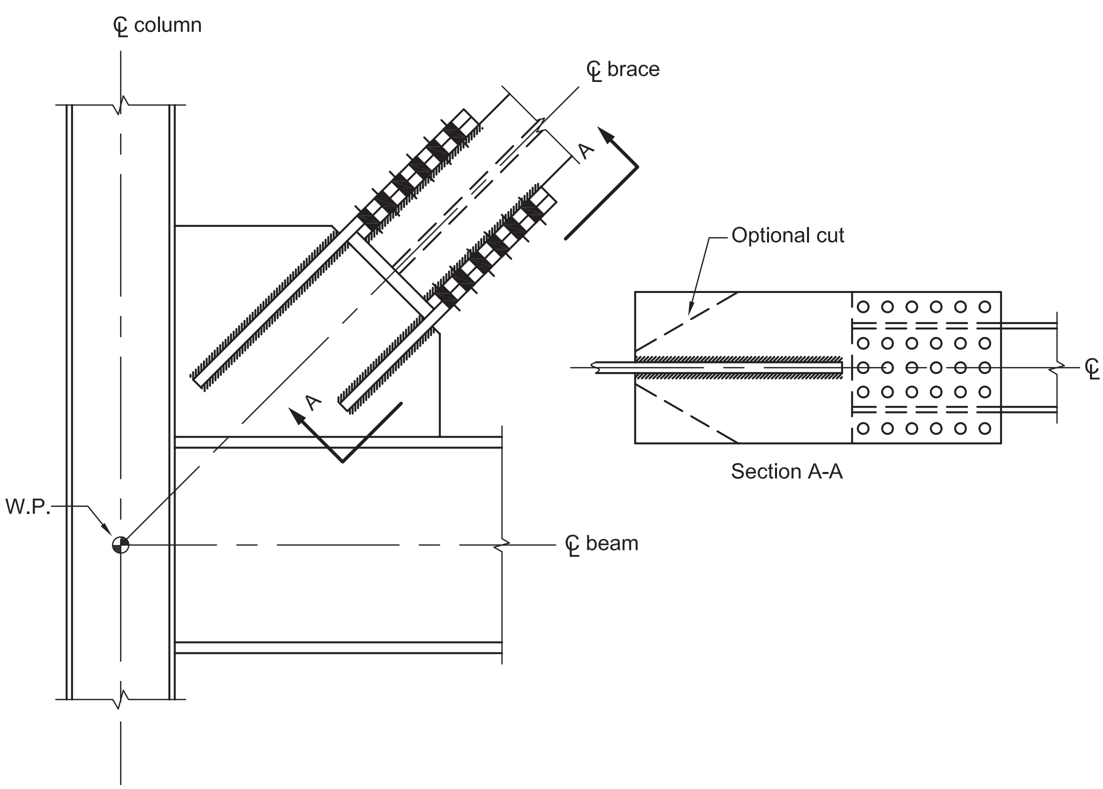


Fig. 3-6. Wide-flange brace connection (flange to view).

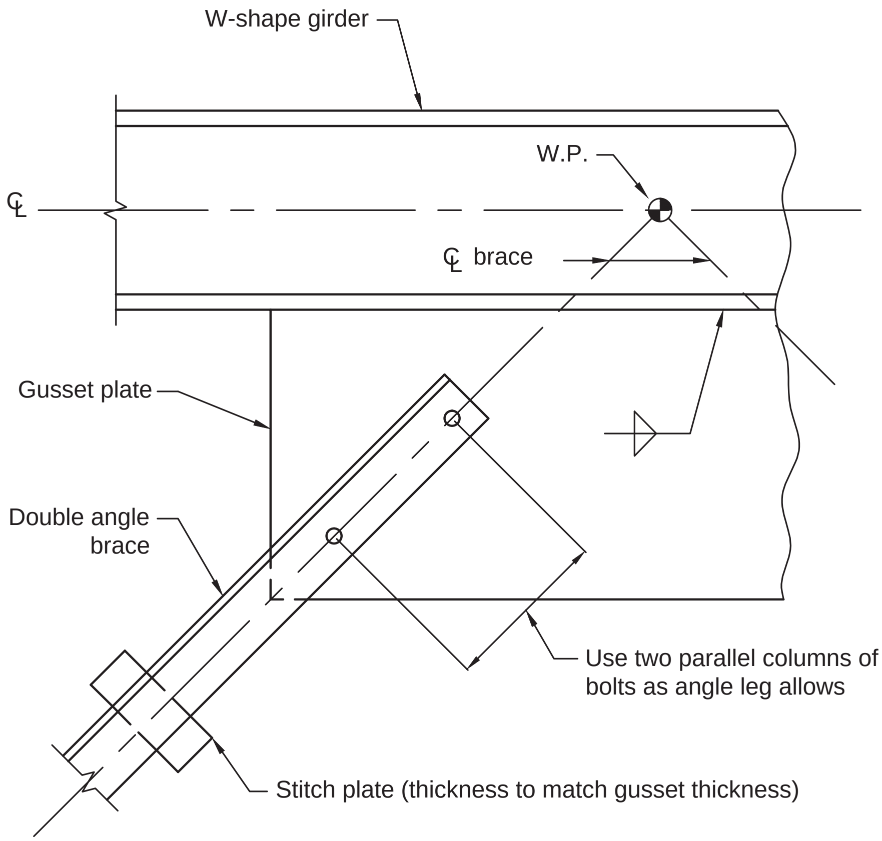


Fig. 3-7. Double-angle brace connection.

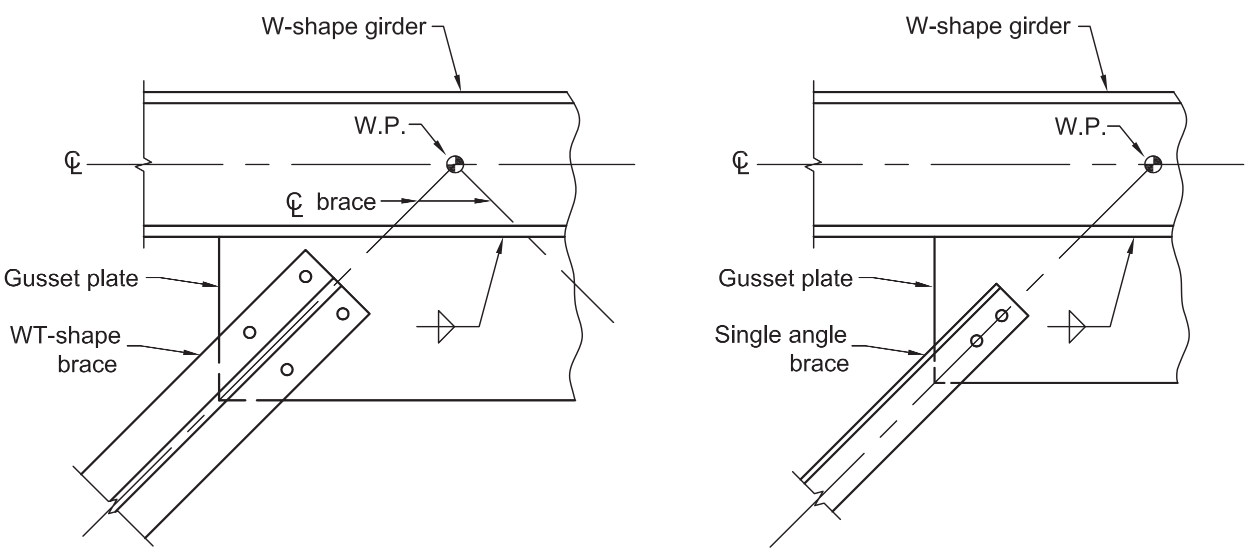


Fig. 3-8. WT- and single-angle brace.

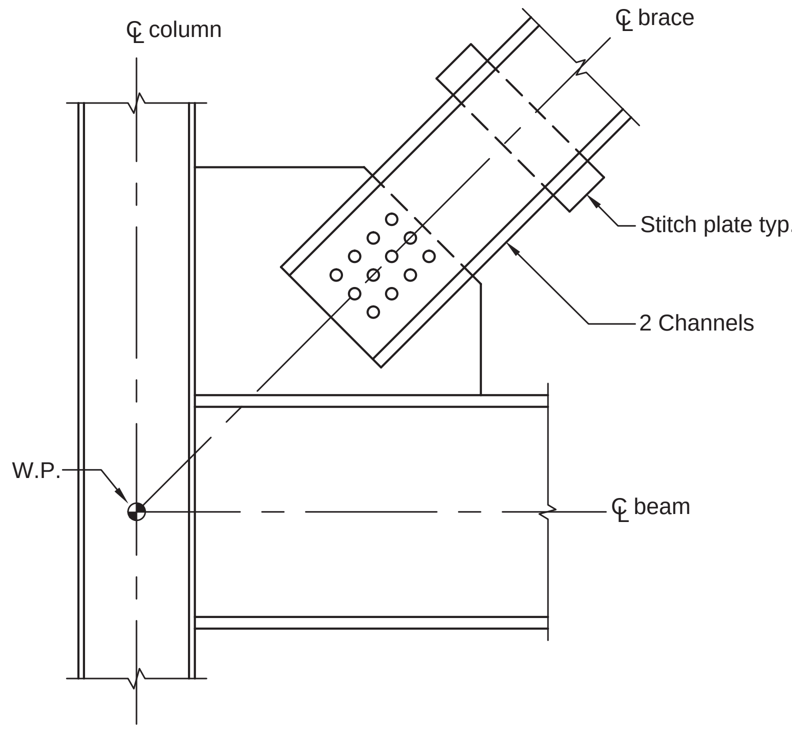


Fig. 3-9. Channel bracing back-to-back.

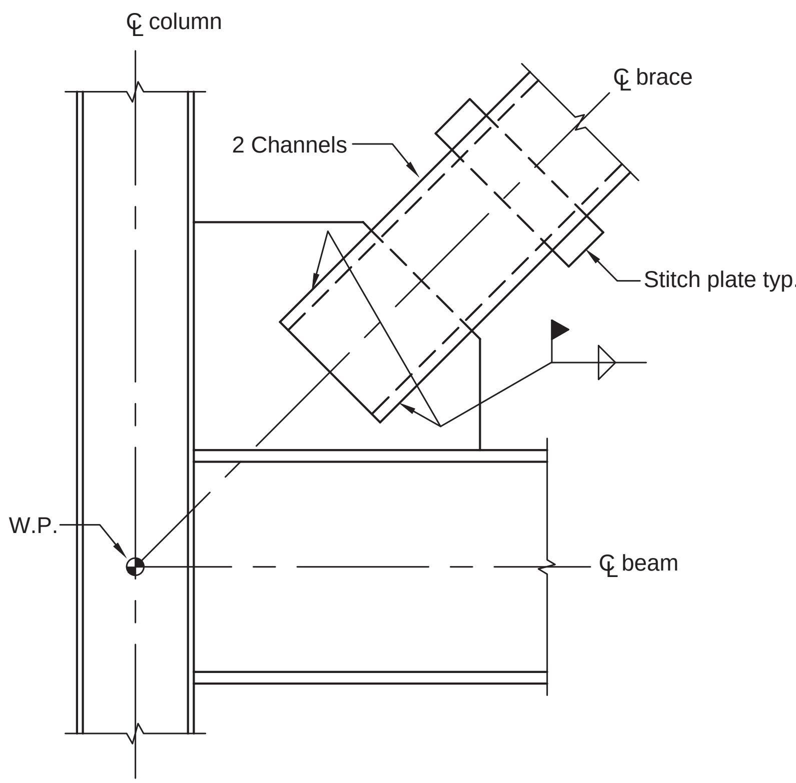


Fig. 3-10. Channel bracing toe-to-toe.

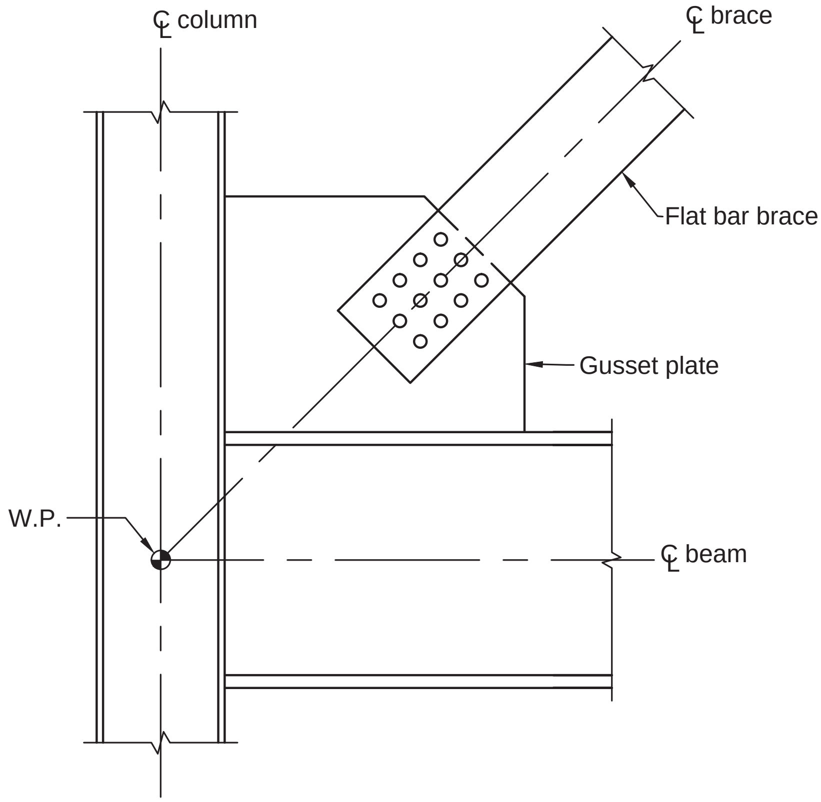
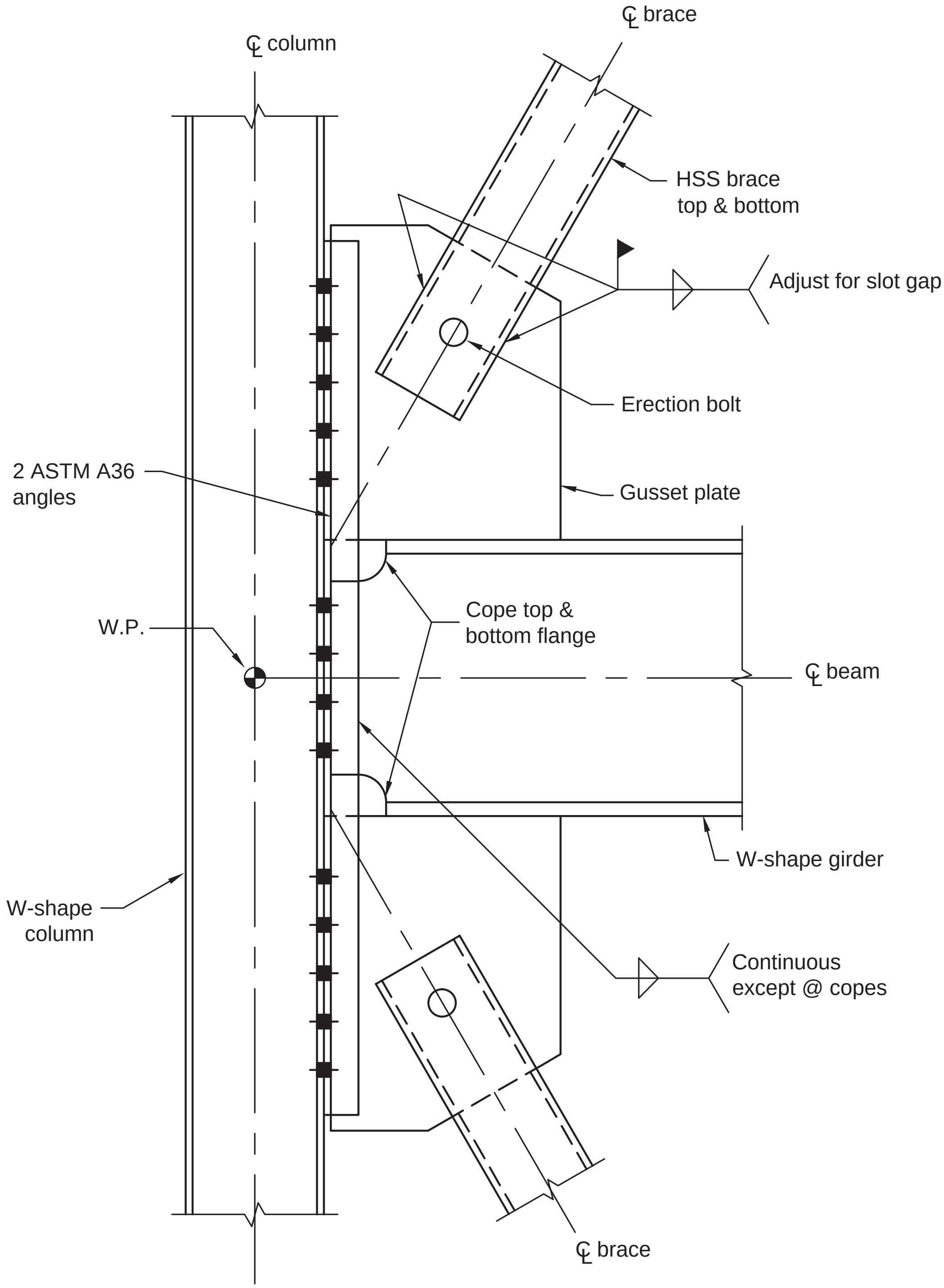


Fig. 3-11. Flat bar brace, tension only.



Note:
Beam copes can be eliminated by using discontinuous connecting angles to column.
Disparity between web thickness and gusset thickness may require fills.

Fig. 3-12. HSS bracing connection.

Chapter 4
Distribution of Forces

Generally, two types of bracing connections are considered in this Design Guide—corner connections and central (or chevron) connections. Of the two types, corner connections are by far the more interesting (i.e., complex) because the distribution of internal forces within the connection cannot be determined by statics alone. Corner connections are statically indeterminate, and there has long been controversy about the correct force distribution that should be assumed for design. Central connections, on the other hand, are statically determinate, making the force distribution in them unique.

The discussion earlier in this Design Guide on the lower bound theorem (LBT) is directed primarily to corner connections. There is no unique solution to the distribution of forces within these connections, but any solution that involves an admissible internal force distribution and satisfies all of the limit states of the configuration (with appropriate steps taken to account for those of limited ductility) is an acceptable solution, in accordance with the LBT.

All structural analysis is based on three fundamental sets of equations:

1. Equilibrium
2. Constitutive (including limit states)
3. Compatibility (deformation and displacement)

The LBT satisfies two of these sets of equations (1 and 2). The upper bound theorem (UBT) also satisfies two of these sets of equations (1 and 3) (Baker et al., 1956). The true solution satisfies all three sets of equations.

The LBT states that among all possible admissible force fields, the one that is closest to the true solution is the one that produces the maximum capacity. The UBT states that among all possible admissible force fields, the one that gives the least capacity is closest to the true solution. The LBT converges to the true solution from below, and the UBT converges to the true solution from above.

It is reasonable to use the fact that, based on the LBT, the method chosen to design a corner connection will be best (closest to the true solution) if it produces the maximum capacity for a given connection. This will be the basis for the following review of methods presented in this Design Guide.

Based on the preceding discussion it is also reasonable to suppose that, among the infinite number of possible admissible force fields, the one providing the greatest lower bound (of the capacity) by the LBT also comes closer than any other to satisfying the compatibility equations. This is another reason to use the LBT as the basis for method choice.

4.1 OVERVIEW OF COMMON METHODS

There are four methods in published literature that are in common use for corner connections. There are many other methods in use by various designers that have not been published, but may be satisfactory. However, only the four common published methods will be considered in this Design Guide. Because they are statically determinate, central connections allow only one of these methods, although some variations are possible, as will be shown.

4.1.1 Corner connections

The four published methods for the design of corner connections are:

1. The KISS (keep it simple, stupid) method (Thornton, 1991; Astaneh-Asl, 1998)
2. The parallel force method (Ricker, 1989; Thornton, 1991; Astaneh-Asl, 1998)
3. The truss analogy method (Astaneh-Asl, 1989)
4. The uniform force method (Thornton, 1991, 1995; Astaneh-Asl, 1998)

KISS Method

The KISS method (Figure 4-1) was formalized by Thornton (1991) but had been used for many years prior and is still in use. The method is indeed simple: the horizontal component of the brace force is assumed to be carried through the gusset-to-beam connection, and the vertical component of the brace force is assumed to be carried through the gusset-to-column connection. It is worth noting that the gusset edge forces (the gusset-beam and gusset-column forces) are not a function of the gusset size. However, Astaneh-Asl (1998) noted that when using this method, many users omit the gusset edge couples. While this may be common practice, unless these couples are included, neither the gusset nor the beam and column will be in equilibrium. Thus, this practice will invalidate the method, because it does not provide an admissible internal force field and the LBT is not applicable. Thornton (1991) has shown that the KISS method does not provide a

greatest lower bound solution, and thus, is less efficient than other available design methods. Therefore, the KISS method is acceptable in its full form (with couples considered in the design) but does not provide the most economical designs.

Parallel Force Method

The parallel force method (PFM) (Figure 4-2a and Figure 4-2b), was first proposed by Ricker (1989) and was published by Thornton (1991). In the PFM, eccentricities are calculated from the brace centerline to the centroids of the gusset plate edges along the beam and column faces. The gusset-to-beam connection is designed for the force P_b, and the gusset-to-column connection is designed for the force P_c. This method results in efficient designs when the gusset is connected to a column flange, but when the gusset is connected to a column web, a large force directed normal to the center of the web will require excessive stiffening, and if the stiffening is not provided the design fails. Therefore, the method is not practical for all design applications. Figure 4-2 illustrates that this method requires a moment, M, to be

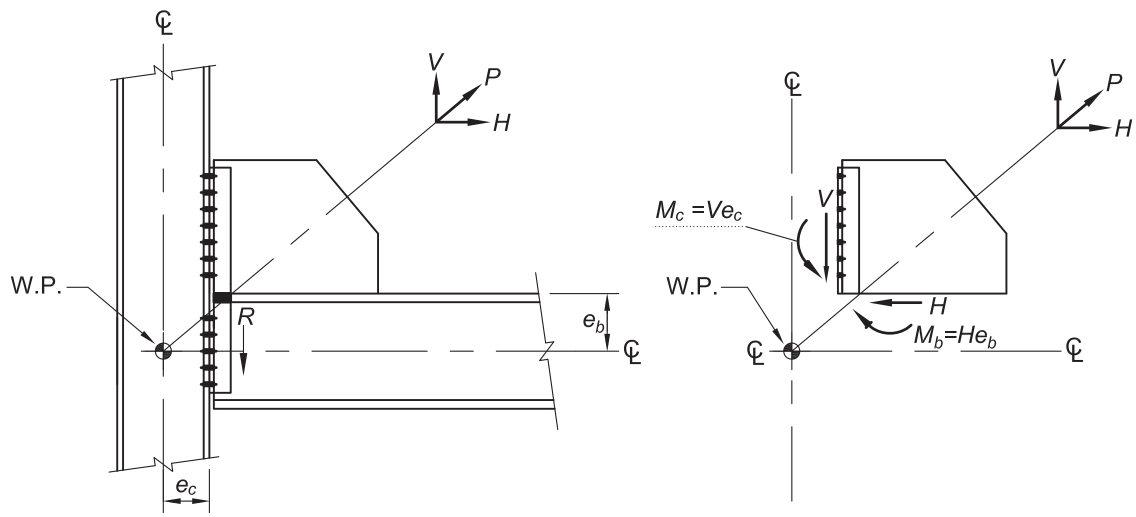


Fig. 4-1. KISS method admissible force field.

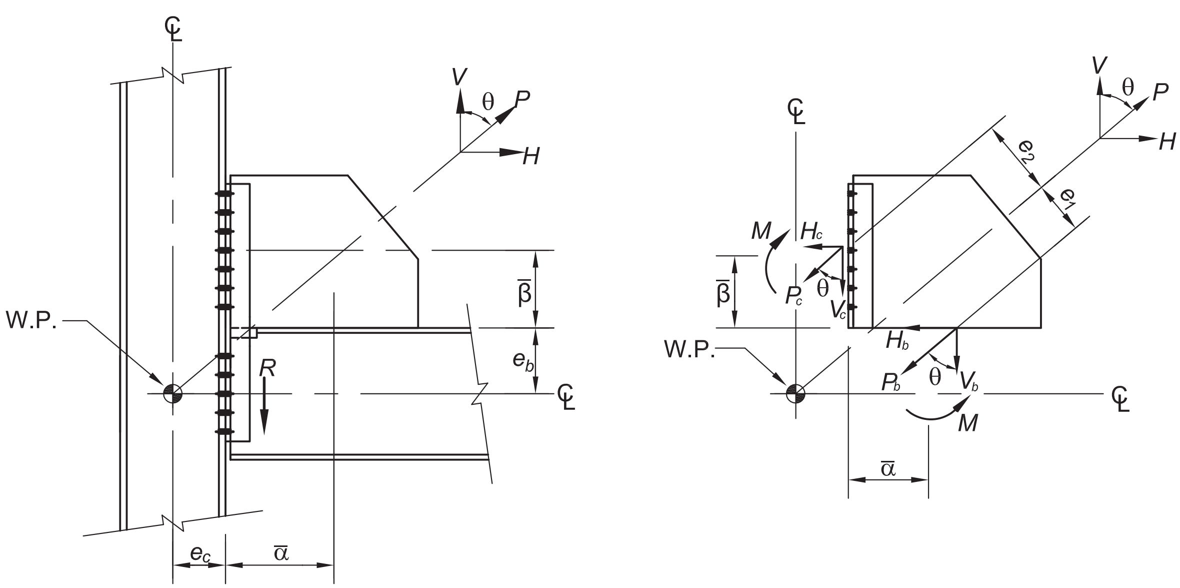


Fig. 4-2a. Parallel force method admissible force field.

applied to either the gusset edges, or to the beam-to-column connection. This couple is not required for gusset equilibrium, but it is required for the equilibrium of the beam and column. It can be shown that when $M = 0$, the PFM reduces to the uniform force method (UFM).

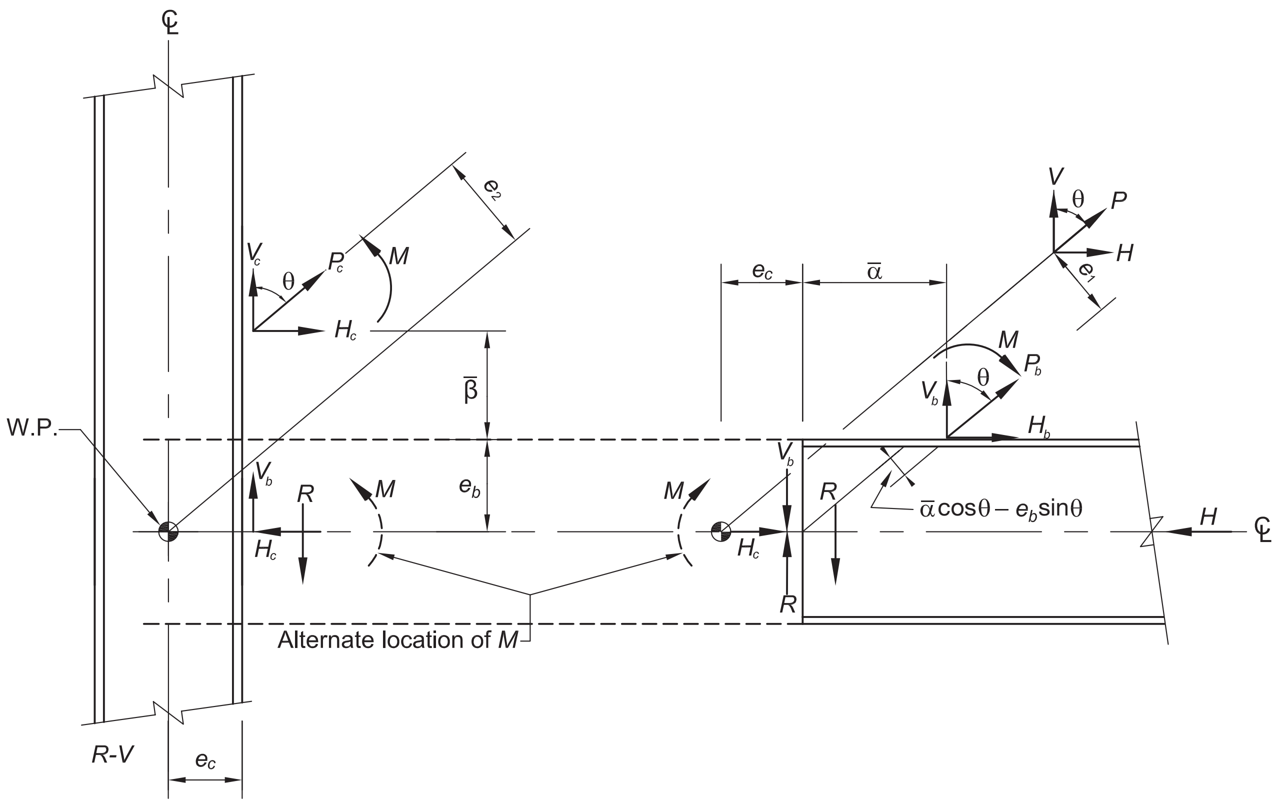
Truss Analogy Method

The truss analogy method (Figure 4-3) was proposed by Astaneh-Asl (1989) as a simplified method to determine gusset edge forces. Unfortunately, this method suffers from limitations similar to the parallel force method. When connecting the gusset to a column web, the large force normal to the web must be accommodated by a series of stiffeners, which would not be required by the KISS method or the uniform force method.

Another problem with this method is that it delivers most of the brace horizontal component to the column, and most of the brace vertical component to the beam. This load path is contrary to what is typically expected of this type of connection and normally requires the beam-to-column connection to be similar to a full strength beam-to-column moment and shear connection.

Uniform Force Method

The uniform force method (Figure 4-4a and Figure 4-4b) was developed by Thornton (1991) based on research performed



$$e_1 = (e_c + \bar{\alpha})\cos\theta - e_b \sin\theta$$

$$e_2 = (e_b + \bar{\beta})\sin\theta - e_b \cos\theta$$

$$P_b = \frac{e_2}{e_1 + e_2} P \qquad P_c = \frac{e_1}{e_1 + e_2}$$

$$M = \frac{e_2}{e_1 + e_2} P(\bar{\alpha}\cos\theta - e_b \sin\theta)$$

Fig. 4-2b. Parallel force method admissible force field.

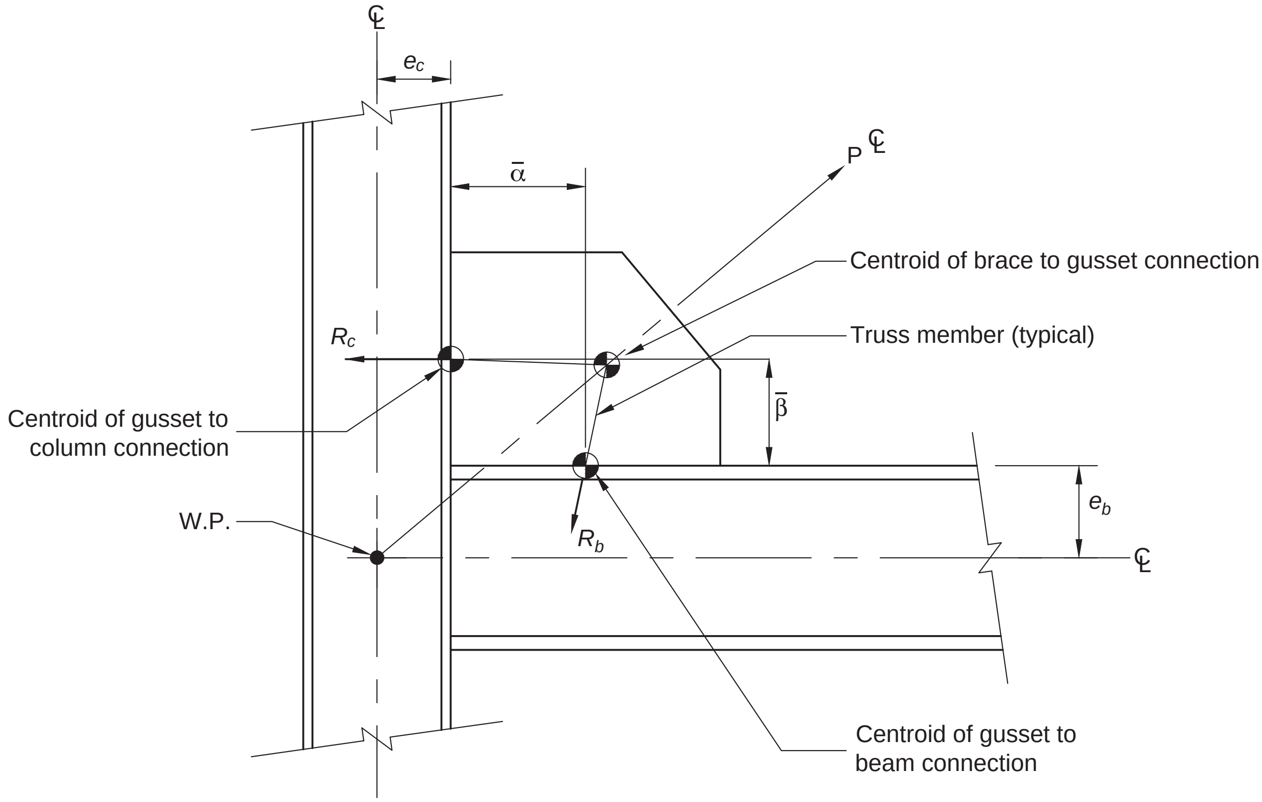


Fig. 4-3. Truss analogy method (Astaneh-Asl, 1989).

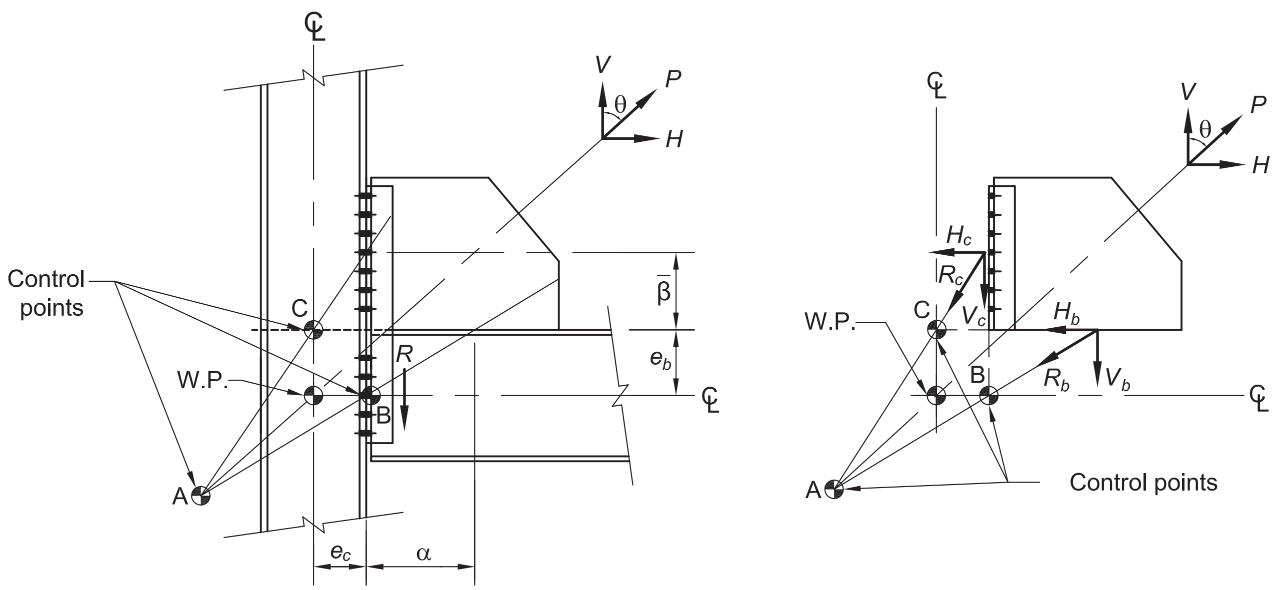


Fig. 4-4a. Uniform force method admissible force field at gusset plate interface.

by Richard (1986). The method is universally applicable and has been shown to produce safe and economical designs by accurately predicting the failure loads of six full-scale physical tests (Thornton, 1991, 1995). Since this method first appeared in the AISC *Manual of Steel Construction* (AISC, 1992), it has been widely used in the industry.

Appendix A gives a derivation and generalization of the uniform force method (UFM). The derivation involves the three control points shown in Figure 4-4a. These are labeled A, B and C. The gusset-to-beam resultant force, R_b, and the gusset-to-column resultant force, R_c, are forced to pass through control points B and C respectively, and they intersect the brace line of action at control point A. The constraint, shown in Figure 4-4b, namely $\alpha - \beta \tan\theta = e_b \tan\theta - e_c$ causes the coincident set of connection forces P, R_b and R_c to exist (i.e., no couples on the connection interfaces). These control points will be explicitly used in the examples.

When all of these four methods are applied to the same completely designed connection, the uniform force method will always produce a design capacity greater than or equal to that produced by any of the other three, thus in keeping with the LBT.

4.1.2 Central or Chevron Connections

Chevron connections are illustrated in Figures 4-5, 4-6 and 4-7 (Thornton, 1987). The force distribution shown in the figures is essentially the only one possible, but some practitioners choose to assume that there is a cut at Section b-b and analyze the connection as two separate connections. This is an acceptable approach, but changes the assumed behavior

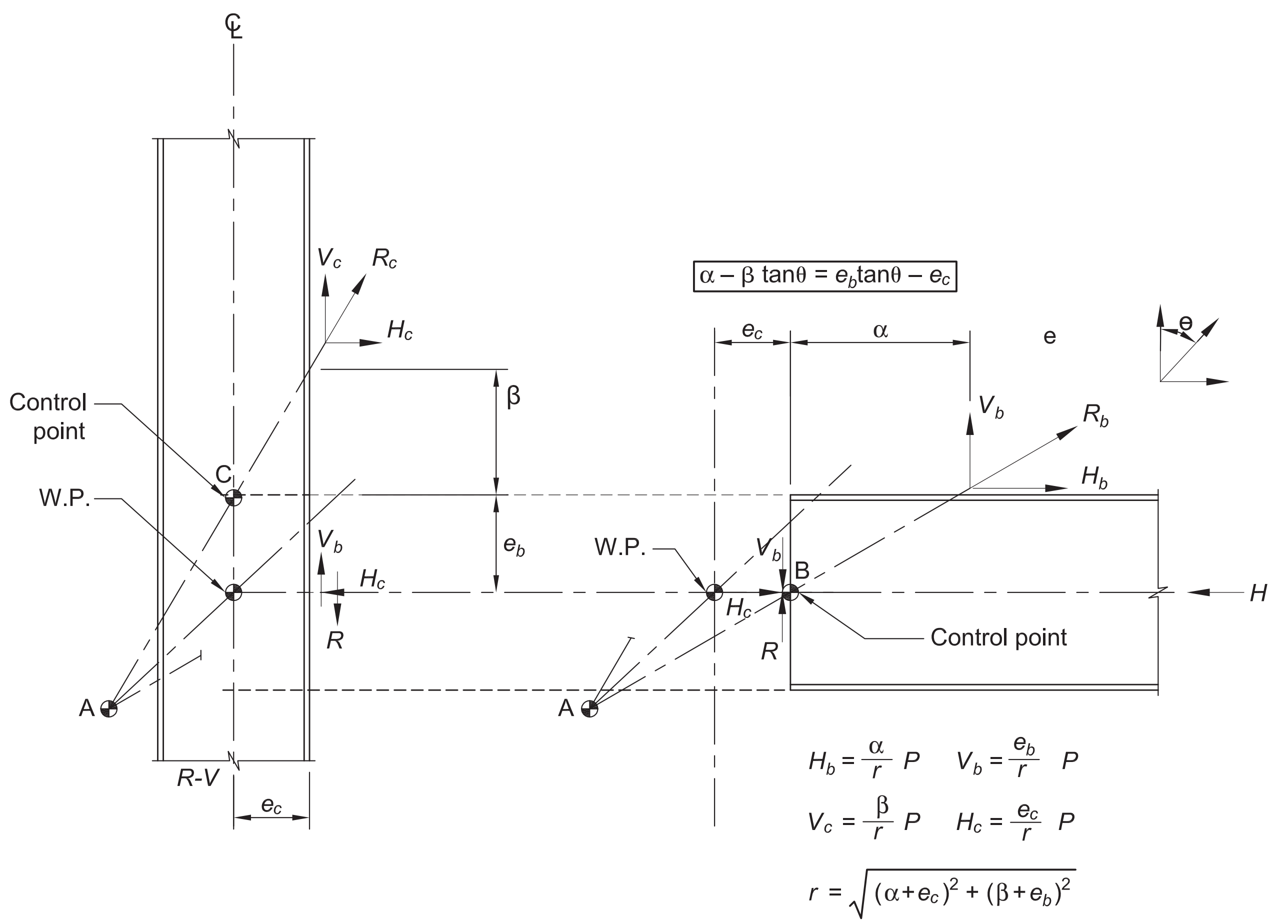


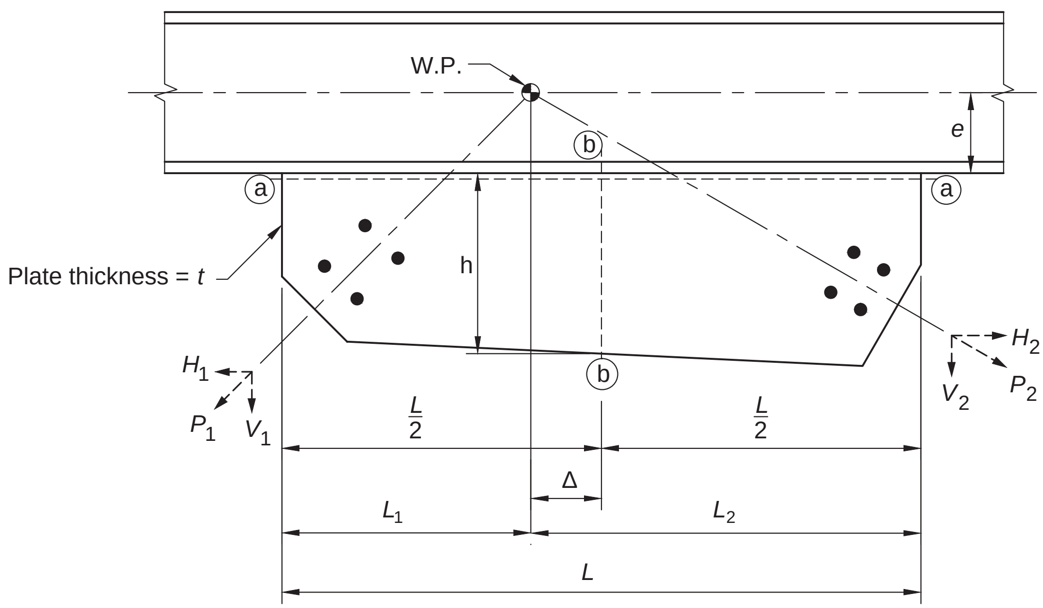
Fig. 4-4b. Uniform force method admissible force field at beam-to-column interface.

of the connection. These changes will mean that the beam must take all of the shear due to the vertical components of the braces through its web, and will more frequently require web doubler plates. Also, since the gusset is cut at Section b-b, the tension brace cannot be used to stabilize the plate at the compression brace. This Design Guide will not use the vertical cut method.

The method developed in Figures 4-5, 4-6 and 4-7 is based on Section a-a, the gusset-to-beam interface. This is the "control" interface, and all the resulting interface force distributions are determined by equilibrium. It is possible to use Section b-b, extended to include the beam web, as the control interface. This is not a very common approach and will not be pursued here.

4.1.3 Comparison of Designs—Uniform Force Method vs. Parallel Force Method

Figure 4-8 shows a typical corner bracing connection to a column web. The arrangement is similar to that of Figure 2-1(a), except the beam and gusset are connected to the column web. There is no transfer force. Figure 4-9 shows the UFM admissible force field, and Figure 4-10 shows the UFM design. The PFM admissible force field is shown in Figure 4-11 and the resulting design is shown in Figure 4-12. It can be seen by a comparison of Figures 4-10 and 4-12 that the PFM gives a very expensive and cumbersome design compared to the UFM. It is not practical or desirable to use the PFM when the connection is to a column web. A similar



Sign convention

P_1, P_2 + for tension, - for compression

If P_1 is +, V_1 and H_1 are + also

If P_1 is -, V_1 and H_1 are - also

Same for P_2 (V_2, H_2)

$$\Delta = \frac{1}{2}(L_2 - L_1) \quad \text{Note } \Delta \text{ is negative if } L_2 < L_1$$

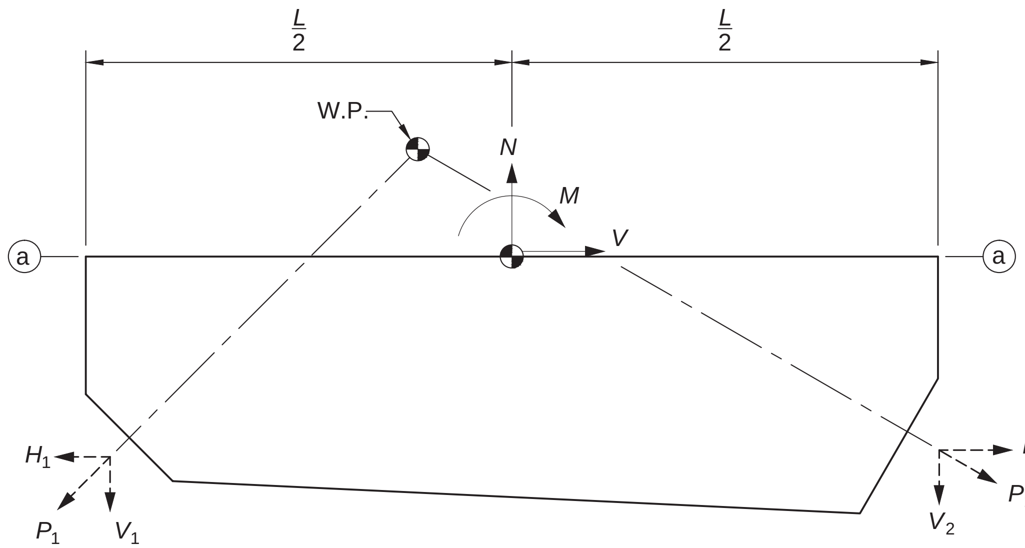
$$M_1 = H_1 e + V_1 \Delta$$

$$M_2 = H_2 e - V_2 \Delta$$

$$M_1' = \frac{1}{8} V_1 L - \frac{1}{4} H_1 h - \frac{1}{2} M_1$$

$$M_2' = \frac{1}{8} V_2 L - \frac{1}{4} H_2 h - \frac{1}{2} M_2$$

Fig. 4-5. Chevron brace gusset forces.



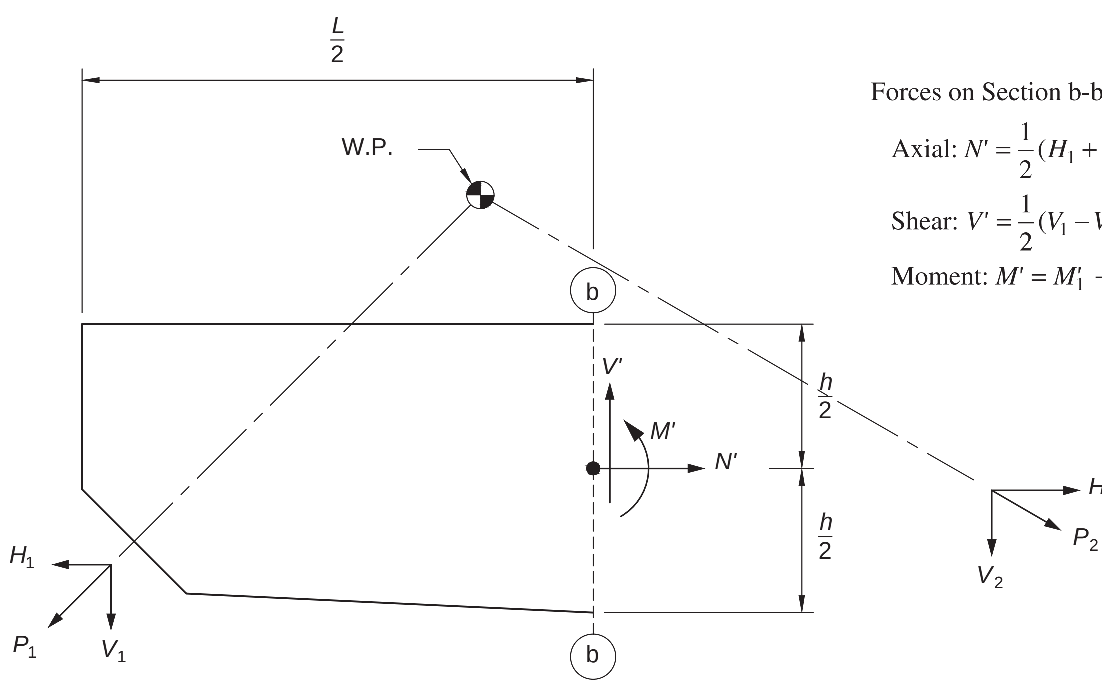
Forces on Section a-a:

Axial: $N = V_1 + V_2$

Shear: $V = H_1 - H_2$

Moment: $M = M_1 - M_2$

Fig. 4-6. Forces on Section a-a (positive directions shown).



Forces on Section b-b:

Axial: $N' = \dfrac{1}{2}(H_1 + H_2)$

Shear: $V' = \dfrac{1}{2}(V_1 - V_2) - \dfrac{2}{L}(M)$

Moment: $M' = M_1' + M_2'$

Fig. 4-7. Forces on Section b-b (positive directions shown).

conclusion would be reached if the problem of Figure 4-8 were designed using the truss analogy method.

4.2 THE UNIFORM FORCE METHOD

The essence of the UFM is the selection of a connection geometry that will not produce moments on the connection interfaces (gusset-to-beam, gusset-to-column, and beam-to-column). In the absence of moment, these connections are designed for shear and normal forces only, hence the origin of the name UFM.

This Design Guide will use the UFM exclusively for examples on the design of corner connections. Of the four methods discussed above, only the UFM and the KISS method are universally applicable. The other two methods will be used in the design examples only to illustrate the designs achieved by them for comparison with UFM designs for the same loads and geometry. The KISS method, while universally applicable, does not generally result in the most economical design, and is not recommended for general use.

4.2.1 The Uniform Force Method—General Case

The UFM formulation shown in Figures 4-4a and 4-4b is the general case. This formulation achieves an admissible internal force field that provides a coincident force field on the gusset, the beam and the column.

The dimensions α and β in Figures 4-4a and 4-4b describe the ideal locations of the centroids of the gusset-to-beam and gusset-to-column connections, respectively. These dimensions must satisfy the constraint of Equation 4-1:

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \tag{4-1}$$

where

e_b = one half of the depth of the beam, in.

e_c = one half of the depth of the column, in. Note that for a column web support, $e_c \approx 0$.

α = distance from the face of the column flange or web to the ideal centroid of the gusset-to-beam connection, in.

β = distance from the face of the beam flange to the ideal centroid of the gusset-to-column connection, in.

θ = angle between the brace axis and vertical

This constraint can always be satisfied for new connections, but in checking existing structures (or for convenience) the connection centroids may not satisfy Equation 4-1. In general, this Design Guide will use the terms $\bar{\alpha}$ and $\bar{\beta}$ to describe the actual locations of the connection centroids. The dimensions α and $\bar{\alpha}$, and β and $\bar{\beta}$ may be the same, but often are different. When the ideal and actual connection centroids are not the same, couples will exist on the gusset edges.

Appendix A contains a generalization of the UFM that dispenses with the constraint of Equation 4-1, then there is no need to distinguish between α and $\bar{\alpha}$ or β and $\bar{\beta}$. However, the examples in this Design Guide, except for the examples

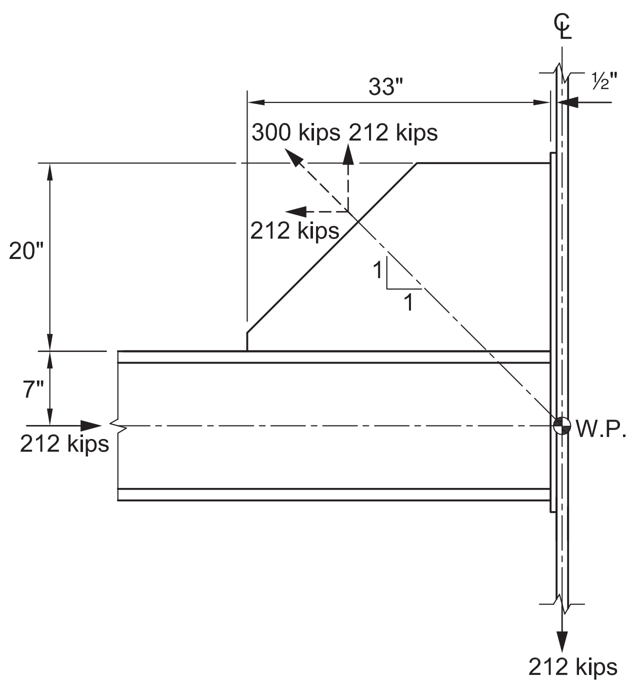


Fig. 4-8. Bracing connection to demonstrate the application of the parallel force method to a column web.

given in Appendix A, will only illustrate the application of the traditional UFM.

There are several ways to approach the design:

1. Lay out the connection geometry to satisfy Equation 4-1. This is the true UFM and will not induce any moment in the connection edges.

2. If the ideal geometry cannot be accommodated, decide which of the two gusset edge connections is stiffer and assign the moment to the stiffer of the two connections. If the gusset-to-column connection is the more flexible of the two, using Equation 4-1, set $\beta = \bar{\beta}$ and calculate α. If $\alpha \neq \bar{\alpha}$, there is a couple on the gusset-to-beam connection equal to:

$$M_b = V_b\left(\alpha - \bar{\alpha}\right) \tag{4-2}$$

If the brace force, P, is tension, this couple is clockwise on the gusset when $\alpha > \bar{\alpha}$ and counterclockwise when $\alpha < \bar{\alpha}$. If the gusset-to-beam connection is the more flexible of the two, using Equation 4-1, set $\alpha = \bar{\alpha}$ and calculate β. If $\beta \neq \bar{\beta}$ there is a couple on the gusset-to-column connection equal to:

$$M_c = H_c\left(\beta - \bar{\beta}\right) \tag{4-3}$$

If the brace force, P, is tension, this couple is counterclockwise on the gusset when $\beta > \bar{\beta}$ and clockwise when $\beta < \bar{\beta}$. The direction of the couple assumes that the gusset is in the first quadrant, where the work point is the origin.

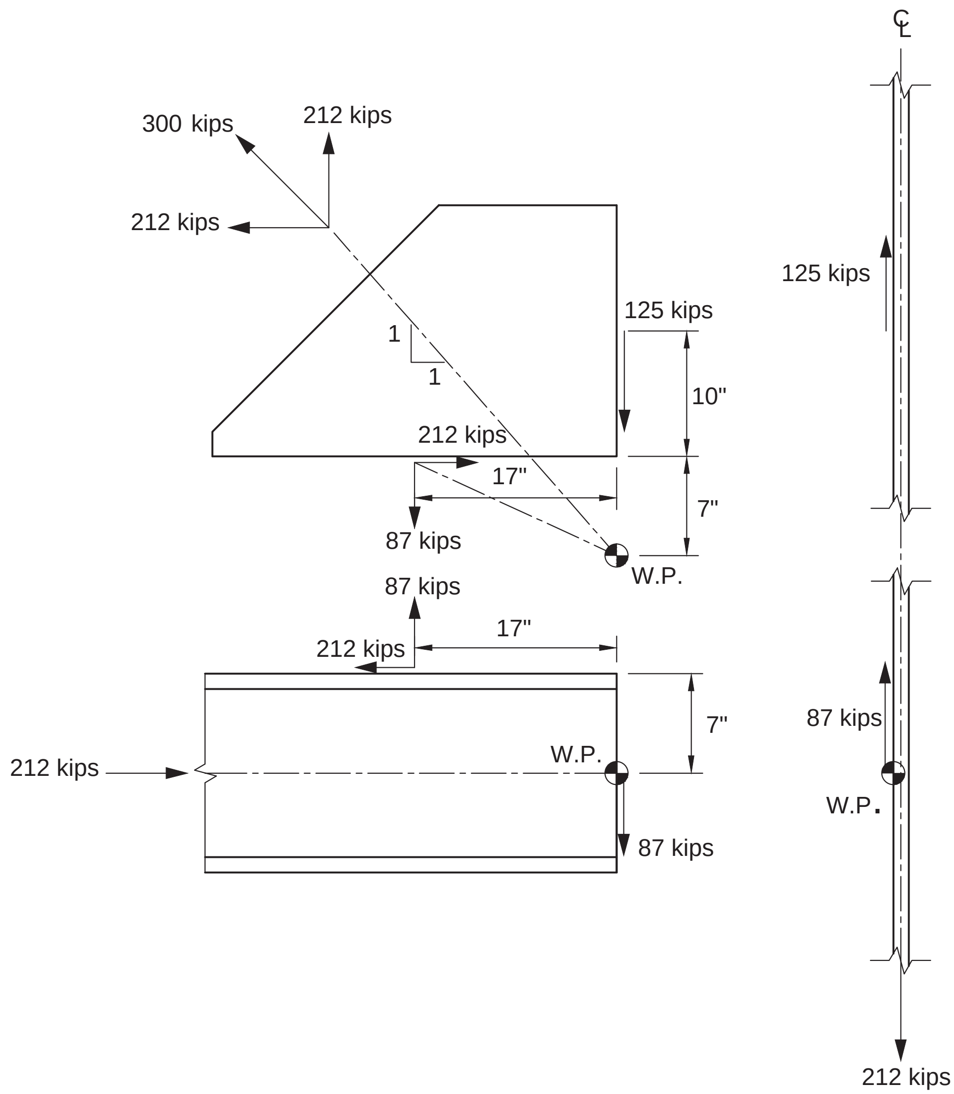


Fig. 4-9. Uniform force method admissible force field.

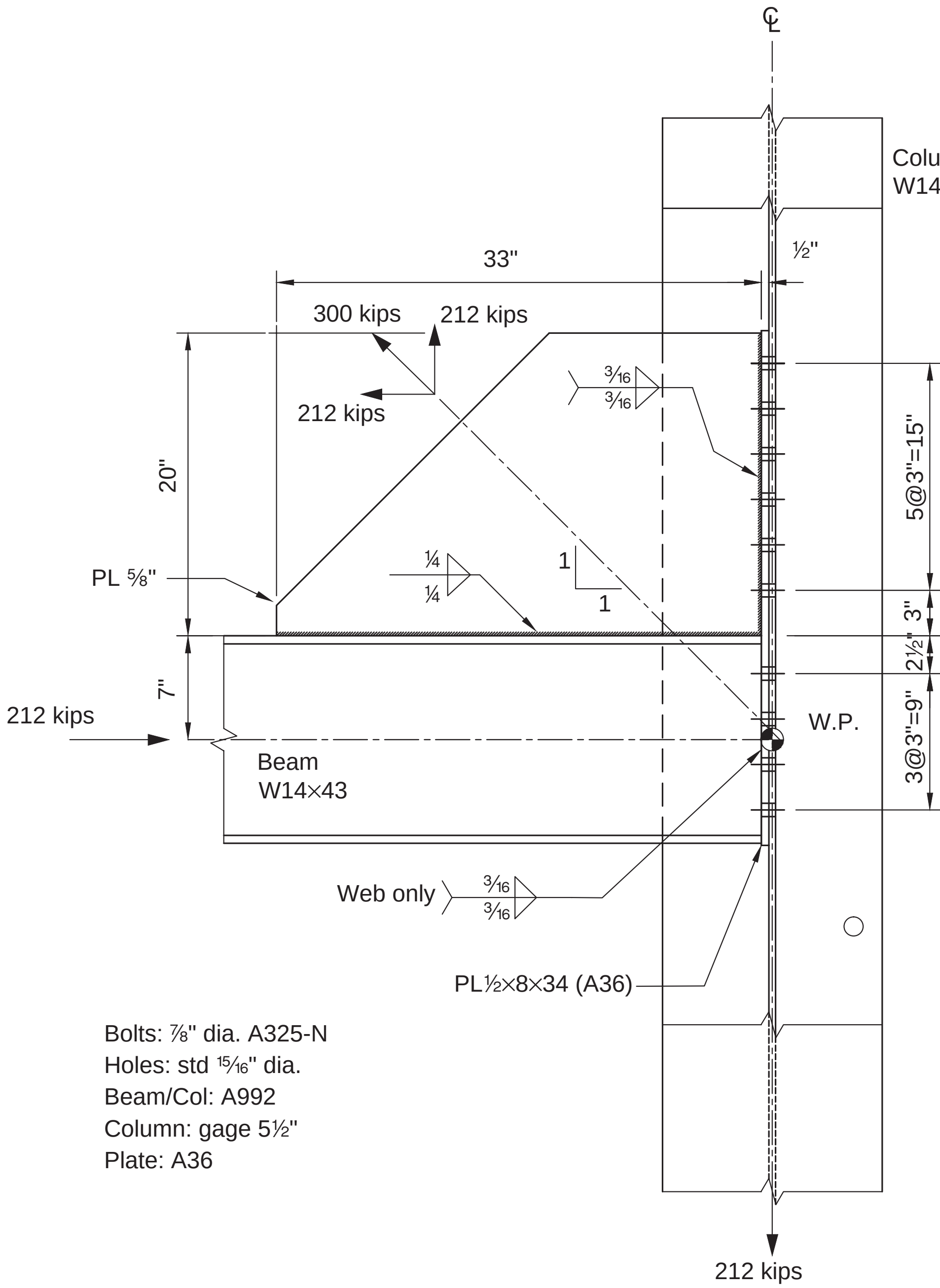


Fig. 4-10. Design by uniform force method.

3. When a decision as to relative stiffness of the two gusset interfaces cannot be made, use the method presented in the AISC *Manual* and distribute the moment based on minimized eccentricities $\alpha - \bar{\alpha}$ and $\beta - \bar{\beta}$, by minimizing the objective function, ξ, as follows:

$$\xi = \left(\frac{\alpha - \bar{\alpha}}{\bar{\alpha}}\right)^2 + \left(\frac{\beta - \bar{\beta}}{\bar{\beta}}\right)^2 - \lambda\left(\alpha - \beta\tan\theta - K\right) \tag{4-4}$$

where

λ is a Lagrange multiplier and

$K = e_b \tan\theta - e_c$

The values of α and β that minimize ξ are:

$$\alpha = \frac{K'\tan\theta + K\left(\frac{\bar{\alpha}}{\bar{\beta}}\right)^2}{D}$$

$$\beta = \frac{K' - K\tan\theta}{D}$$

$$K' = \bar{\alpha}\left(\tan\theta + \frac{\bar{\alpha}}{\bar{\beta}}\right)$$

$$D = \tan^2\theta + \left(\frac{\bar{\alpha}}{\bar{\beta}}\right)^2$$

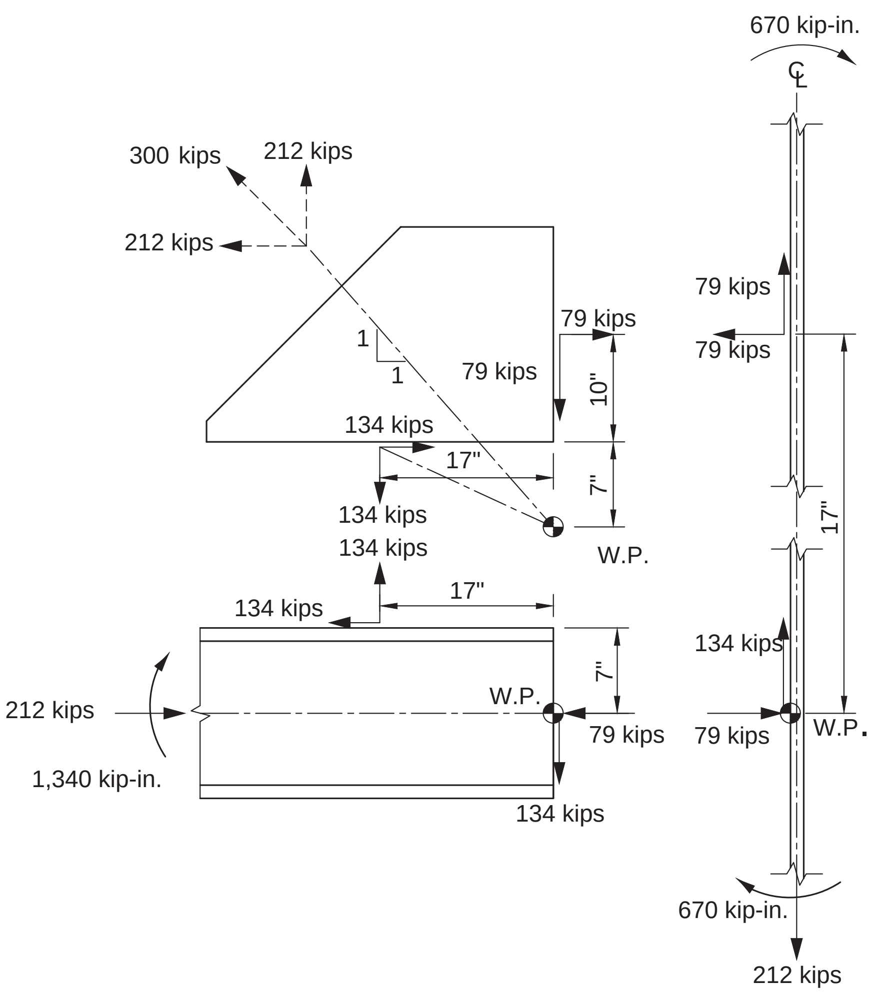


Fig. 4-11. Parallel force method admissible force field.

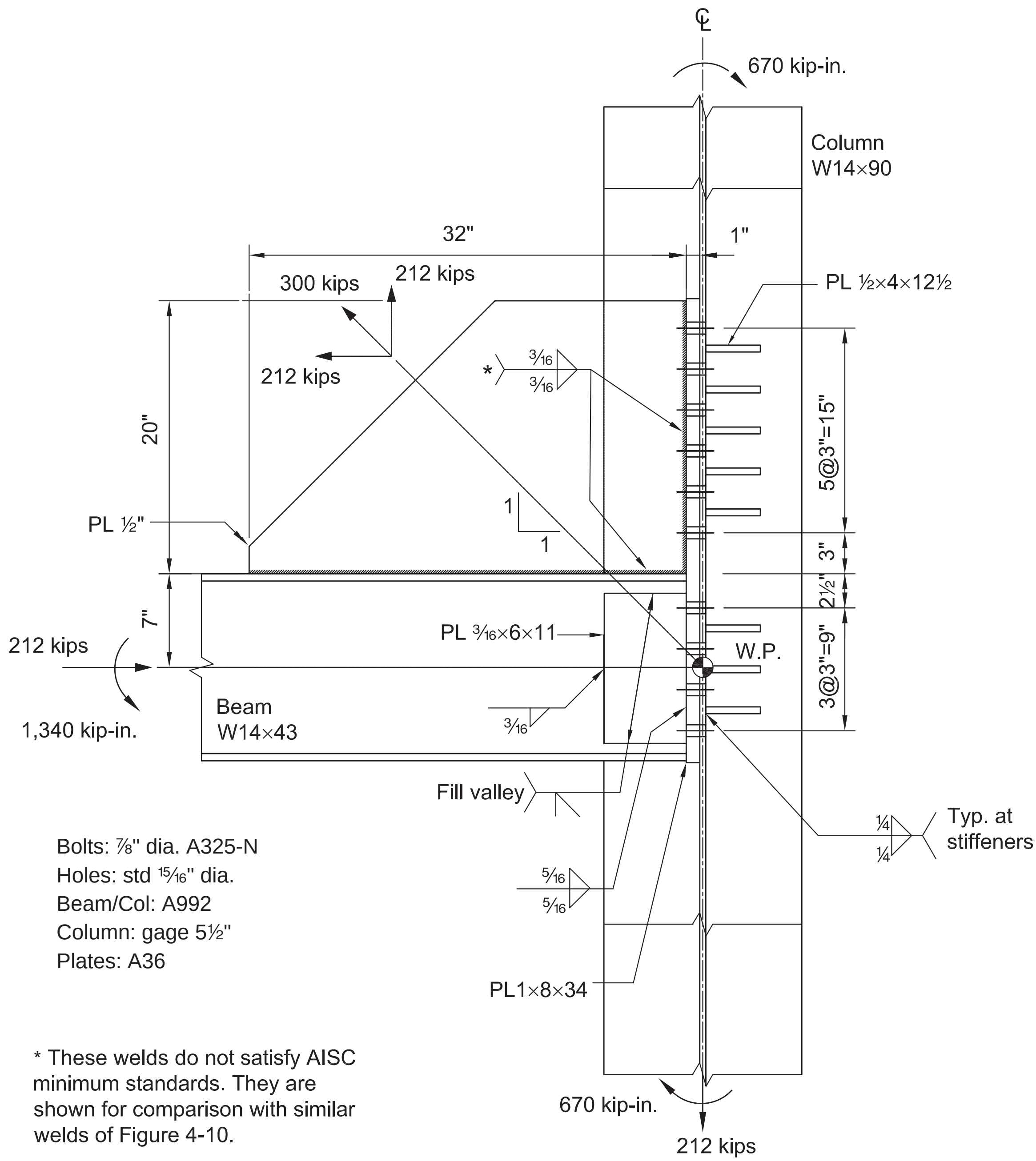


Fig. 4-12. Design by parallel force method.

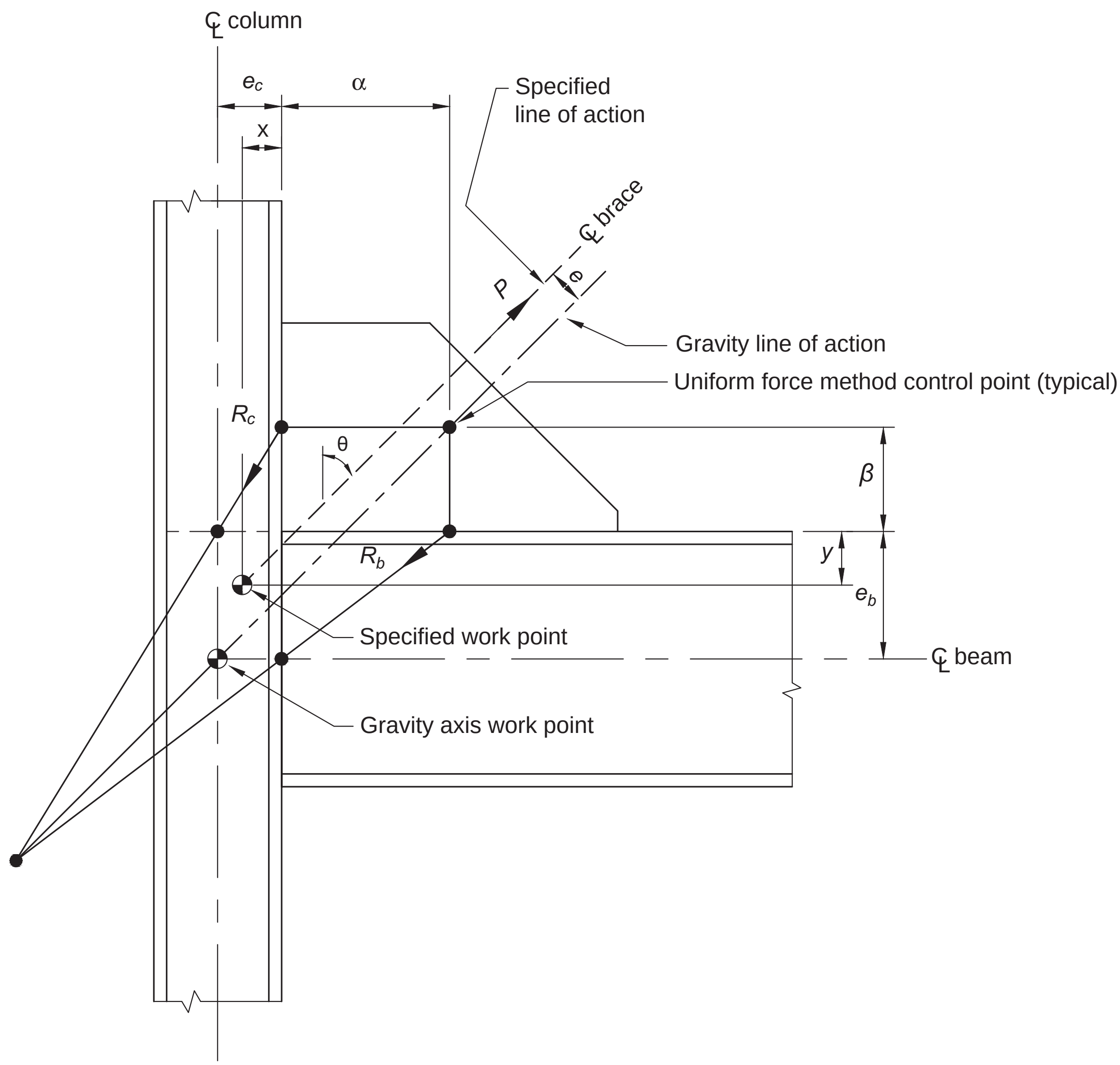
4.2.2 Nonconcentric Brace Force—Special Case 1

The case where the work point is located at the gusset corner, instead of at the intersection of the beam and column centerlines, is referred to as Special Case 1 in Figure 13-3 of the AISC *Steel Construction Manual* (AISC, 2011). The force distribution shown in *Manual* Figure 13-3 is admissible, but it does not agree with analytical results obtained by Richard (1986). A better approach is shown in Figure 4-13. Here, the work point can be located at the gusset corner or at any other reasonable location, defined by the coordinates x and y. The corner gusset location is defined by the coordinates $x = 0$ and $y = 0$. Figure 4-14 shows the force distribution that results from the eccentricity, e, with the work point at the gusset corner. The forces in Figure 4-14 are given by:

$$H' = \frac{(1-\eta)M}{\bar{\beta} + e_b} \tag{4-5}$$

$$V' = \frac{M - H'\bar{\beta}}{\bar{\alpha}} \tag{4-6}$$

$$M = Pe \tag{4-7}$$



$$e = (e_b - y)\sin\theta - (e_c - x)\cos\theta$$

Fig. 4-13. Nonconcentric uniform force method.

where η is the fraction of the moment that is distributed to the beam. η can be estimated at ultimate load by:

$$\eta = \frac{Z_{beam}}{Z_{beam} + \sum Z_{column}} \tag{4-8}$$

or at service loads by:

$$\eta = \frac{\left(I/L\right)_{beam}}{\left(I/L\right)_{beam} + \sum \left(I/L\right)_{column}} \tag{4-9}$$

Note that when the connection is to a column web, η = 1 and all of the moment, M, is in the beam. In this case, the column stiffness or strength is not mobilized because of web distortion. This is the distribution found by Gross (1990) in essentially full scale physical tests.

The forces H' and V' are superimposed on the UFM forces on the same interface, and the connection is designed for the resultant forces. The beam and column must be checked for ηM and $\left(\frac{1-\eta}{2}\right)M$, respectively.

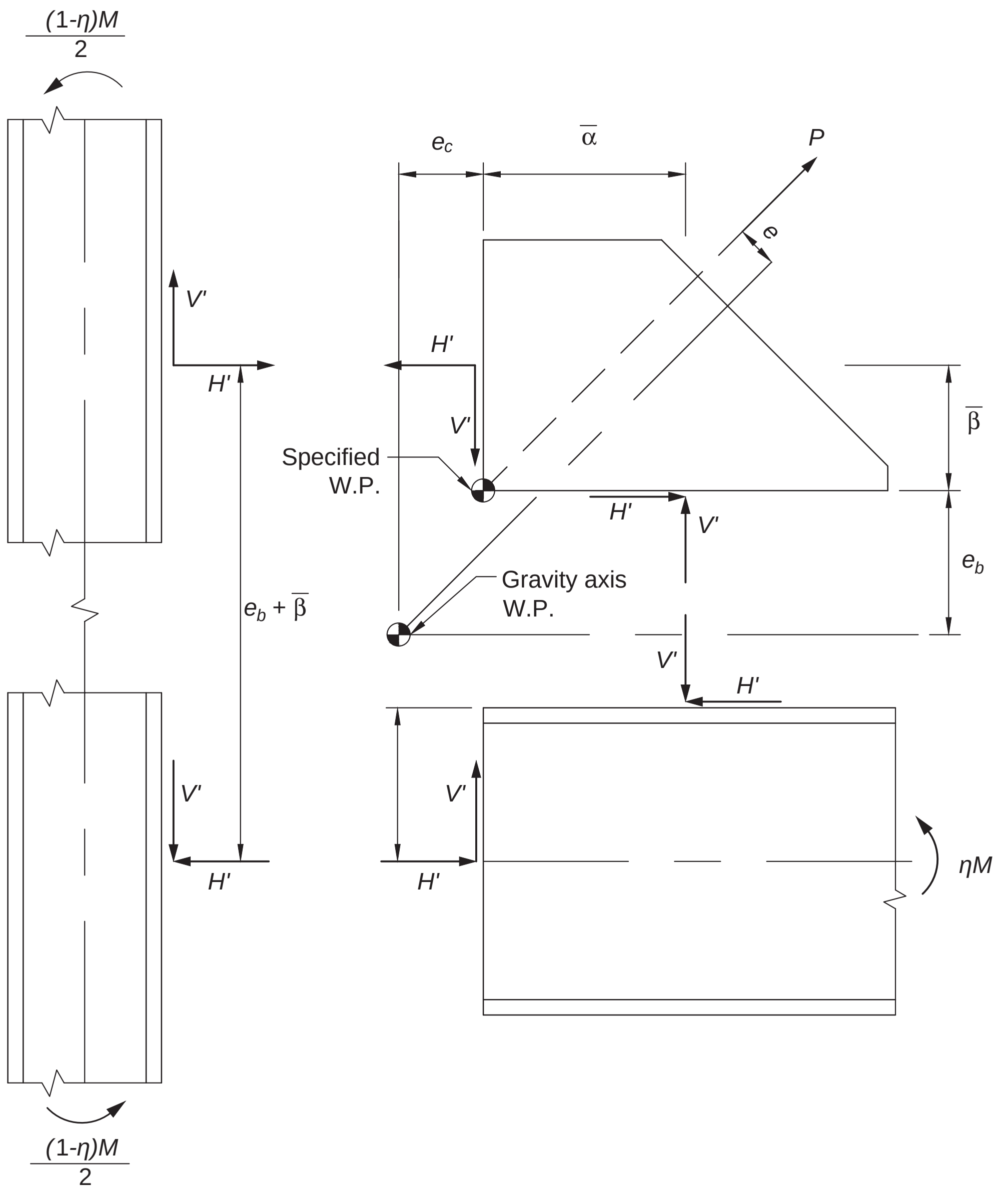


Fig. 4-14. Extra forces due to nonconcentric work point.

4.2.3 Reduced Vertical Brace Shear Force in Beam-to-Column Connection—Special Case 2

In many cases, large braces are connected to similarly sized columns, but the beams are essentially struts. This often happens at the outside walls of buildings and elsewhere where gravity loads are small. This is often due to the use of computer analysis to design the members. Computer models for a concentrically braced frame usually have all members meeting at a node. How these members are to be connected to each other is not considered until the connections are designed. Small beams or struts generally cannot carry much, if any, of the brace vertical component, because they were not designed for this load. To resolve this problem, some or all of the brace vertical component that the UFM places on the gusset-to-beam interface, and hence on the beam-to-column connection, can be removed by introducing a parameter ΔV_b. ΔV_b acts at the gusset-to-beam connection centroid, located at a distance $\bar{\alpha}$ from the beam end, in a direction opposite to V_b as shown in Figure 4-15. Then, the total vertical force on the gusset-to-column interface is $V_c + \Delta V_c$, and on the gusset-to-beam interface the force is $V_b - \Delta V_b$. In Figure 4-15, the general case of $\alpha \neq \bar{\alpha}$ and $\beta \neq \bar{\beta}$

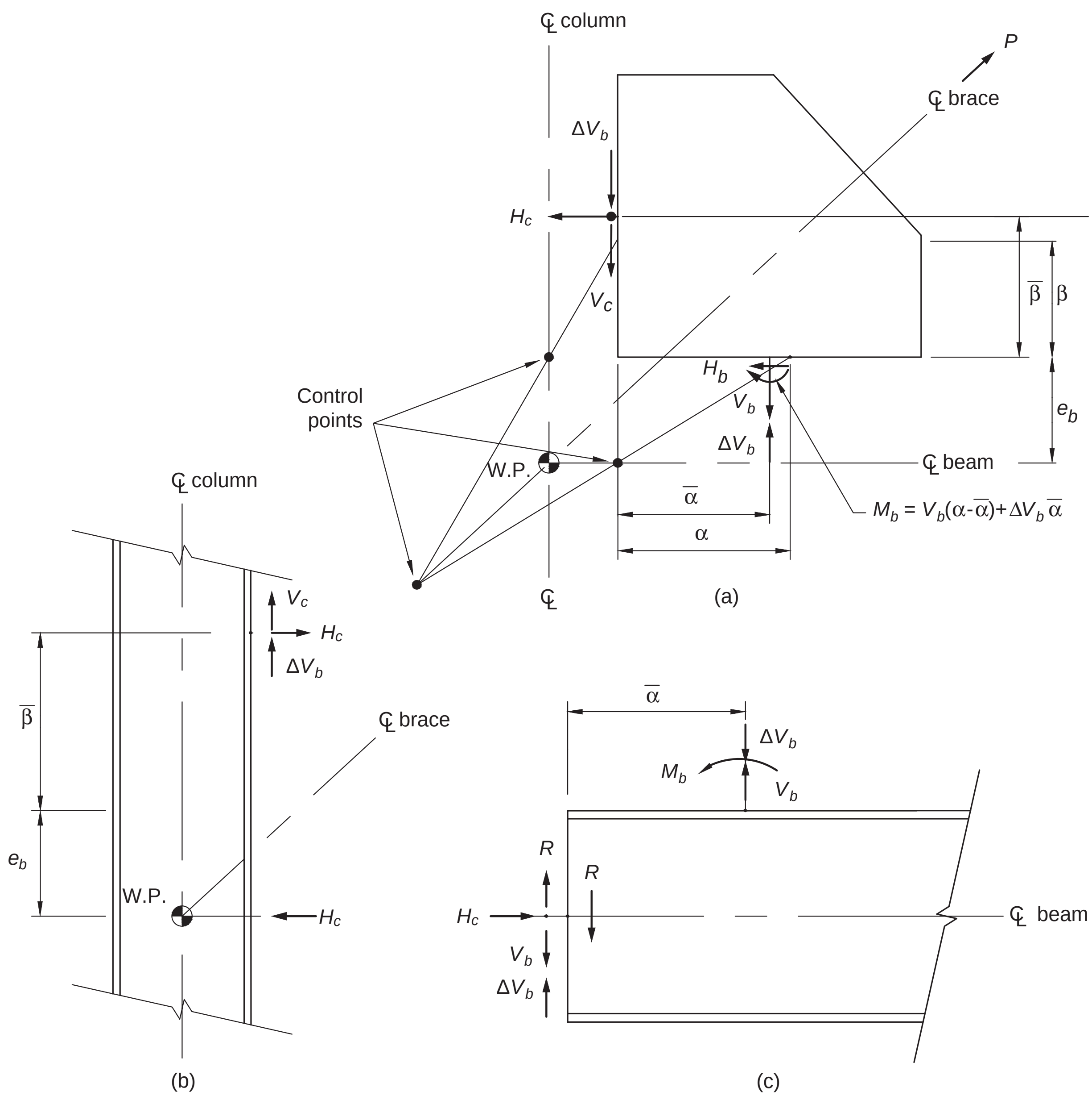


Fig. 4-15. Special Case 2.

is assumed. The introduction of ΔV_b in Figure 4-15 produces a moment equal to $\Delta V_b\bar{\alpha}$ on the gusset-to-beam interface; therefore, the total moment there is:

$$M_b = V_b\left(\alpha - \bar{\alpha}\right) + \Delta V_b\bar{\alpha} \tag{4-10}$$

which is clockwise on the gusset when the brace force, P, is tension and $\alpha > \bar{\alpha}$. When $\Delta V_b = V_b$, it is referred to as Special Case 2 in the AISC *Manual*, i.e., no vertical force on the beam-to-column interface due to the brace force, and Equation 4-10 becomes:

$$M_b = V_b\alpha = H_b e_b \tag{4-11}$$

Note that the force component H_c still acts as an axial force on the beam-to-column interface.

4.2.4 No Gusset-to-Column Connection— Special Case 3

This case occurs when the brace bevel is shallow, i.e., approximately $\theta > 60°$. Figure 4-16 shows the general case and the admissible force field. Setting $\beta = 0$, $\alpha = e_b\tan\theta - e_c$. If $\alpha \neq \bar{\alpha}$, a moment $M_b = V(\bar{\alpha} - \alpha)$ exists on the beam-to-gusset interface, which is counterclockwise when $\bar{\alpha} > \alpha$, the usual case. Note also that there is a moment on the beam-to-column interface, $M_{bc} = Ve_c$, in addition to the vertical brace component V. In this case, it is impossible to have uniform

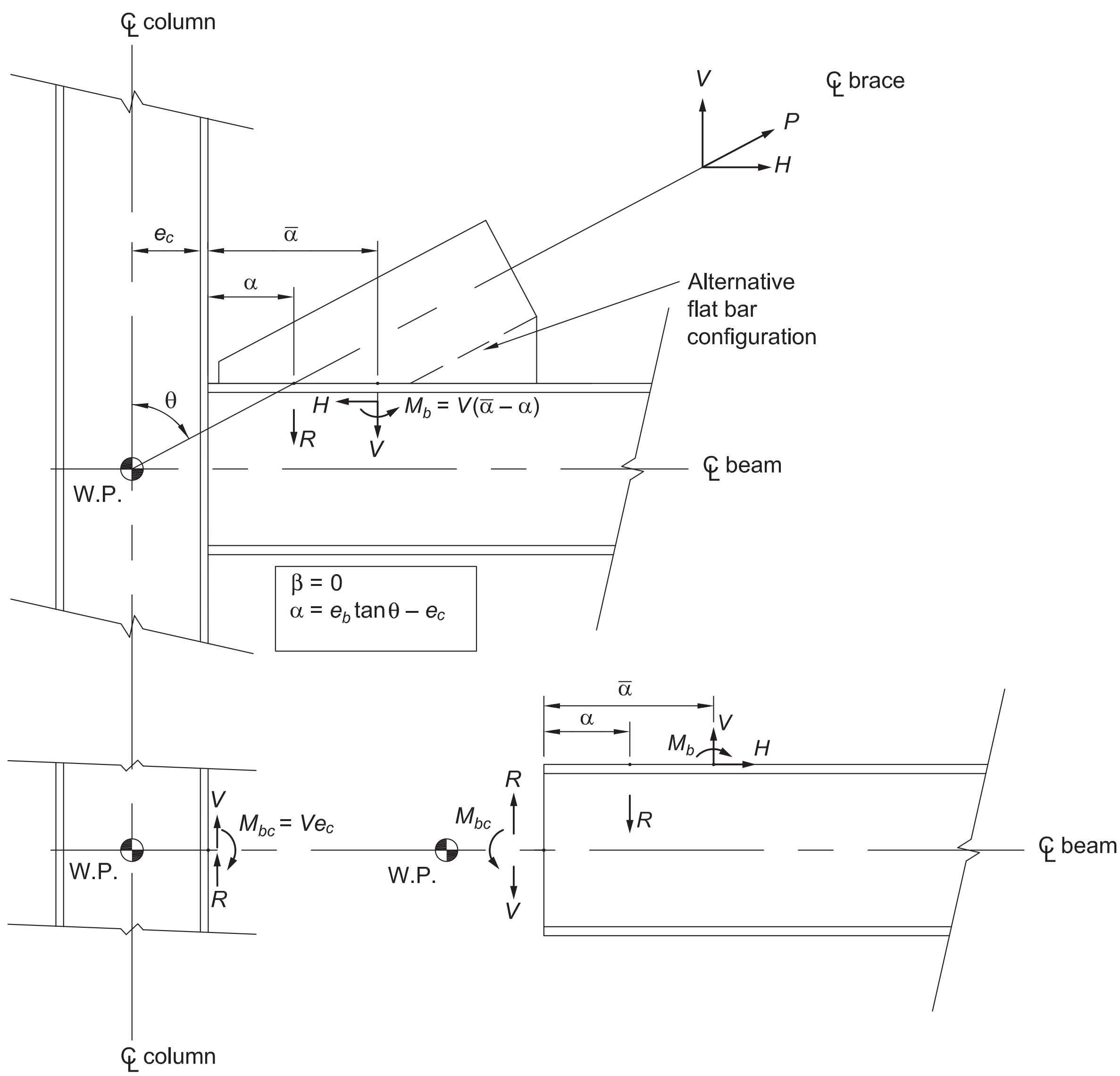


Fig. 4-16. Special Case 3.

forces on all interfaces unless the connection is to a column web, where $e_c = 0$ and thus $M_{bc} = 0$. If the brace bevel is steep (i.e., $\theta < 30°$), a similar formulation for the gusset connected only to the column can be developed. Note that the force distribution of Figure 4-16 can be determined less formally by statics alone.

4.2.5 Nonorthogonal Corner Connections

Figure 4-17 shows a generalization of the UFM to nonorthogonal corner connections. These occur when the beam or column is sloping, and also in trusses with a sloping top chord. The angle γ in Figure 4-17 is positive when the angle between the beam and column is less than 90°, and negative otherwise. A design example of this type of connection is presented later in this Design Guide.

$$\boxed{\alpha - \beta(\cos\gamma\tan\theta - \sin\gamma) = e_b(\tan\theta - \tan\gamma) - \frac{e_c}{\cos\gamma}}$$

$$V_b = \frac{e_b}{r}P \qquad H_b = \frac{\alpha + e_b\tan\gamma}{r}P$$

$$V_c = \frac{\beta\cos\gamma}{r}P \qquad H_c = \left(\beta\sin\gamma + \frac{e_c}{\cos\gamma}\right)\frac{P}{r}$$

$$Q = H_c - P\cos\theta\tan\gamma$$

$$r = \sqrt{\left(\alpha + e_b\tan\gamma + \beta\sin\gamma + \frac{e_c}{\cos\gamma}\right)^2 + \left(e_b + \beta\cos\gamma\right)^2}$$

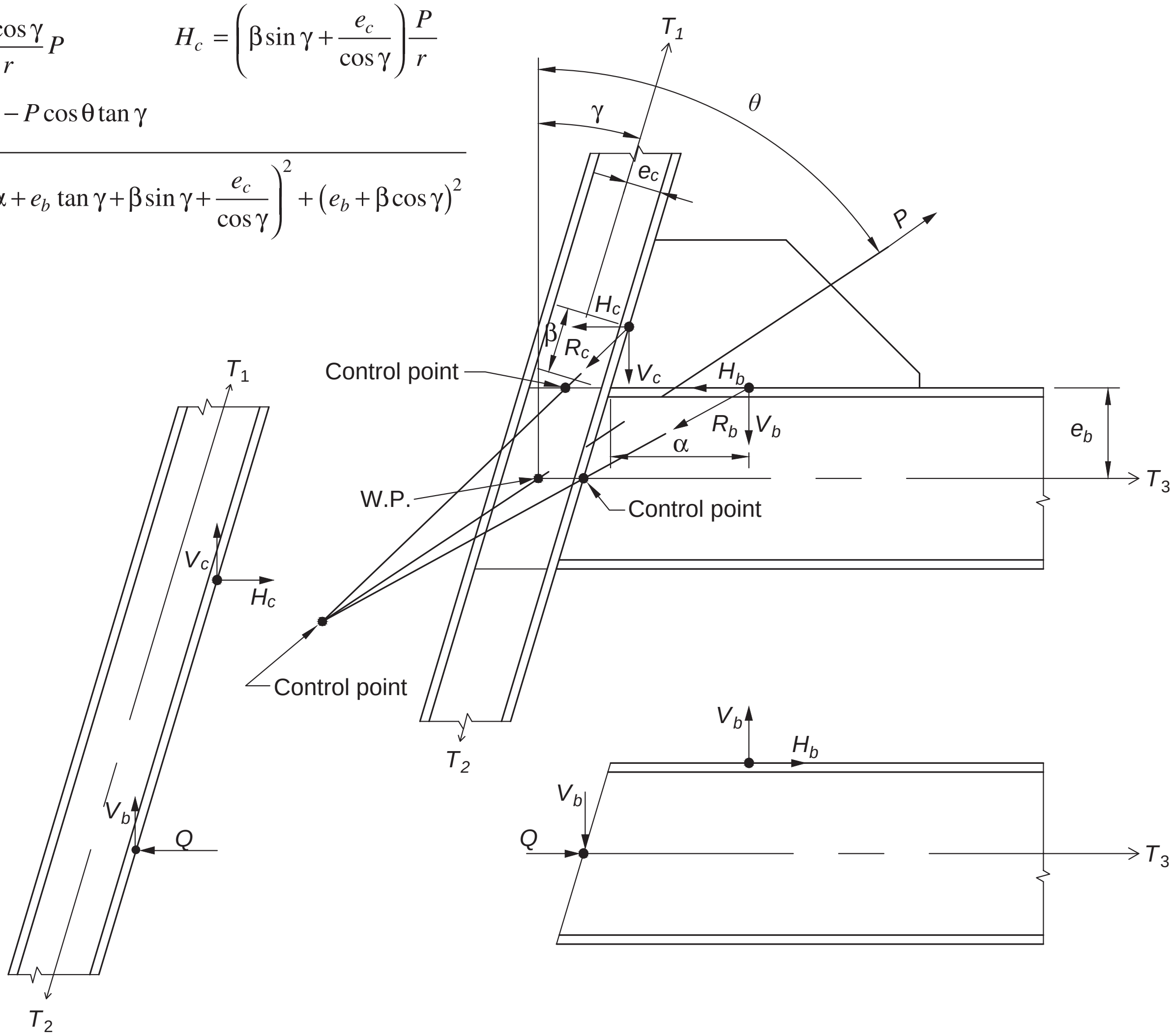


Fig. 4-17. Nonorthogonal uniform force method.

4.2.6 Effect of Frame Distortion

As was mentioned earlier, braced frames are usually analyzed as pin-connected members but, obviously, the connections are not pins. This gives rise to additional forces in the connections known as distortional forces. An admissible distribution of distortional forces is presented in Figure 4-18. If an elastic analysis is used, an estimate of the moment in the beam can be determined from Tamboli (2010, pp. 2–72):

$$M_D = 6\left(\frac{P}{Abc}\right)\left(\frac{I_b I_c}{\dfrac{I_b}{b}+\dfrac{2I_c}{c}}\right)\left(\frac{b^2+c^2}{bc}\right) \tag{4-12}$$

where

P = brace force, kips
I_b = moment of inertia of the beam, in.^{4}
I_c = moment of inertia of the column, in.^{4}
A = brace area, in.^{2}
D = subscript denoting distortion
b = length of beam to inflection point, in. (assumed at beam midpoint)
c = length of column to inflection point, in. (assumed at column midlength)

This formula is limited to bracing arrangements such as those given in Figure 2-1, except those of Figure 2-1(c) at column line D and Figure 2-1(e). Similar formulas can be derived for these two cases and others that occur in practice. Alternatively, a computer-based frame analysis can be used to produce M_D for design.

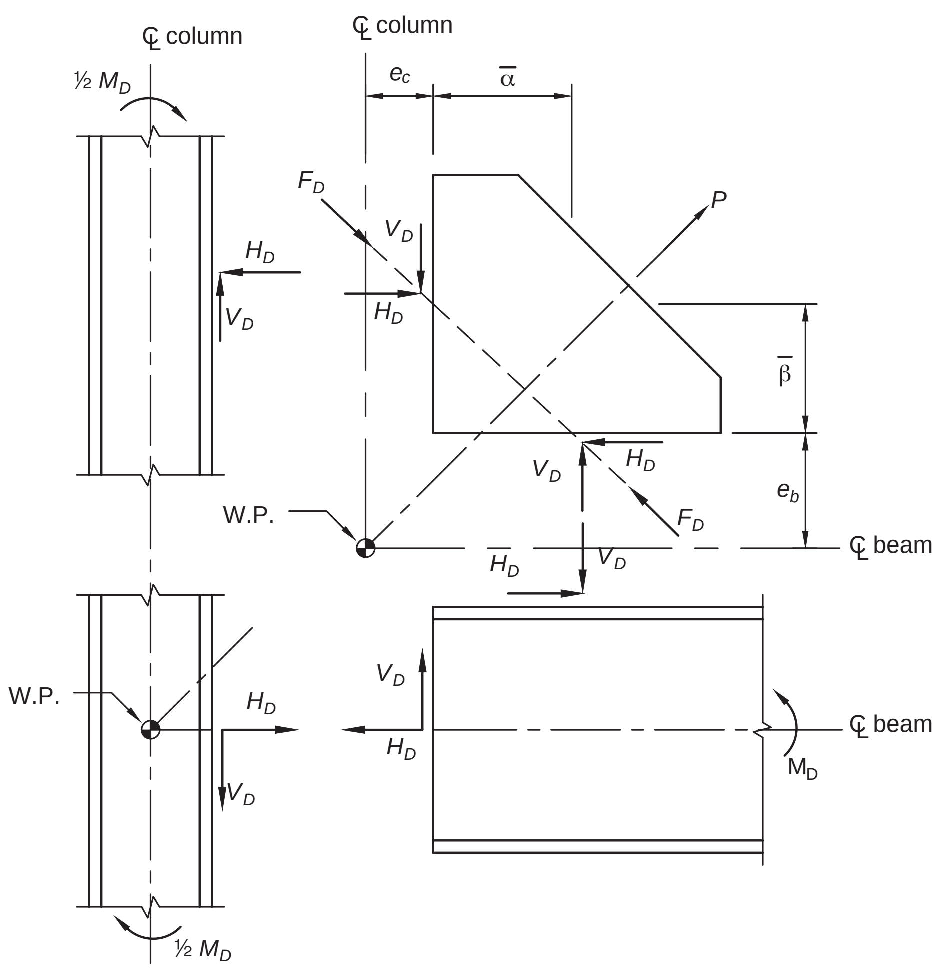


Fig. 4-18. Admissible distribution of distortional forces.

If M_D, previously calculated, exceeds the elastic flexural strength of the beam, or $\frac{M_D}{2}$ exceeds the elastic flexural strength of the column, then Equation 4-12 is no longer valid. The plastic moments of the beam or column could be used as an upper bound on M_D:

$$M_D = \min\left\{\phi M_{p(beam)}, \sum \phi M_{p(column)}\right\} \tag{4-13}$$

In either case, the forces of Figure 4-18 are calculated as:

$$H_D = \frac{M_D}{\bar{\beta} + e_b} \tag{4-14}$$

$$V_D = \frac{\bar{\beta}}{\bar{\alpha}} H_D \tag{4-15}$$

$$F_D = \sqrt{H_D^2 + V_D^2} \tag{4-16}$$

Note that when P is tension, F_D is compression, and therefore it is possible for the gusset plate to buckle when the

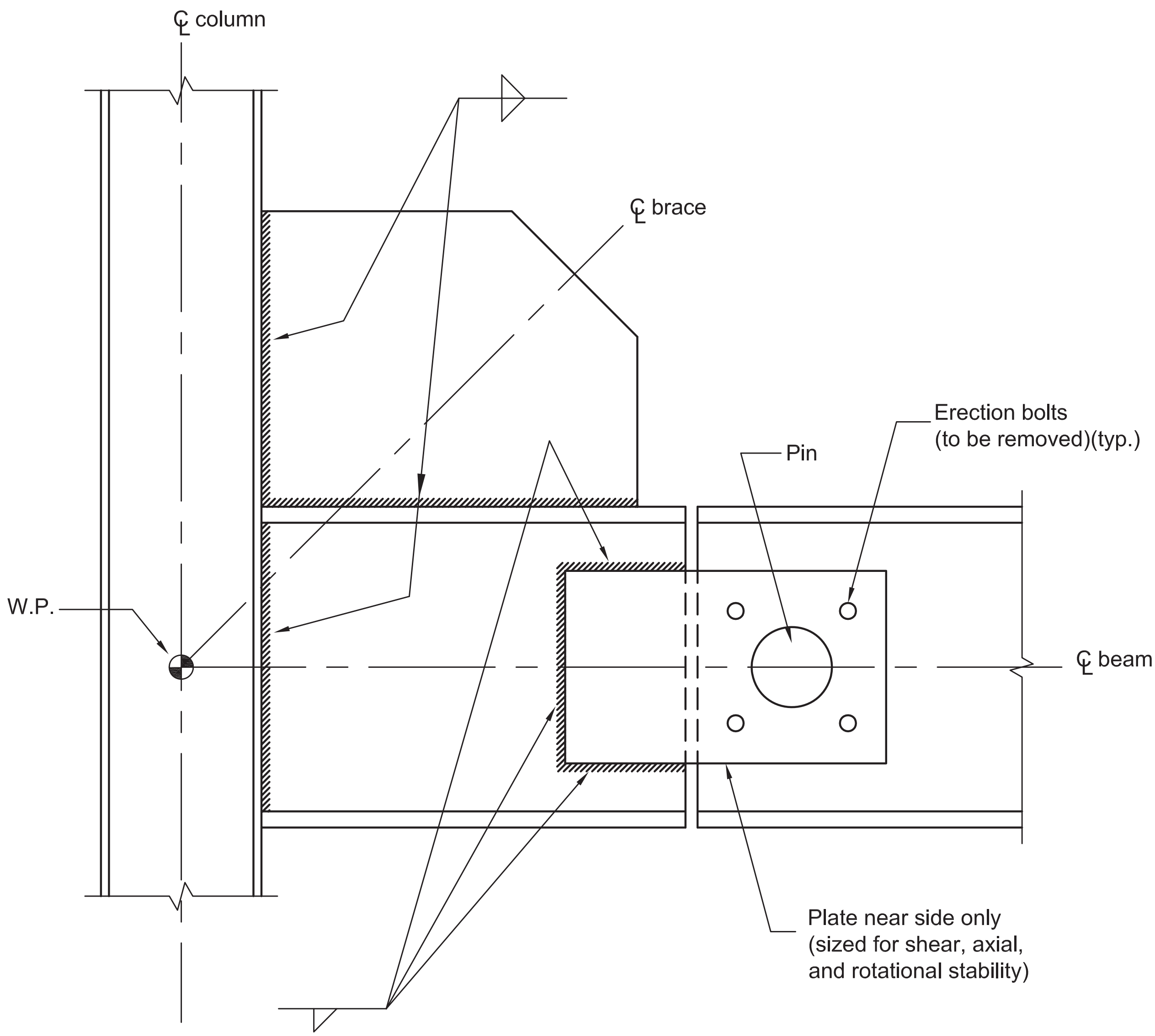


Fig. 4-19. Connection to minimize distortional forces.

brace is in tension. This is called "gusset pinching" and has been observed in physical testing (Lopez et al., 2004). Note also that the distortional forces can be controlled by controlling M_D. If an actual physical pin were introduced into the beam as shown in Figure 4-19, the distortional forces would be theoretically reduced to zero. Short of using a pin as shown in Figure 4-19, a shear connection (Figure 4-20) or a reduced beam section (Figure 4-21) can be introduced at this point to control the distortional forces. An idea similar to this has been investigated in recent research at Lehigh (Fahnestock, 2005). A well-thought-out detail to decouple the moment frame, which produces the distortional forces, from the braced frame is given by Walters et al. (2004). Another way to consider distortional forces is to design the connection ignoring them, then determine the maximum distortional forces that can be developed by the existing design. The connection is then checked using the combined original and distortional forces. This method will be demonstrated in this Design Guide, where the distortional forces are controlled via a defined hinge in the beam.

Figures 4-18 through 4-21 all illustrate connections to column flanges. When the connection is made to a column web, the web itself distorts and prevents the development of significant distortional forces in the gusset plate. Therefore, $H_D = 0$, $V_D = 0$ and $M_D = 0$. This was shown in physical testing reported by Gross (1990).

Typically, distortional forces have been ignored in bracing connection design, just as they have been ignored in truss

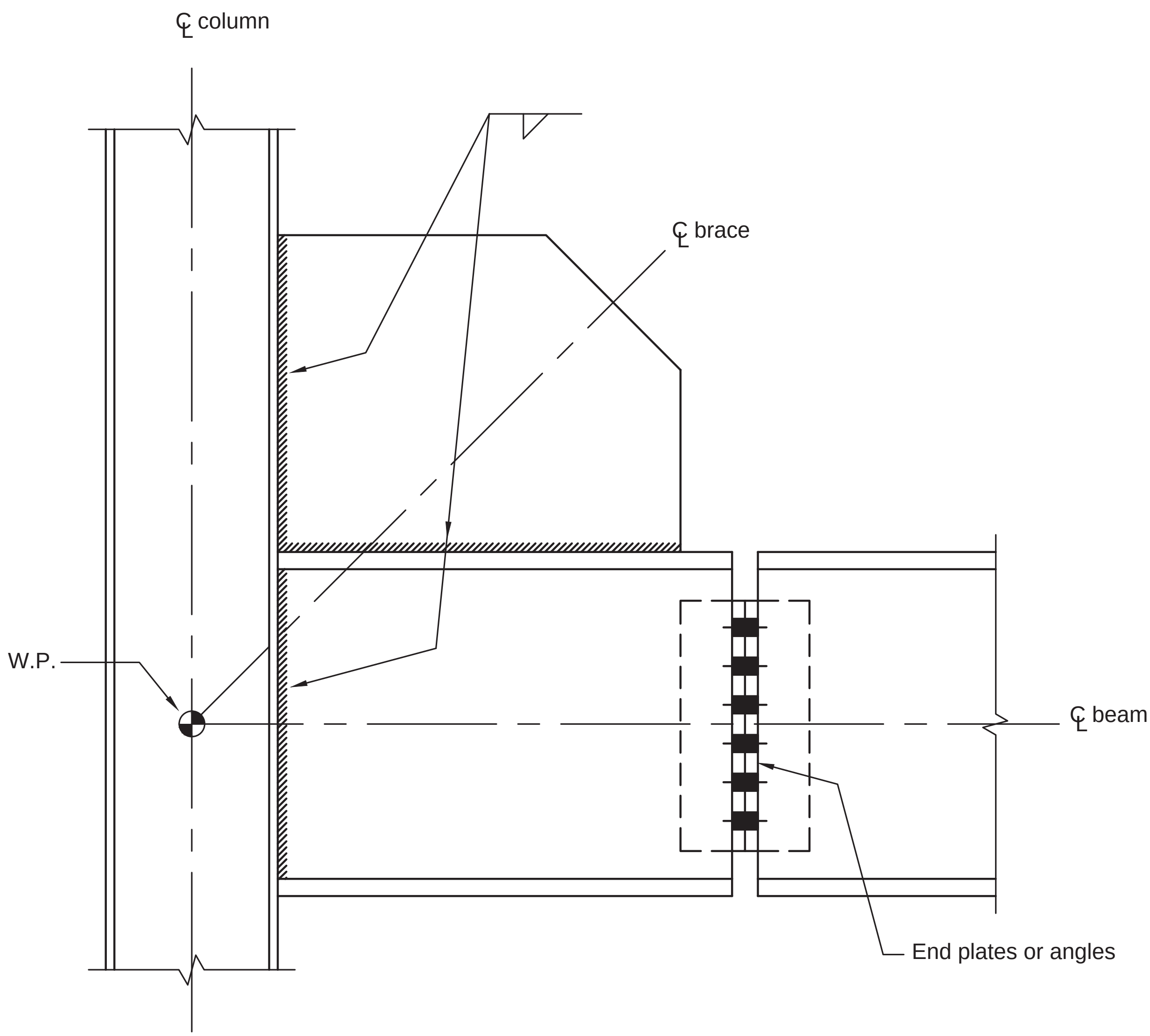


Fig. 4-20. Shear splice to control distortional forces.

design, with no known negative consequences for nonseismic and low-seismic ($R = 3$) applications. However, because of the high drift ratios of about 2 to 2.5%, which produce very high distortional forces, these forces cannot be ignored in high-seismic design ($R > 3$). When comparing the uniform force method to nonseismic physical results (Gross, 1990), which included distortional forces (frame action), Thornton (1995) showed that the uniform force method produced conservative results. Because of this, and the uncertainties involved in their determination, distortional forces are generally not included in the examples presented in this Design Guide, except for the high-seismic design example of Section 6.1.

4.3 BRACING CONNECTIONS TO COLUMN BASE PLATES

A bracing connection to a column base is shown to the column strong axis in Figure 4-22, along with an admissible force field. As with the UFM, the admissible force field of Figure 4-22 produces uniform forces on the gusset edges, as shown. The work point is shown at a distance, e, above the top of the base plate because this position is favored by many designers to keep some or all of the brace above the surface of the finished floor. The ideal location of the work point is at the top surface of the base plate, because this location eliminates bending and shear forces on the column cross section.

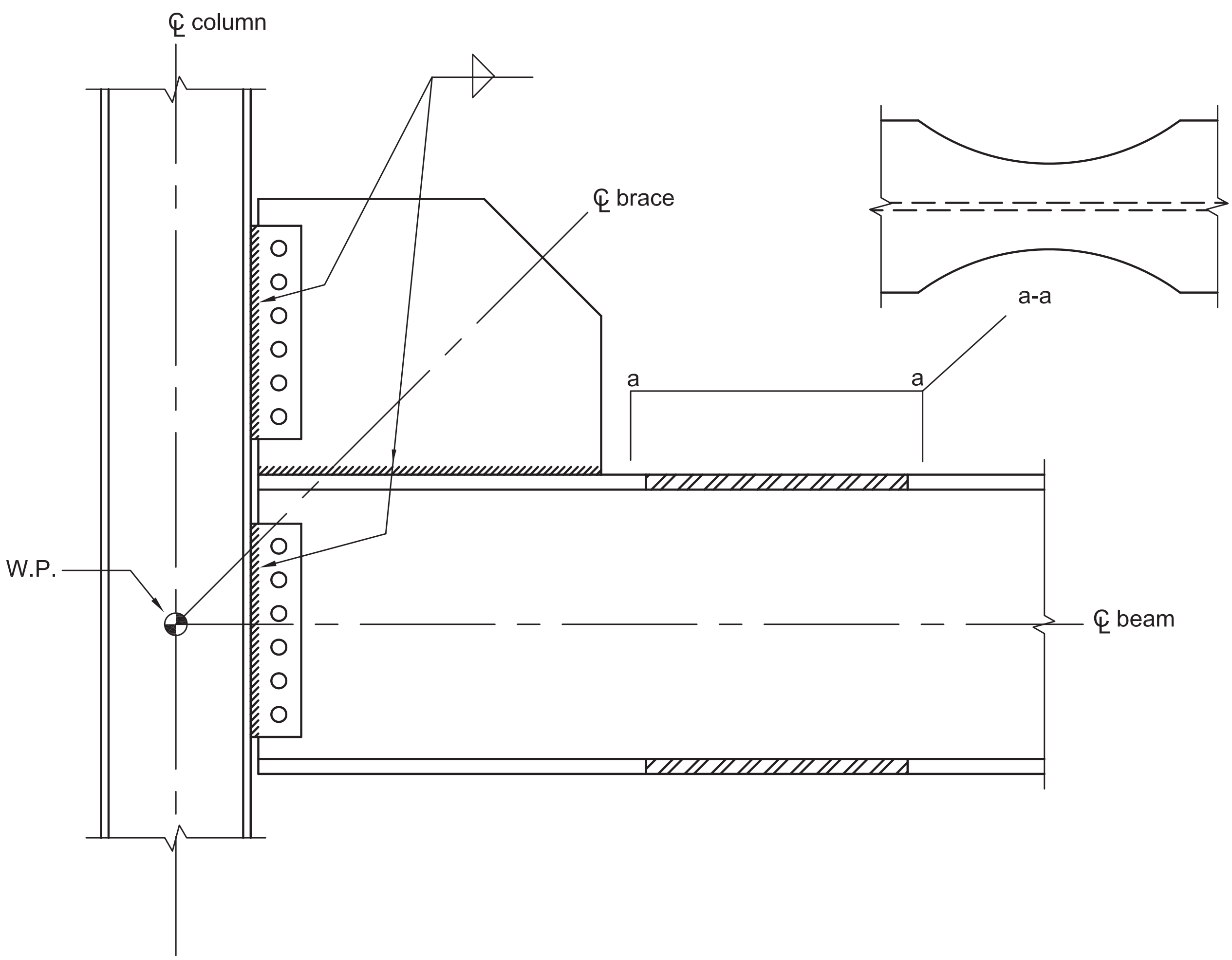


Fig. 4-21. Reduced beam section (RBS) to control distortional forces.

In the position shown at e above the base plate, the column will be subjected to shear force, H_c, and moment, $H_c e$, in addition to any other column loads. This force and moment (H_c and $H_c e$) are probably negligible, but the designer should be aware of them.

The work point is often specified to be at the bottom of the base plate, at $e = -t_{bp}$. This point is very convenient for layout geometry because the base plate thickness is not required to determine bevels (brace inclinations). If e is large, it may be desirable to attach the gusset to the column only.

Figure 4-22 shows the gusset connected to a column flange. When the connection is to a column web, $e_c = 0$, and Figure 4-23 results. It is recommended that the work point be located at the top of the base plate ($e = 0$) whenever possible. It can be seen that if e is not equal to zero, there will be a force, H_c, acting normal to the column web. This force can be split between the optional stiffener of Figure 4-23 with ½H_c added to H_b at the base plate. However, the column web is capable of carrying some or all of H_c, as can be determined by a yield line analysis. Figure 4-24 shows an appropriate yield line pattern. From Abolitz and Warner (1965), the moment that can be carried by the column web, with the gusset pivoting about point A of Figure 4-24, is:

$$M_n = k m_p L \tag{4-17}$$

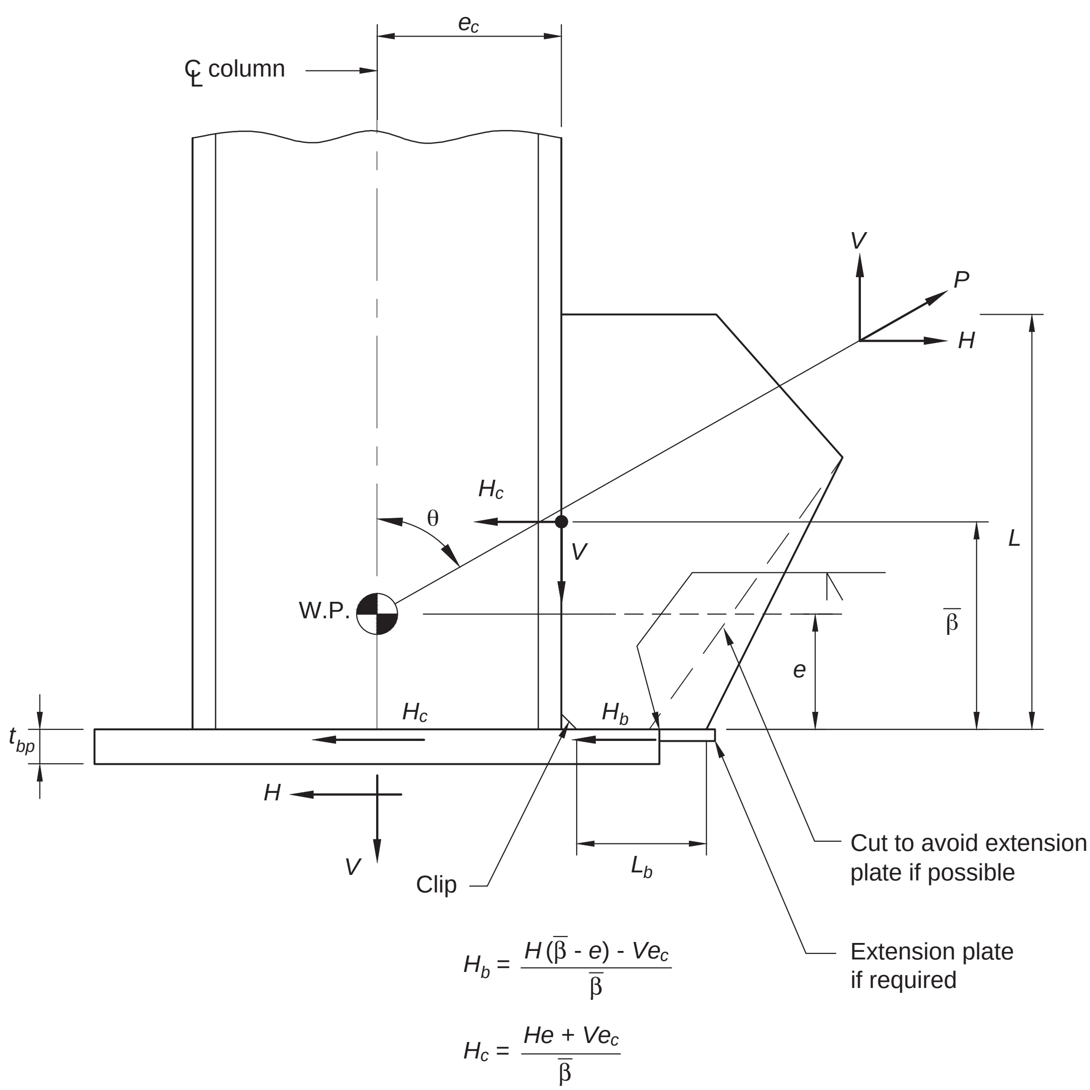


Fig. 4-22. Gusset-to-base plate geometry and admissible force field, strong-axis case.

where

L = depth of gusset plate, in.

$k = 4 + 2\sqrt{2} + 6(L/h) + (h/L)$

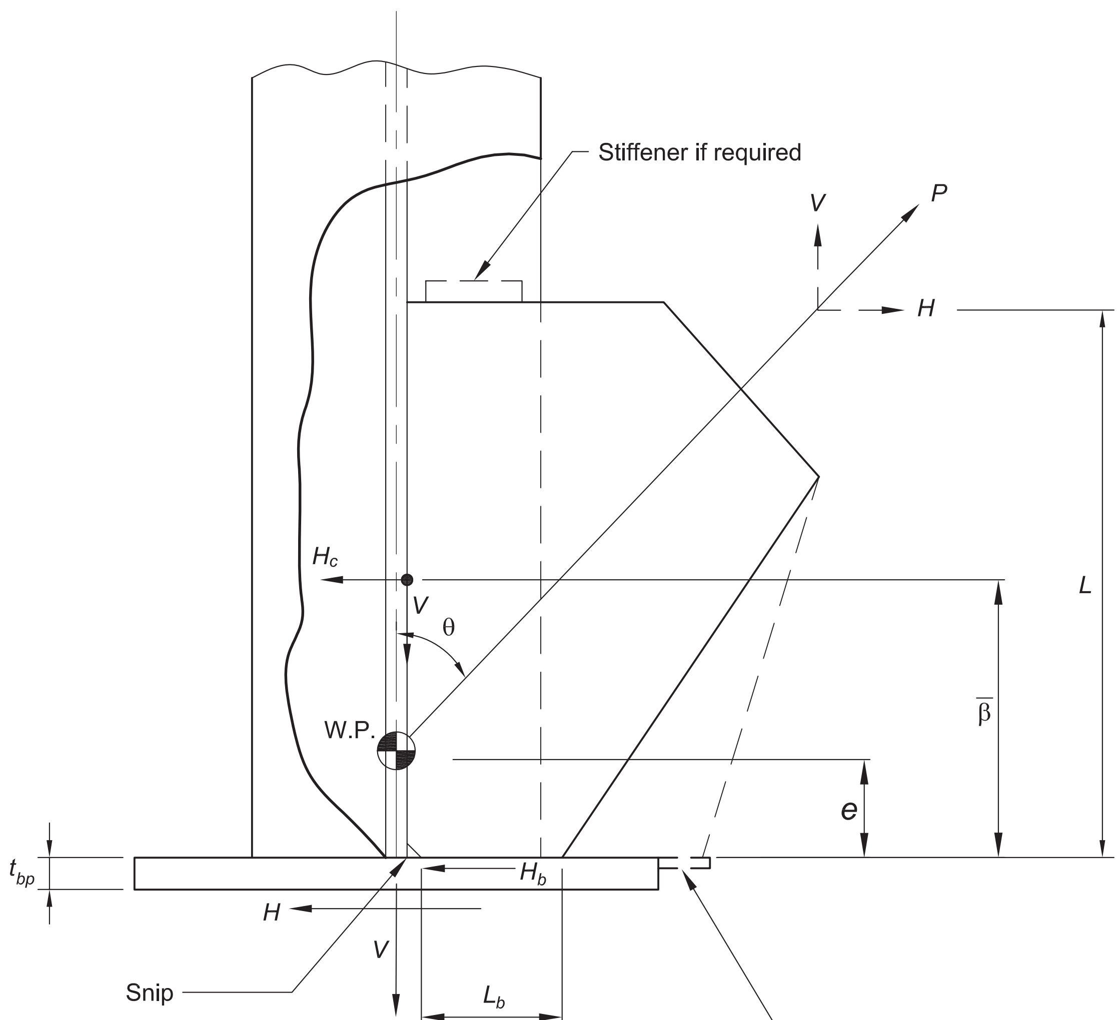
$m_p = (1/4) F_y\, t_w^2$, kips (for the column)

h = clear distance between web to flange fillets, in.

The expression for k is the average of the k values for the web pinned at the flanges and fixed at the flanges. The average is used because at the base plate, the flanges (being welded to the base plate) cannot rotate. At some point above the base plate, the flanges can rotate. The average of these two values is a reasonable approximate value to use in lieu of a more detailed analysis. For most cases, L will be larger than h and k will be larger than 16. Using $k = 16$:

$$\begin{aligned} M_n &= 16(1/4)F_y t_w^2 L \\ &= 4F_y t_w^2 L \end{aligned} \tag{4-18}$$

For an applied moment of $H_c\bar{\beta}$ for ASD, the web will be



$$H_b = \frac{H(\bar{\beta} - e) - Ve_c}{\bar{\beta}}$$

$$H_c = \frac{He + Ve_c}{\bar{\beta}}$$

Fig. 4-23. Gusset-to-base plate geometry and admissible force field, weak-axis case.

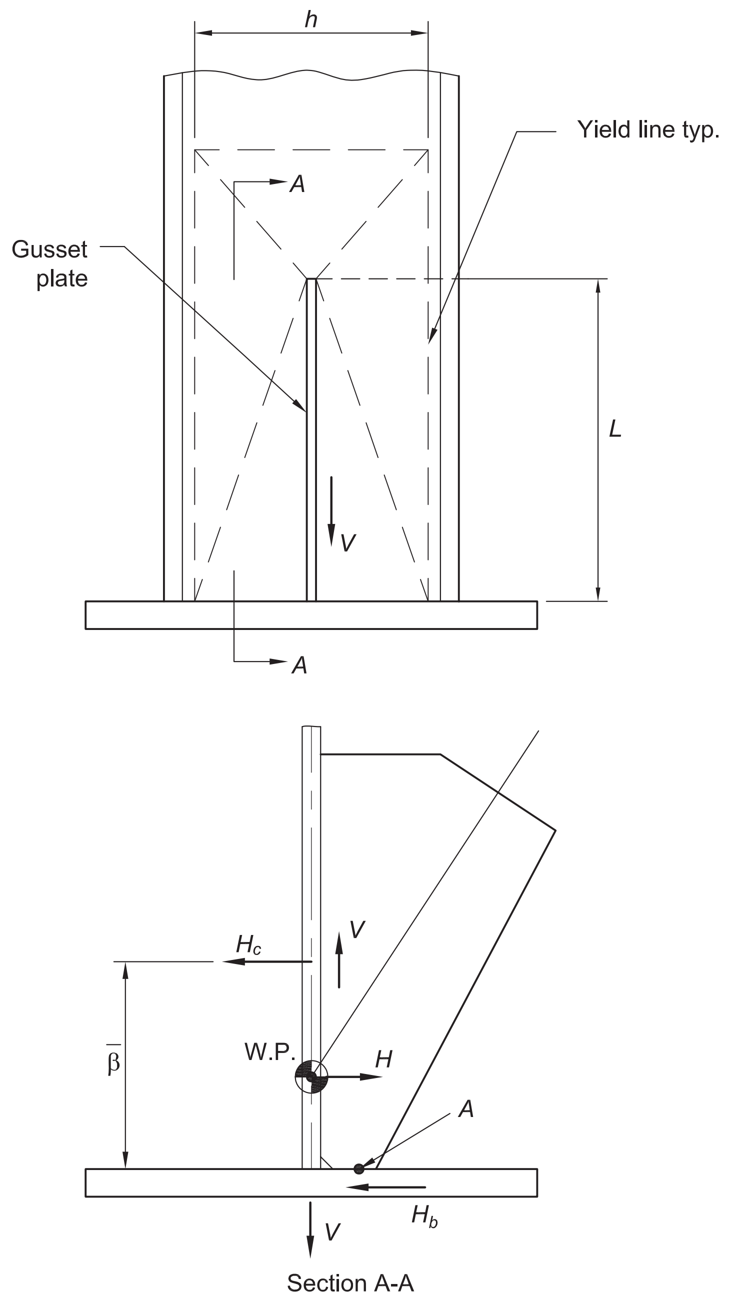


Fig. 4-24. Yield line for column web at base plate.

adequate to carry H_c if:

$$H_c \bar{\beta} \le \frac{M_n}{\Omega} = \frac{4F_y t_w^2 L}{\Omega}$$

or

$$H_c \le \frac{1}{\Omega}\left[\frac{4F_y t_w^2 L}{\bar{\beta}}\right] \tag{4-19}$$

For LRFD, the web will be adequate to carry H_c if:

$$H_c \bar{\beta} \le \phi M_n = \phi\left(4F_y t_w^2 L\right)$$

or

$$H_c \le \phi\left(\frac{4F_y t_w^2 L}{\bar{\beta}}\right) \tag{4-20}$$

If H_c exceeds the ASD strength for ASD formulations or the LRFD required strength for LRFD formulations, the optional stiffener of Figure 4-23 should be used. Note that while the same symbol, H_c, is used for both ASD and LRFD formulations, they are not the same number due to the different load combinations used for ASD versus LRFD

Because of web flexibility, the fillet weld of the gusset plate to the base plate at point A of Figure 4-24 will be subjected to a rotation that could cause a weld fracture. To avoid this, the fillet weld can be made large enough to force the gusset to yield before the weld fractures. From the AISC *Manual* discussion of single-plate connections, the fillet weld between the plate and the support is sized to be ⅝ of the plate thickness. This provision can be extended to the gusset plate to column web connection. Alternatively, the optional top stiffener can be used as shown in Figure 4-23.

Note that the above analysis is for wide-flange columns. For HSS columns, a k-value for fixed flanges would be appropriate.

Chapter 5
Design Examples

The design examples of this section are worked in a generally complete manner and the design approach for each connection type is presented in the example. Design aid tables that are currently printed in the AISC *Steel Construction Manual* (AISC, 2011), hereafter referred to as the AISC *Manual*, have been used where possible. Examples 5.1 through 5.4 address a corner bracing connection with the gusset plate connected to the column flange (strong-axis bracing connection). Examples 5.5 through 5.8 address a corner bracing connection with the gusset plate connected to the column web (weak-axis bracing connection). For each configuration, four different uniform force method (UFM) procedures are exemplified: general UFM, Special Case 1, Special Case 2, and Special Case 3. Example 5.9 demonstrates the design of a chevron bracing connection. Nonorthogonal bracing connections, truss connections, and brace-to-column base plate connections are addressed in Examples 5.10, 5.11 and 5.12, respectively.

Example 5.1—Corner Connection-to-Column Flange: General Uniform Force Method

Given:

Design the corner bracing connection shown in Figure 5-1 given the listed members, geometry and loads. The bay width is 25 ft. The connection designed in this problem is shown in Figure 5-1, in completed form (note that the final bolt type changes in the course of the design example).

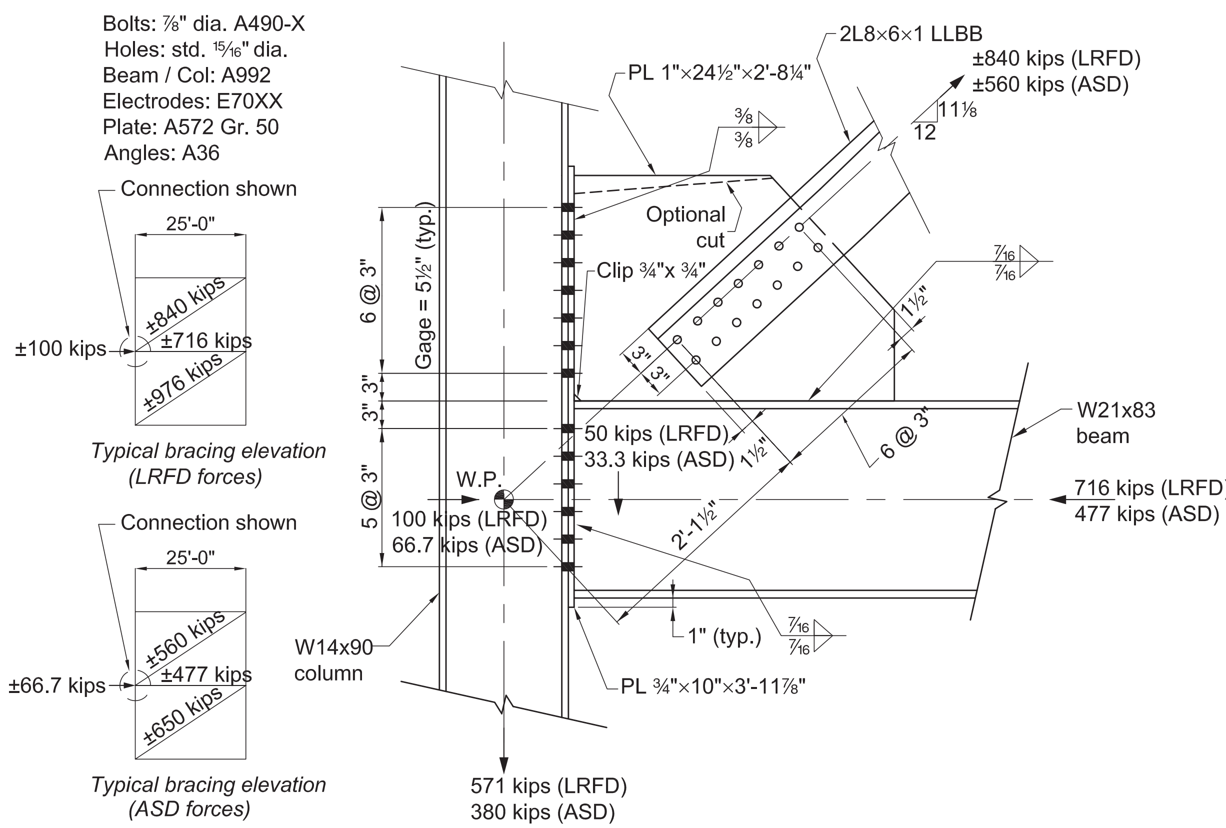


Fig. 5-1. Strong axis bracing connection—general uniform force method.

The required strengths of the brace connection were chosen to force many limit states to be critical. The required strength of the brace-to-gusset connection is:

LRFD	ASD
$P_u = 840$ kips	$P_a = 560$ kips

The transfer force, as shown in the elevation in Figure 5-1, is:

LRFD	ASD
$A_{ub} = 100$ kips	$A_{ab} = 66.7$ kips

The beam shear end reaction is:

LRFD	ASD
$V_u = 50.0$ kips	$V_a = 33.3$ kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A572 Grade 50
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W21×83
$d = 21.4$ in. $\quad t_w = 0.515$ in. $\quad b_f = 8.36$ in. $\quad t_f = 0.835$ in. $\quad k_{des} = 1.34$ in. $\quad k_1 = 7/8$ in. $\quad I_x = 1{,}830$ in.^{4}

Column
W14×90
$d = 14.0$ in. $\quad t_w = 0.440$ in. $\quad b_f = 14.5$ in. $\quad t_f = 0.710$ in. $\quad I_x = 999$ in.^{4}

Brace
2L8×6×1 LLBB
$A_g = 26.2$ in.^{2} $\quad \bar{x} = 1.65$ in. (single angle)

Brace-to-Gusset Connection

The brace-to-gusset connection should be designed first so that a minimum required size of the gusset plate can be determined. In order to facilitate the connection design, a sketch of the connection should be drawn to scale (Figure 5-2). From this sketch, the important dimensions can be checked graphically (either manually or on a computer) at the same time as they are being calculated analytically.

Determine required number of bolts

The preliminary design uses ⅞-in.-diameter ASTM A325-X bolts. The calculations begin with the assumption that A325-X bolts will work. However, calculations later in this example will show that the beam-to-column connection requires ⅞-in.-diameter ASTM A490-X bolts. It is not advisable to use different grade bolts of the same diameter, so ASTM A490-X bolts are used for all ⅞-in.-diameter bolts as shown in Figure 5-1. For now, proceed with ASTM A325-X bolts.

From the AISC *Specification for Structural Steel Buildings* (AISC, 2010c), hereafter referred to as the AISC *Specification,* Section J3.6, the available shear strength (in double shear) and available tensile strength are determined using Equation J3-1 and Table J3.2, as follows:

LRFD	ASD
$\phi r_{nv} = \phi F_n A_b$ $= 2(0.75)(68 \text{ ksi})\left[\pi(\tfrac{7}{8}\text{ in.})^2/4\right]$ $= 61.3 \text{ kips}$ $\phi r_{nt} = \phi F_n A_b$ $= 0.75(90 \text{ ksi})\left[\pi(\tfrac{7}{8}\text{ in.})^2/4\right]$ $= 40.6 \text{ kips}$	$\dfrac{r_{nv}}{\Omega} = \dfrac{F_n A_b}{\Omega}$ $= \dfrac{2(68 \text{ ksi})\left[\pi(\tfrac{7}{8}\text{ in.})^2/4\right]}{2.00}$ $= 40.9 \text{ kips}$ $\dfrac{r_{nt}}{\Omega} = \dfrac{F_n A_b}{\Omega}$ $= \dfrac{(90 \text{ ksi})\left[\pi(\tfrac{7}{8}\text{ in.})^2/4\right]}{2.00}$ $= 27.1 \text{ kips}$

Alternatively, the available shear and tensile strengths can be determined directly from AISC *Manual* Tables 7-1 and 7-2. The minimum number of ⅞-in.-diameter ASTM A325-X bolts in double shear required to develop the required strength is:

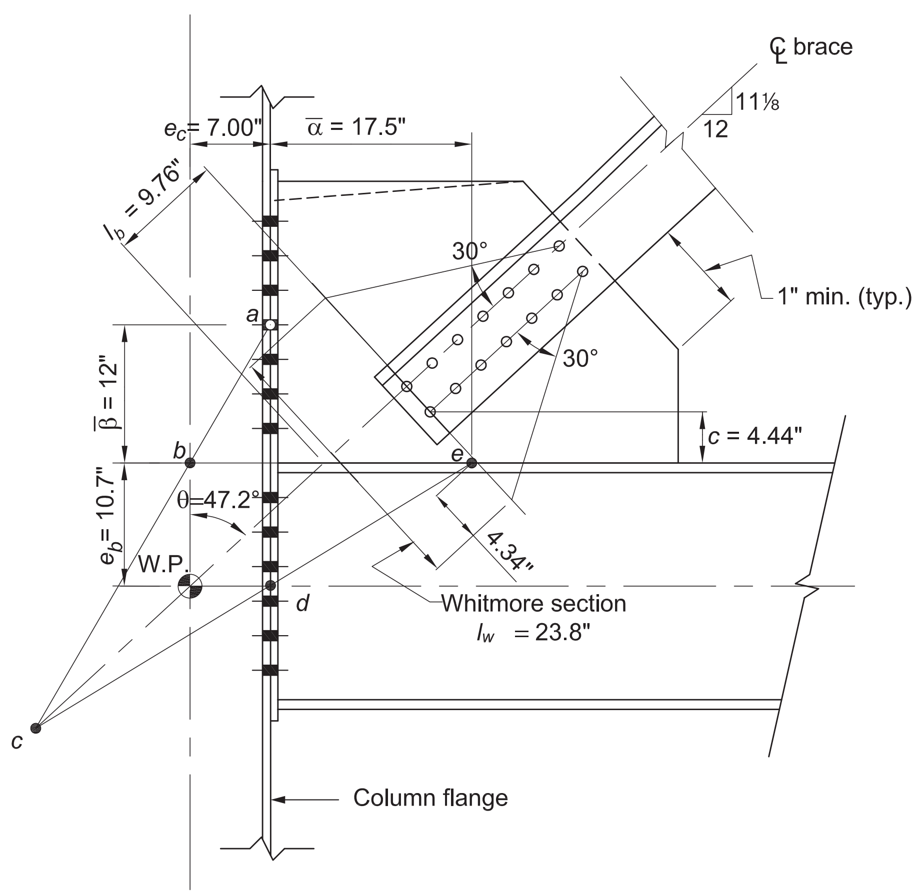


Fig. 5-2. Geometry for Example 5.1 calculations.

LRFD	ASD
$N_b = \dfrac{P_u}{\phi P_n}$ $= \dfrac{840\text{ kips}}{61.3\text{ kips/bolt}}$ $= 13.7\text{ bolts}$	$N_b = \dfrac{P_a}{\left(P_n/\Omega\right)}$ $= \dfrac{560\text{ kips}}{40.9\text{ kips/bolt}}$ $= 13.7\text{ bolts}$

Use two rows of seven bolts with 3-in. spacing, 3-in. pitch, and 1½-in. edge distance as shown in Figure 5-1.

Check tensile yielding on the brace gross section

From AISC *Specification* Section D2(a), use Equation D2-1 to determine the available tensile yielding strength of the double-angle brace:

LRFD	ASD
$\phi P_n = \phi F_y A_g$ $= 0.90\left(36\text{ ksi}\right)\left(26.2\text{ in.}^2\right)$ $= 849\text{ kips} > 840\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{\left(36\text{ ksi}\right)\left(26.2\text{ in.}^2\right)}{1.67}$ $= 565\text{ kips} > 560\text{ kips}$ **o.k.**

Alternatively, from AISC *Manual* Table 5-8, the available tensile yielding strength of the double-angle brace is:

LRFD	ASD
$\phi P_n = 849\text{ kips} > 840\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = 565\text{ kips} > 560\text{ kips}$ **o.k.**

Check tensile rupture on the brace net section

The net area of the double-angle brace is determined in accordance with AISC *Specification* Section B4.3, with the bolt hole diameter, d_h = $^{15}\!/_{16}$ in., from AISC *Specification* Table J3.3:

$$
\begin{aligned}
A_n &= A_g - 4t\left(d_h + {}^{1}\!/_{16}\text{ in.}\right) \\
&= 26.2\text{ in.}^2 - 4\left(1.00\text{ in.}\right)\left({}^{15}\!/_{16}\text{ in.} + {}^{1}\!/_{16}\text{ in.}\right) \\
&= 22.2\text{ in.}^2
\end{aligned}
$$

Because the outstanding legs of the double angle are not connected to the brace, an effective net area of the double angle needs to be determined. From AISC *Specification* Section D3 and Table D3.1, Case 2, the effective net area is:

$$A_e = A_n U \qquad (\textit{Spec}.\text{ Eq. D3-1})$$

where

$$U = 1 - \frac{\bar{x}}{l}$$

$$l = 6(3.00 \text{ in.})$$
$$= 18.0 \text{ in.}$$

$$U = 1 - \frac{1.65 \text{ in.}}{18.0 \text{ in.}}$$
$$= 0.908$$

$$A_e = (22.2 \text{ in.}^2)(0.908)$$
$$= 20.2 \text{ in.}^2$$

From AISC *Specification* Section D2(b), the available tensile rupture strength of the double-angle brace is:

LRFD	ASD
$\phi P_n = \phi F_u A_e$ $= 0.75(58 \text{ ksi})(20.2 \text{ in.}^2)$ $= 879 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega} = \frac{F_u A_e}{\Omega}$ $= \frac{(58 \text{ ksi})(20.2 \text{ in.}^2)}{2.00}$ $= 586 \text{ kips} > 560 \text{ kips}$ **o.k.**

Alternatively, because $A_e > 0.75A_g$, AISC *Manual* Table 5-8 could be used conservatively to determine the available tensile rupture strength. The calculated values provide a more precise solution, however.

Check block shear rupture on the brace

The block shear rupture failure path is assumed as shown in Figure 5-2a. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \qquad (Spec. \text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = 2(1.00 \text{ in.})[6(3.00 \text{ in.}) + 1.50 \text{ in.}]$$
$$= 39.0 \text{ in.}^2$$

$$0.60F_y A_{gv} = 0.60(36\text{ksi})(39.0 \text{ in.}^2)$$
$$= 842 \text{ kips}$$

Shear rupture component:

$$A_{nv} = 39.0 \text{ in.}^2 - 6.5(2)(1.00 \text{ in.})(1.00 \text{ in.})$$
$$= 26.0 \text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(58 \text{ ksi})(26.0 \text{ in.}^2)$$
$$= 905 \text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = 2(1.00 \text{ in.})[(3.00 \text{ in.} + 2.00 \text{ in.}) - 1.5(\tfrac{7}{8} \text{ in.} + \tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})]$$
$$= 7.00 \text{ in.}^2$$

$$U_{bs}F_u A_{nt} = 1(58 \text{ ksi})(7.00 \text{ in.}^2)$$
$$= 406 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_u A_{nv} + U_{bs}F_u A_{nt} = 905 \text{ kips} + 406 \text{ kips}$$
$$= 1{,}310 \text{ kips}$$

$$0.60F_y A_{gv} + U_{bs}F_u A_{nt} = 842 \text{ kips} + 406 \text{ kips}$$
$$= 1{,}250 \text{ kips}$$

Because 1,310 kips > 1,250 kips, use R_n = 1,250 kips.

LRFD	ASD
$\phi R_n = 0.75(1{,}250 \text{ kips})$ $= 938 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1{,}250 \text{ kips}}{2.00}$ $= 625 \text{ kips} > 560 \text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

Assume that the gusset plate is 1 in. thick and verify the assumption later. The block shear rupture failure path is assumed as shown in Figure 5-2b.

Shear yielding component:

$$A_{gv} = 2(1.00 \text{ in.})(19.5 \text{ in.})$$
$$= 39.0 \text{ in.}^2$$

$$0.60F_yA_{gv} = 0.60(50 \text{ ksi})\left(39.0 \text{ in.}^2\right)$$
$$= 1{,}170 \text{ kips}$$

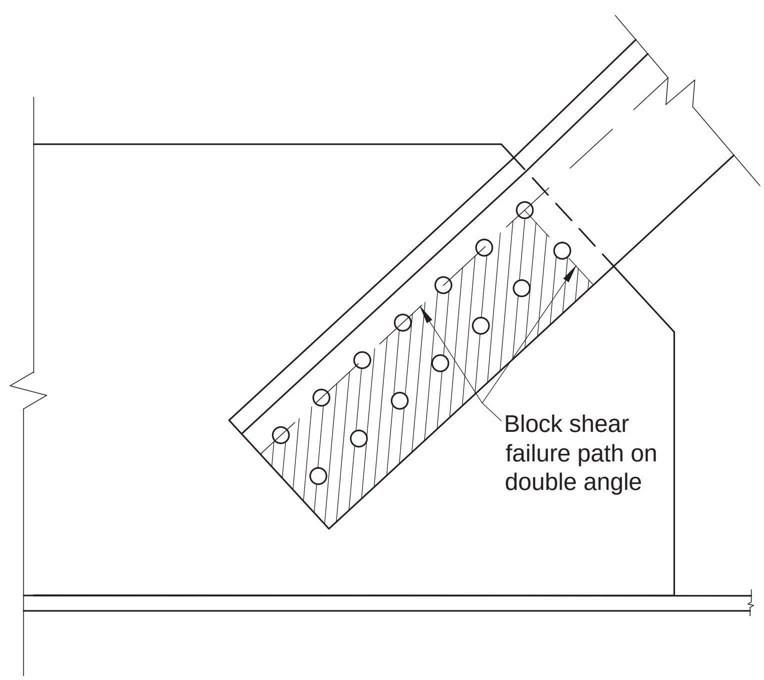


Fig. 5-2a. Block shear rupture failure path on double-angle brace.

Shear rupture component:

$$A_{nv} = 39.0 \text{ in.}^2 - 6.5(1.00 \text{ in.})(1.00 \text{ in.})(2)$$
$$= 26.0 \text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(65 \text{ ksi})\left(26.0 \text{ in.}^2\right)$$
$$= 1{,}010 \text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = (1.00 \text{ in.})(3.00 \text{ in.}) - 2(0.5)(1.00 \text{ in.})(1.00 \text{ in.})$$
$$= 2.00 \text{ in.}^2$$

$$U_{bs}F_u A_{nt} = 1(65 \text{ ksi})\left(2.00 \text{ in.}^2\right)$$
$$= 130 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_u A_{nv} + U_{bs}F_u A_{nt} = 1{,}010 \text{ kips} + 130 \text{ kips}$$
$$= 1{,}140 \text{ kips}$$

$$0.60F_y A_{gv} + U_{bs}F_u A_{nt} = 1{,}170 \text{ kips} + 130 \text{ kips}$$
$$= 1{,}300 \text{ kips}$$

Therefore, $R_n = 1{,}140$ kips.

From AISC *Specification* Section J4.3, the available block shear rupture strength is:

LRFD	ASD
$\phi R_n = 0.75(1{,}140 \text{ kips})$ $= 855 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1{,}140 \text{ kips}}{2.00}$ $= 570 \text{ kips} > 560 \text{ kips}$ **o.k.**

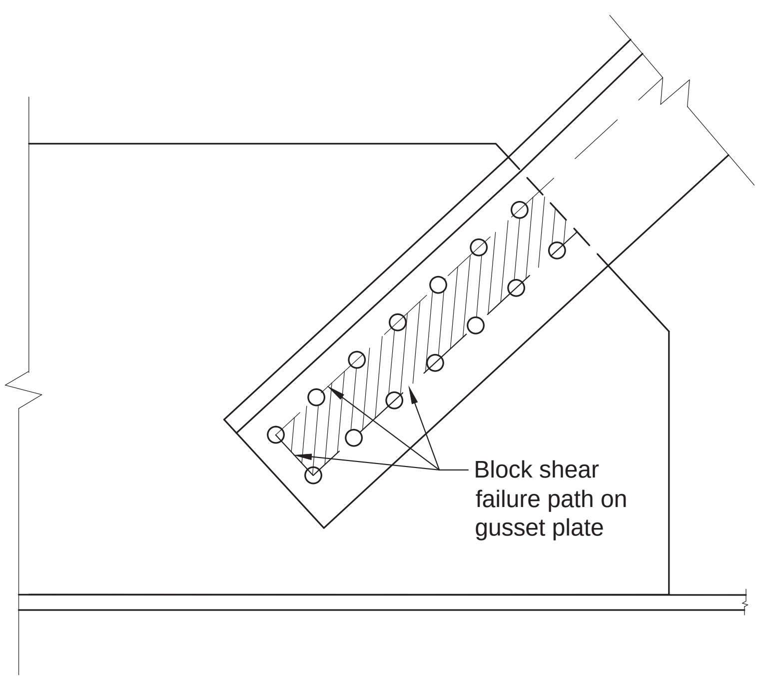


Fig. 5-2b. Block shear rupture failure path on gusset plate.

Check bolt bearing on the gusset plate

Bearing strength at bolt holes on the gusset plate will control over bearing strength at bolt holes on the brace because the brace has two angles for every bolt hole; therefore, only the gusset plate will be checked here. Assume the gusset plate is 1 in. thick. Standard size holes are used in the brace. From AISC *Specification* Table J3.3, for a ⅞-in.-diameter bolt, d_h = $^{15}\!/_{16}$ in.

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(\textit{Spec}. Eq. J3-6a)}$$

For the inner bolts, the clear distance is:

$l_c = 3.00 \text{ in.} - 1.0 d_h$
$= 3.00 \text{ in.} - 1.0(^{15}\!/_{16} \text{ in.})$
$= 2.06 \text{ in.}$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06 \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})$ $= 121 \text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(⅞ \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})$ $= 102 \text{ kips}$ Therefore, $\phi r_n = 102$ kips.	$1.2 l_c t F_u/\Omega = 1.2(2.06 \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})/2.00$ $= 80.3 \text{ kips}$ $2.4 d t F_u/\Omega = 2.4(⅞ \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})/2.00$ $= 68.3 \text{ kips}$ Therefore, $r_n/\Omega = 68.3$ kips.

Because the available bolt shear strength determined previously (61.3 kips for LRFD and 40.9 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the inner bolts.

For the end bolts:

$l_c = 1.50 \text{ in.} - 0.5 d_h$
$= 1.50 \text{ in.} - 0.5(^{15}\!/_{16} \text{ in.})$
$= 1.03 \text{ in.}$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.03 \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})$ $= 60.3 \text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(⅞ \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})$ $= 102 \text{ kips}$ Therefore, $\phi r_n = 60.3$ kips.	$1.2 l_c t F_u/\Omega = 1.2(1.03 \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})/2.00$ $= 40.2 \text{ kips}$ $2.4 d t F_u/\Omega = 2.4(⅞ \text{ in.})(1.00 \text{ in.})(65 \text{ ksi})/2.00$ $= 68.3 \text{ kips}$ Therefore, $r_n/\Omega = 40.2$ kips.

For the end bolts, the available bearing strength governs over the available bolt shear strength determined previously. To determine the available strength of the bolt group, sum the individual effective strengths for each bolt.

The total available strength of the bolt group is:

LRFD	ASD
$\phi R_n = (2 \text{ bolts})(60.3 \text{ kips}) + (12 \text{ bolts})(61.3 \text{ kips})$ $= 856 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (2 \text{ bolts})(40.2 \text{ kips}) + (12 \text{ bolts})(40.9 \text{ kips})$ $= 571 \text{ kips} > 560 \text{ kips}$ **o.k.**

Therefore, the 1-in.-thick gusset plate is o.k. Additional checks are required as follows.

Check the gusset plate for tensile yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section is:

$$l_w = 3.00 \text{ in.} + 2(18.0 \text{ in.}) \tan 30^\circ$$
$$= 23.8 \text{ in.}$$

Approximately 4.70 in. of this length runs into the beam web, as shown in Figure 5-2, which is thinner than the gusset. AISC *Manual* Part 9 states that the Whitmore section may spread across the joint between connected elements, which in this case includes the beam web. Thus, the effective area of the Whitmore section is:

$$A_w = (23.8 \text{ in.} - 4.70 \text{ in.})(1.00 \text{ in.}) + (4.70 \text{ in.})(0.515 \text{ in.})$$
$$= 21.5 \text{ in.}^2$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(50 \text{ ksi})(21.5 \text{ in.}^2)$ $= 968 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{(50 \text{ ksi})(21.5 \text{ in.}^2)}{1.67}$ $= 644 \text{ kips} > 560 \text{ kips}$ **o.k.**

Check the gusset plate for compression buckling on the Whitmore section

The available compressive strength of the gusset plate based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4, using an effective length factor, K, of 0.50 as established by full scale tests on bracing connections (Gross, 1990). Note that this K value requires the gusset to be supported on both edges. Alternatively, the effective length factor for gusset buckling could be determined according to Dowswell (2006). In this case, because KL/r is found to be less than 25 assuming $K = 0.50$, the same conclusion, that buckling does not govern, will be reached using either method.

$$r = \frac{t_g}{\sqrt{12}}$$
$$= \frac{1.00 \text{ in.}}{\sqrt{12}}$$
$$= 0.289 \text{ in.}$$

From Figure 5-2, the gusset plate unbraced length along the axis of the brace has been determined graphically to be 9.76 in.

$$\frac{KL}{r} = \frac{0.50(9.76 \text{ in.})}{0.289 \text{ in.}}$$
$$= 16.9$$

Because $\dfrac{KL}{r} < 25$, AISC *Specification* Equation J4-6 is applicable, and the available compressive strength is:

LRFD	ASD
$\phi P_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(20.9 \text{ in.}^2)$ $= 941 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50 \text{ ksi})(20.9 \text{ in.}^2)}{1.67}$ $= 626 \text{ kips} > 560 \text{ kips}$ **o.k.**

This completes the brace-to-gusset connection calculations and the gusset plate width and depth size can be determined as shown in Figure 5-1. Squaring off the gusset on top shows that seven rows of bolts will fit.

Connection Interface Forces

The forces at the gusset-to-beam and gusset-to-column interfaces are determined using the general case of the UFM as discussed in AISC *Manual* Part 13 and this Design Guide:

$$\begin{aligned} e_b &= \frac{d_b}{2} \\ &= \frac{21.4 \text{ in.}}{2} \\ &= 10.7 \text{ in.} \\ e_c &= \frac{d_c}{2} \\ &= \frac{14.0 \text{ in.}}{2} \\ &= 7.00 \text{ in.} \end{aligned}$$

From Figure 5-2:

$$\begin{aligned} \tan\theta &= \frac{12}{11\frac{1}{8}} \\ \theta &= 47.2^\circ \end{aligned}$$

Choose $\overline{\beta} = 12$ in. Then, the constraint $\alpha - \beta\tan\theta = e_b\tan\theta - e_c$ from AISC *Manual* Part 13, with $\beta = \overline{\beta}$, yields:

$$\begin{aligned} \alpha &= (10.7 \text{ in.} + 12.0 \text{ in.})(1.08) - 7.00 \text{ in.} \\ &= 17.5 \text{ in.} \end{aligned}$$

This can also be done graphically as shown in Figure 5-2, with $\beta = \overline{\beta} = 12$ in. Start at point *a* and construct a line through *b* to intersect the brace line at *c*. From *c*, construct a line through point *d* until the top flange of the beam is intersected at point *e*. The distance from point *e* to the face of the column flange is α. The points *b*, *c* and *d*, are known as the "control points" of the uniform force method.

Arrange the horizontal edge of the gusset so that $\alpha = \overline{\alpha}$ and from Figure 5-1:

$$\overline{\alpha} = \frac{l_h + \frac{3}{4} \text{ in.}}{2} + t_p$$

where t_p is the yet to be determined end-plate thickness, ¾ in. is the gusset corner clip thickness, and l_h is the horizontal gusset length.

Assume $t_p = 1$ in. and solving for l_h:

$$\begin{aligned} l_h &= 2(17.5 \text{ in.}) - 2(1.00 \text{ in.}) - \tfrac{3}{4} \text{ in.} \\ &= 32.3 \text{ in.} \end{aligned}$$

So, the tentative gusset plate size is 1 in. × 24½ in. × 2 ft 8¼ in. The top edge of the gusset could be shaped as shown by the broken line to reduce the excessive (but acceptable) edge distance. Proceeding with $\alpha = \overline{\alpha} = 17.5$ in. and $\beta = \overline{\beta} = 12$ in., there will be no couples on any connection interface. This is the general case of the UFM.

Using the previously determined variables and guidelines from AISC *Manual* Part 13:

$$\begin{aligned} r &= \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \\ &= \sqrt{(17.5\text{ in.} + 7.00\text{ in.})^2 + (12.0\text{ in.} + 10.7\text{ in.})^2} \\ &= 33.4\text{ in.} \end{aligned}$$ (*Manual* Eq. 13-6)

The required shear force at the gusset-to-column connection and required normal force at the gusset-to-beam connection are determined as follows:

$$V_c = \frac{\beta}{r} P$$ (*Manual* Eq. 13-2)

$$V_b = \frac{e_b}{r} P$$ (*Manual* Eq. 13-4)

LRFD	ASD
$V_{uc} = \frac{\beta}{r} P_u$ $= \left(\frac{12.0\text{ in.}}{33.4\text{ in.}}\right)(840\text{ kips})$ $= 302\text{ kips}$ $V_{ub} = \frac{e_b}{r} P_u$ $= \left(\frac{10.7\text{ in.}}{33.4\text{ in.}}\right)(840\text{ kips})$ $= 269\text{ kips}$	$V_{ac} = \frac{\beta}{r} P_a$ $= \left(\frac{12.0\text{ in.}}{33.4\text{ in.}}\right)(560\text{ kips})$ $= 201\text{ kips}$ $V_{ab} = \frac{e_b}{r} P_a$ $= (10.7\text{ in.})\left(\frac{560\text{ kips}}{33.4\text{ in.}}\right)$ $= 179\text{ kips}$

Verify that the sum of the vertical gusset forces equals the vertical component of the brace force:

LRFD	ASD
$\Sigma(V_{uc} + V_{ub}) = 302\text{ kips} + 269\text{ kips}$ $= 571\text{ kips}$ $P_u(\cos\theta) = (840\text{ kips})(\cos 47.2°)$ $= 571\text{ kips}$ **o.k.**	$\Sigma(V_{ac} + V_{ab}) = 201\text{ kips} + 179\text{ kips}$ $= 380\text{ kips}$ $P_a(\cos\theta) = (560\text{ kips})(\cos 47.2°)$ $= 380\text{ kips}$ **o.k.**

The required normal force at the gusset-to-column connection and required shear force at the gusset-to-beam connection are determined as follows:

$$H_c = \frac{e_c}{r} P$$ (*Manual* Eq. 13-3)

$$H_b = \frac{\alpha}{r} P$$ (*Manual* Eq. 13-5)

LRFD	ASD
$H_{uc} = \frac{e_c}{r} P_u$ $= \left(\frac{7.00 \text{ in.}}{33.4 \text{ in.}}\right)(840 \text{ kips})$ $= 176 \text{ kips}$ $H_{ub} = \frac{\alpha}{r} P_u$ $= \left(\frac{17.5 \text{ in.}}{33.4 \text{ in.}}\right)(840 \text{ kips})$ $= 440 \text{ kips}$	$H_{ac} = \frac{e_c}{r} P_a$ $= \left(\frac{7.00 \text{ in.}}{33.4 \text{ in.}}\right)(560 \text{ kips})$ $= 117 \text{ kips}$ $H_{ab} = \frac{\alpha}{r} P_a$ $= \left(\frac{17.5 \text{ in.}}{33.4 \text{ in.}}\right)(560 \text{ kips})$ $= 293 \text{ kips}$

The total horizontal force at the brace-to-gusset connection is:

LRFD	ASD
$\Sigma(H_{uc} + H_{ub}) = 176 \text{ kips} + 440 \text{ kips}$ $= 616 \text{ kips}$ Check that the sum of the horizontal gusset forces equals the brace horizontal component $\Sigma(H_{uc} + H_{ub}) = (840 \text{ kips})(\sin 47.2°)$ $= 616 \text{ kips}$ **o.k.**	$\Sigma(H_{ac} + H_{ab}) = 117 \text{ kips} + 293 \text{ kips}$ $= 410 \text{ kips}$ Check that the sum of the horizontal gusset forces equals the brace horizontal component $\Sigma(H_{ac} + H_{ab}) = (560 \text{ kips})(\sin 47.2°)$ $= 411 \text{ kips}$ **o.k.**

Figures 5-3a and 5-3b show the free body diagram (admissible force field) determined by the UFM.

Gusset-to-Beam Connection

Using the same symbols that were used in determining the UFM forces, the required strengths and weld length at the gusset-to-beam interface are:

LRFD	ASD
Required shear strength, $H_{ub} = 440$ kips Required normal strength, $V_{ub} = 269$ kips Length of weld, $l = 32\frac{1}{4}$ in. $-$ $\frac{3}{4}$ in. $= 31.5$ in.	Required shear strength, $H_{ab} = 293$ kips Required normal strength, $V_{ab} = 179$ kips Length of weld, $l = 32\frac{1}{4}$ in. $-$ $\frac{3}{4}$ in. $= 31.5$ in.

Check gusset plate for shear yielding and tensile yielding along the beam flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1, as follows:

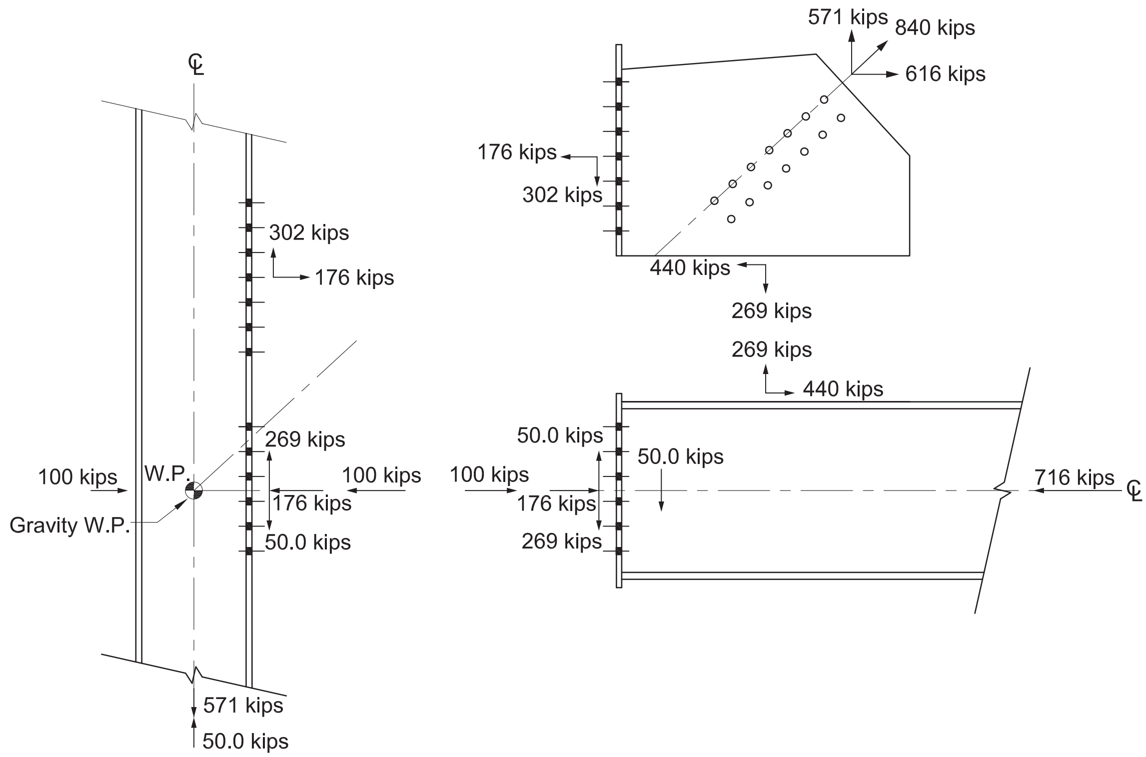


Fig. 5-3a. Admissible force field for Example 5.1—LRFD.

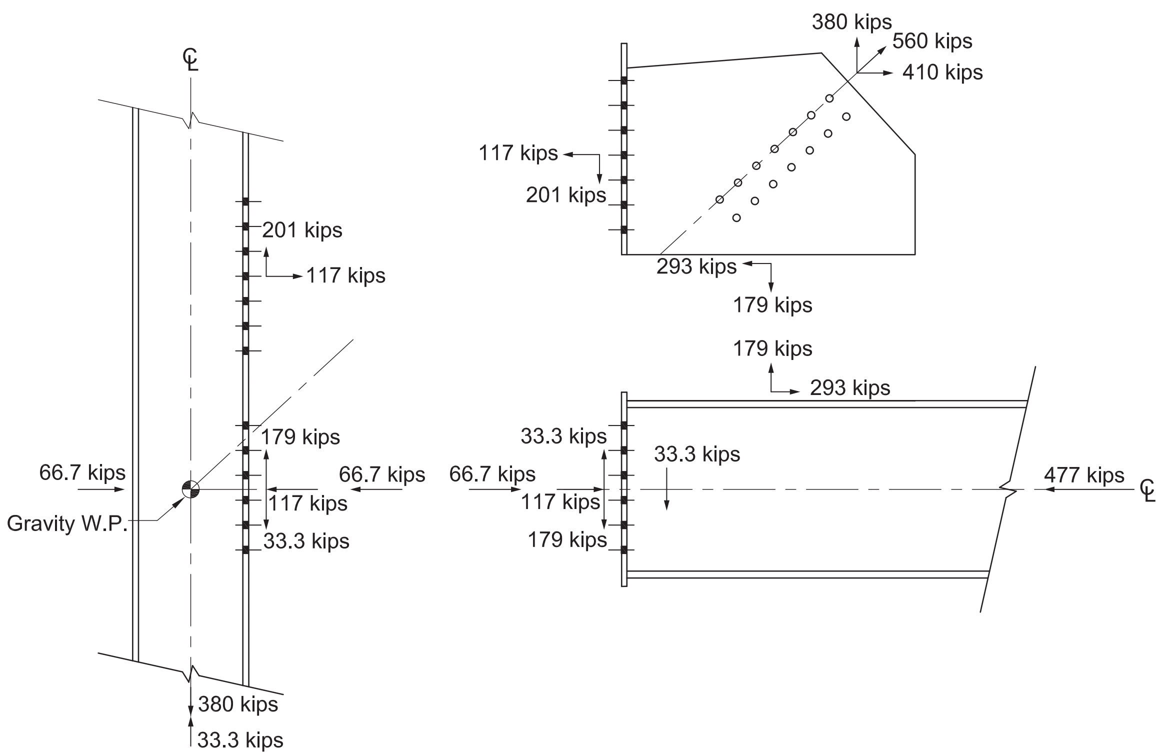


Fig. 5-3b. Admissible force field for Example 5.1—ASD.

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(1.00 \text{ in.})(31.5 \text{ in.})$ $= 945 \text{ kips} > 440 \text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(1.00 \text{ in.})(31.5 \text{ in.})$ $= 1{,}420 \text{ kips} > 269 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(1.00 \text{ in.})(31.5 \text{ in.})}{1.50}$ $= 630 \text{ kips} > 293 \text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50 \text{ ksi})(1.00 \text{ in.})(31.5 \text{ in.})}{1.67}$ $= 943 \text{ kips} > 179 \text{ kips}$ **o.k.**

Consider force interaction for gusset plate

It is usually suggested that the von Mises yield criterion be used for checking interaction; however, in its underlying theory the von Mises criterion requires three stresses at a point and only two stresses (normal and shear) are available. A better choice of interaction check is the equation derived from plasticity theory (Neal, 1977) and suggested by Astaneh-Asl (1998) as:

LRFD	ASD
$\left(\dfrac{M_{ub}}{\phi M_n}\right) + \left(\dfrac{V_{ub}}{\phi N_n}\right)^2 + \left(\dfrac{H_{ub}}{\phi V_n}\right)^4 \le 1$ From AISC *Specification* Equation F2-1 $\phi M_n = \phi M_p$ $= 0.90 F_y Z_x$ $= 0.90 F_y \left(\dfrac{t_g l^2}{4}\right)$ $= 0.90(50 \text{ ksi})\left[\dfrac{(1.00 \text{ in.})(31.5 \text{ in.})^2}{4}\right]$ $= 11{,}200 \text{ kip-in.}$ Incorporating the previously determined values $\left(\dfrac{0 \text{ kip-in.}}{11{,}200 \text{ kip-in.}}\right) + \left(\dfrac{269 \text{ kips}}{1{,}420 \text{ kips}}\right)^2$ $+ \left(\dfrac{440 \text{ kips}}{945 \text{ kips}}\right)^4 = 0.0829 < 1.0$ **o.k.**	$\left[\dfrac{M_{ab}}{(M_n/\Omega)}\right] + \left[\dfrac{V_{ab}}{(N_n/\Omega)}\right]^2 + \left[\dfrac{H_{ab}}{(V_n/\Omega)}\right]^4 \le 1$ From AISC *Specification* Equation F2-1 $\dfrac{M_n}{\Omega} = \dfrac{M_p}{\Omega}$ $= \dfrac{F_y Z_x}{\Omega}$ $= \left(\dfrac{F_y}{1.67}\right)\left(\dfrac{t_g l^2}{4}\right)$ $= \left(\dfrac{50 \text{ ksi}}{1.67}\right)\left[\dfrac{(1.00 \text{ in.})(31.5 \text{ in.})^2}{4}\right]$ $= 7{,}430 \text{ kip-in.}$ Incorporating the previously determined values $\left(\dfrac{0 \text{ kip-in.}}{7{,}430 \text{ kip-in.}}\right) + \left(\dfrac{179 \text{ kips}}{943 \text{ kips}}\right)^2$ $+ \left(\dfrac{293 \text{ kips}}{630 \text{ kips}}\right)^4 = 0.0828 < 1.0$ **o.k.**

Therefore, the 1-in.-thick gusset plate is adequate.

Note that interaction of the forces at the gusset-to-beam interface is generally not a concern in gusset design, as can be seen from this result. The AISC *Specification* and *Manual* generally do not require that this interaction be checked.

Design weld at gusset-to-beam flange connection

The forces involved are:

$V_{ub} = 269$ kips and $V_{ab} = 179$ kips
$H_{ub} = 440$ kips and $H_{ab} = 293$ kips

The UFM postulates uniform forces on each connection interface. For this to be possible, sufficient ductility must be present in the system. As discussed in Section 1.2, most connection limit states have some ductility, or force redistribution can be accomplished either by support or element flexibility or by increasing the strength of nonductile elements to allow redistribution to occur without premature local fracture. To allow for redistribution of stress in welded gusset connections, the weld is designed for a stress of f_{peak} or $1.25f_{avg}$, as defined in the following. The 1.25 factor is the weld ductility factor discussed in the AISC *Manual* Part 13 (Hewitt and Thornton, 2004), which is actually an overstrength factor, and is intended to allow redistribution to occur before fracture of nonductile elements.

The required shear force and normal force per linear inch of weld on the gusset-to-beam flange interface are:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{l} = \dfrac{269 \text{ kips}}{31.5 \text{ in.}} = 8.54 \text{ kip/in.}$	$f_{aa} = \dfrac{V_{ab}}{l} = \dfrac{179 \text{ kips}}{31.5 \text{ in.}} = 5.68 \text{ kip/in.}$
$f_{uv} = \dfrac{H_{ub}}{l} = \dfrac{440 \text{ kips}}{31.5 \text{ in.}} = 14.0 \text{ kip/in.}$	$f_{av} = \dfrac{H_{ab}}{l} = \dfrac{293 \text{ kips}}{31.5 \text{ in.}} = 9.30 \text{ kip/in.}$

Use a vector sum (square root of the sum of the squares) to combine the shear, axial and bending stresses on the gusset-to-beam interface. The peak and average stresses are:

$$f_{peak} = \sqrt{\left(f_a + f_b\right)^2 + f_v^2}$$

$$f_{avg} = \frac{1}{2}\left[\sqrt{\left(f_a - f_b\right)^2 + f_v^2} + \sqrt{\left(f_a + f_b\right)^2 + f_v^2}\right]$$

Because f_b, the bending stress, is zero in this case, the average stress on the gusset-to-beam flange interface is:

LRFD	ASD
$f_{u\,avg} = f_{u\,peak} = \sqrt{\left(8.54 \text{ kip/in.}\right)^2 + \left(14.0 \text{ kip/in.}\right)^2} = 16.4 \text{ kip/in.}$	$f_{a\,avg} = f_{a\,peak} = \sqrt{\left(5.68 \text{ kip/in.}\right)^2 + \left(9.30 \text{ kip/in.}\right)^2} = 10.9 \text{ kip/in.}$
The resultant load angle is:	The resultant load angle is:
$\theta = \tan^{-1}\left(\dfrac{8.54 \text{ kip/in.}}{14.0 \text{ kip/in.}}\right) = 31.4°$	$\theta = \tan^{-1}\left(\dfrac{5.68 \text{ kip/in.}}{9.30 \text{ kip/in.}}\right) = 31.4°$

According to AISC *Manual* Part 13, because the gusset is directly welded to the beam, the weld is designed for the larger of the peak stress and 1.25 times the average stress:

LRFD	ASD
$f_{u\,weld} = \max\left(1.25 f_{u\,avg}, f_{u\,peak}\right)$ $= 1.25(16.4\text{ kip/in.})$ $= 20.5\text{ kip/in.}$	$f_{a\,weld} = \max\left(1.25 f_{a\,avg}, f_{a\,peak}\right)$ $= 1.25(10.9\text{ kip/in.})$ $= 13.6\text{ kip/in.}$

The strength of fillet welds defined in AISC *Specification* Section J2.4 can be simplified, as explained in AISC *Manual* Part 8, to AISC *Manual* Equations 8-2a and 8-2b. Also, incorporating the increased strength due to the load angle from AISC *Specification* Equation J2-5, the required weld size is:

LRFD	ASD
$D = \dfrac{f_{u\,weld}}{2(1.392\text{ kip/in.})\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{20.5\text{ kip/in.}}{2(1.392\text{ kip/in.})\left(1.0+0.50\sin^{1.5}31.4^\circ\right)}$ $= 6.20\text{ sixteenths}$	$D = \dfrac{f_{a\,weld}}{2(0.928\text{ kip/in.})\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{13.6\text{ kip/in.}}{2(0.928\text{ kip/in.})\left(1.0+0.50\sin^{1.5}31.4^\circ\right)}$ $= 6.17\text{ sixteenths}$

From AISC *Specification* Table J2.4, the minimum fillet weld required is 5⁄16 in., which does not control in this case. Therefore, use a two-sided 7⁄16-in. fillet weld to connect the gusset plate to the beam.

Check beam web local yielding

The normal force is applied at 16.8 in. away from the beam end (assuming a ¾-in.-thick end plate), which is less than the beam depth of 21.4 in. Therefore, use AISC *Specification* Equation J10-3 as follows:

LRFD	ASD
$\phi R_n = \phi F_{yw} t_w (2.5k + l_b)$ $= 1.00(50\text{ ksi})(0.515\text{ in.})\left[2.5(1.34\text{ in.}) + 31.5\text{ in.}\right]$ $= 897\text{ kips} > 269\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_{yw} t_w \left(2.5k + l_b\right)}{\Omega}$ $= \dfrac{(50\text{ ksi})(0.515\text{ in.})\left[2.5(1.34\text{ in.})+31.5\text{ in.}\right]}{1.50}$ $= 598\text{ kips} > 179\text{ kips}$ **o.k.**

Alternatively, AISC *Manual* Table 9-4 may be used to determine the available strength of the beam due to web local yielding. When the compressive force is applied at a distance less than the beam depth, use AISC *Manual* Equation 9-45 as follows:

LRFD	ASD
$\phi R_n = \phi R_1 + l_b(\phi R_2)$ $= 86.3\text{ kips} + (31.5\text{ in.})(25.8\text{ kip/in.})$ $= 899\text{ kips} > 269\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{R_1}{\Omega} + l_b\left(\dfrac{R_2}{\Omega}\right)$ $= 57.5\text{ kips} + (31.5\text{ in.})(17.2\text{ kip/in.})$ $= 599\text{ kips} > 179\text{ kips}$ **o.k.**

The differences in the calculated strength versus the table values are due to rounding.

Check equivalent normal force, N_e

There is no moment on the gusset-to-beam connection, so there is no additional normal force due to the moment. If there were a moment on the gusset-to-beam connection, the following calculation could be used to determine the equivalent normal force:

LRFD	ASD
$N_{ue} = V_{ub} + \dfrac{2M_{ub}}{\left(\dfrac{l}{2}\right)}$ $= 269 \text{ kips} + \dfrac{2(0 \text{ kip-in.})}{\left(\dfrac{31.5 \text{ in.}}{2}\right)}$ $= 269 \text{ kips}$	$N_{ae} = V_{ab} + \dfrac{2M_{ab}}{\left(\dfrac{l}{2}\right)}$ $= 179 \text{ kips} + \dfrac{2(0 \text{ kip-in.})}{\left(\dfrac{31.5 \text{ in.}}{2}\right)}$ $= 179 \text{ kips}$

Check beam web local crippling

The normal force is applied at 16.8 in. away from the beam end (assuming a ¾-in.-thick end plate), which is greater than $d/2$. Therefore, AISC *Specification* Equation J10-4 is applicable:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $\phi R_n = 0.75(0.80)(0.515 \text{ in.})^2\left[1 + 3\left(\dfrac{31.5 \text{ in.}}{21.4 \text{ in.}}\right)\left(\dfrac{0.515 \text{ in.}}{0.835 \text{ in.}}\right)^{1.5}\right] \times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.835 \text{ in.})}{0.515 \text{ in.}}}$ $= 766 \text{ kips} > 269 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}}{\Omega}$ $\dfrac{R_n}{\Omega} = \dfrac{\left[(0.80)(0.515 \text{ in.})^2\left[1 + 3\left(\dfrac{31.5 \text{ in.}}{21.4 \text{ in.}}\right)\left(\dfrac{0.515 \text{ in.}}{0.835 \text{ in.}}\right)^{1.5}\right] \times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.835 \text{ in.})}{0.515 \text{ in.}}}\right]}{2.00}$ $= 511 \text{ kips} > 179 \text{ kips}$ **o.k.**

Alternatively, AISC *Manual* Table 9-4 and Equation 9-49 (for $x \geq d/2$) may be used to determine the available strength of the beam for web local crippling:

LRFD	ASD
$\phi R_n = 2\left[(\phi R_3) + l_b(\phi R_4)\right]$ $= 2\left[122 \text{ kips} + (31.5 \text{ in.})(8.28 \text{ kip/in.})\right]$ $= 766 \text{ kips} > 269 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\left[(R_3/\Omega) + l_b(R_4/\Omega)\right]$ $= 2\left[81.3 \text{ kips} + (31.5 \text{ in.})(5.52 \text{ kip/in.})\right]$ $= 510 \text{ kips} > 179 \text{ kips}$ **o.k.**

The differences in the calculated strength versus the table values are due to rounding.

Gusset-to-Column Connection

The required strengths at the gusset-to-column interface are:

LRFD	ASD
Required normal strength, $H_{uc} = 176$ kips Required shear strength, $V_{uc} = 302$ kips	Required axial strength, $H_{ac} = 117$ kips Required shear strength, $V_{ac} = 201$ kips

Use an end-plate connection between the gusset plate and the column flange and the beam and the column flange, where the end plate is continuous.

Design bolts at gusset-to-column connection

Use ⅞-in.-diameter ASTM A325-X bolts in standard holes for preliminary analysis. If this bolt type proves to be insufficient, use ⅞-in.-diameter ASTM A490-X bolts with standard holes. Note that if A490 ⅞-in.-diameter bolts are used for this part of the connection, they should be used everywhere in this connection and on this job. It is not good practice to use different grades of the same size bolts on a specific job.

For preliminary design, assume two rows, seven bolts per row, in the gusset-to-column portion of the connection.

The available shear and tensile strengths per bolt, from AISC *Manual* Tables 7-1 and 7-2, and the required shear and tensile strengths per bolt are:

LRFD	ASD
$\phi r_{nv} = 30.7$ kips $\phi r_{nt} = 40.6$ kips Required shear strength per bolt $r_{uv} = \dfrac{302 \text{ kips}}{14 \text{ bolts}}$ $= 21.6$ kips/bolt < 30.7 kips **o.k.** Required tensile strength per bolt $r_{ut} = \dfrac{176 \text{ kips}}{14} = 12.6 \text{ kips} < 40.6 \text{ kips}$ **o.k.**	$\dfrac{r_{nv}}{\Omega} = 20.4$ kips $\dfrac{r_{nt}}{\Omega} = 27.1$ kips Required shear strength per bolt $r_{av} = \dfrac{201 \text{ kips}}{14 \text{ bolts}}$ $= 14.4$ kips/bolt < 20.4 kips **o.k.** Required tensile strength per bolt $r_{at} = \dfrac{117 \text{ kips}}{14} = 8.36 \text{ kips} < 27.1 \text{ kips}$ **o.k.**

The tensile strength requires an additional check due to the combination of tension and shear. From AISC *Specification* Section J3.7, the available tensile strength of the bolts subject to combined tension and shear is:

LRFD	ASD
Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3a $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ Note that the AISC *Specification* Commentary Section J3 has an alternative elliptical formula, Equation C-J3-8a, that can be used.	Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3b $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ Note that the AISC *Specification* Commentary Section J3 has an alternative elliptical formula, Equation C-J3-8b, that can be used.

LRFD	ASD
From AISC *Specification* Table J3.2 $F_{nt} = 90$ ksi $F_{nv} = 68$ ksi $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(68 \text{ ksi})}\left(\dfrac{21.6 \text{ kips}}{0.601 \text{ in.}^2}\right)$ $= 53.6 \text{ ksi} < 90 \text{ ksi}$ **o.k.** The reduced available tensile strength per bolt due to combined forces is: $\phi r_{nt} = \phi F'_{nt} A_b$ $= 0.75(53.6 \text{ kips})(0.601 \text{ in.}^2)$ $= 24.2 \text{ kips} > 12.6 \text{ kips}$ **o.k.**	From AISC *Specification* Table J3.2 $F_{nt} = 90$ ksi $F_{nv} = 68$ ksi $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{68 \text{ ksi}}\left(\dfrac{14.4 \text{ kips}}{0.601 \text{ in.}^2}\right)$ $= 53.6 \text{ ksi} < 90 \text{ ksi}$ **o.k.** The reduced available tensile strength per bolt due to combined forces is: $\dfrac{r_{nt}}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ $= \dfrac{(53.6 \text{ kips})(0.601 \text{ in.}^2)}{2.00}$ $= 16.1 \text{ kips} > 8.36 \text{ kips}$ **o.k.**

A 1-in.-end-plate thickness, t_p, has been assumed. The gage of the bolts is taken as 5½ in., which is the standard gage for a W14×90 and will work with a 1-in.-thick gusset plate and assuming ½-in. fillet welds. From Figure 5-4a, the clear distance from the center of the bolt hole to the toe of the fillet is 1¾ in. From AISC *Manual* Table 7-15, the entering clearance for aligned ⅞-in.-diameter ASTM A325 and A490 bolts is ⅞ in. < 1¾ in. The next step is to determine the fillet weld size of the end plate-to-gusset plate connection to ensure that the 5½-in. gage will work.

Design gusset-to-end plate weld

The resultant load, R, and its angle with the vertical, θ, are:

LRFD	ASD
$R_u = \sqrt{(176 \text{ kips})^2 + (302 \text{ kips})^2}$ $= 350$ kips $\theta = \tan^{-1}\left(\dfrac{176 \text{ kips}}{302 \text{ kips}}\right)$ $= 30.2°$	$R_a = \sqrt{(117 \text{ kips})^2 + (201 \text{ kips})^2}$ $= 233$ kips $\theta = \tan^{-1}\left(\dfrac{117 \text{ kips}}{201 \text{ kips}}\right)$ $= 30.2°$

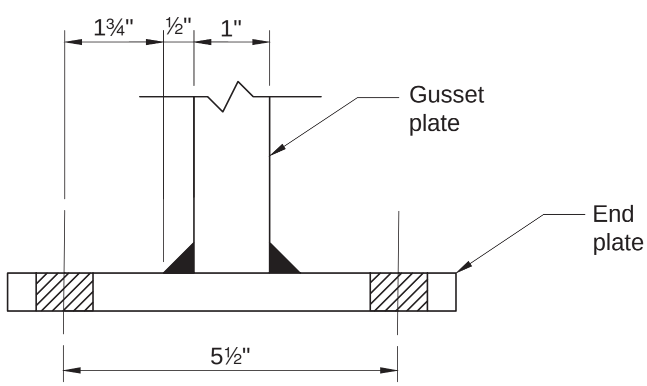


Fig. 5-4a. Washer clearance for end-plate bolts.

The weld size is determined from AISC *Manual* Equation 8-2, including the increase in strength allowed by AISC *Specification* Section J2.4 when the angle of loading is not along the weld longitudinal axis. In the following, only the weld length tributary to the group of bolts is used; therefore, $l = 7(3.0 \text{ in.}) = 21.0 \text{ in.}$

LRFD	ASD
Rearranging AISC *Manual* Equation 8-2a, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $D_{req'd} = \dfrac{350 \text{ kips}}{2(1.392 \text{ kip/in.})(21.0 \text{ in.})(1.0+0.50\sin^{1.5} 30.2°)} = 5.08$	Rearranging AISC *Manual* Equation 8-2b, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $D_{req'd} = \dfrac{233 \text{ kips}}{2(0.928 \text{ kip/in.})(21.0 \text{ in.})(1.0+0.50\sin^{1.5} 30.2°)} = 5.07$

Therefore, use a two-sided ⅜-in. fillet weld at the gusset-to-end plate connection. Note that this exceeds the minimum size fillet weld of 5⁄16 in. required by AISC *Specification* Table J2.4. Note that in the equation for $D_{req'd}$, no ductility factor has been used. This is because the flexibility of the end plate will allow nonuniform gusset edge forces to be redistributed. If the edge force is high at one point, the flexible end plate will deform and shed load to less highly loaded locations. Since the required fillet weld of ⅜-in. is less than the assumed ½-in. fillet weld in Figure 5-4a, the assumed 5½-in. gage can be used.

Check gusset plate tensile and shear yielding at the gusset-to-end-plate interface

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(1.00 \text{ in.})(23.8 \text{ in.})$ $= 714 \text{ kips} > 302 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(1.00 \text{ in.})(23.8 \text{ in.})}{1.50}$ $= 476 \text{ kips} > 201 \text{ kips}$ **o.k.**
$\phi N_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(1.00 \text{ in.})(23.8 \text{ in.})$ $= 1{,}070 \text{ kips} > 176 \text{ kips}$ **o.k.**	$\dfrac{N_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50 \text{ ksi})(1.00 \text{ in.})(23.8 \text{ in.})}{1.67}$ $= 713 \text{ kips} > 117 \text{ kips}$ **o.k.**

Check prying action on bolts at the end plate

Using AISC *Manual* Part 9, determine the effect of prying action on the end-plate connection. The end plate is initially assumed to be 1 in. thick × 10 in. wide. The final end plate thickness may be different.

From AISC *Manual* Part 9 and Figure 5-4a:

$$b = \frac{5\tfrac{1}{2} \text{ in.} - 1.00 \text{ in.}}{2} = 2.25 \text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$

$$= 2.25 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2} = 1.81 \text{ in.}$$

$$a = \frac{10.0\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 2.25\text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= \left(2.25\text{ in.} + \frac{\tfrac{7}{8}\text{ in.}}{2}\right) \le \left[1.25(2.25\text{ in.}) + \frac{\tfrac{7}{8}\text{ in.}}{2}\right]$$
$$= 2.69\text{ in.} < 3.25\text{ in.} \qquad \textbf{o.k.}$$

From AISC *Specification* Table J3.3 for a ⅞-in.-diameter bolt, the standard hole dimension is $^{15}\!/_{16}$ in.

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{1.81\text{ in.}}{2.69\text{ in.}}$$
$$= 0.673$$

p = pitch of bolts
= 3 in.

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-24)}$$
$$= 1 - \frac{^{15}\!/_{16}\text{ in.}}{3.00\text{ in.}}$$
$$= 0.688$$

From AISC *Manual* Part 9, the available tensile strength, including prying action effects, is:

LRFD	ASD
From AISC *Manual* Equation 9-30a, with $B = \phi r_{nt}$ = 24.2 kips previously determined: $$t_c = \sqrt{\frac{4Bb'}{\phi p F_u}}$$ $$= \sqrt{\frac{4(24.2\text{ kips})(1.81\text{ in.})}{0.90(3.00\text{ in.})(65\text{ ksi})}}$$ $$= 0.999\text{ in.}$$ $$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ $$= \frac{1}{0.688(1+0.673)}\left[\left(\frac{0.999\text{ in.}}{1.00\text{ in.}}\right)^2 - 1\right]$$ $$= 0.00$$	From AISC *Manual* Equation 9-30b, with $B = r_{nt}/\Omega$ = 16.1 kips previously determined: $$t_c = \sqrt{\frac{\Omega 4Bb'}{pF_u}}$$ $$= \sqrt{\frac{1.67(4)(16.1\text{ kips})(1.81\text{ in.})}{(3.00\text{ in.})(65\text{ ksi})}}$$ $$= 0.999\text{ in.}$$ $$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ $$= \frac{1}{0.688(1+0.673)}\left[\left(\frac{0.999\text{ in.}}{1.00\text{ in.}}\right)^2 - 1\right]$$ $$= 0.00$$

LRFD	ASD
Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33: $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta\alpha') \\ &= \left(\frac{1.00\text{ in.}}{0.999\text{ in.}}\right)^2 \left[1+0.688(0.00)\right] \\ &= 1.00 \end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned} T_{avail} &= BQ \\ &= (24.2\text{ kips})(1.00) \\ &= 24.2\text{ kips} > 12.6\text{ kips} \quad \textbf{o.k.} \end{aligned}$$	Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33: $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta\alpha') \\ &= \left(\frac{1.00\text{ in.}}{0.999\text{ in.}}\right)^2 \left[1+0.688(0.00)\right] \\ &= 1.00 \end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned} T_{avail} &= BQ \\ &= (16.1\text{ kips})(1.00) \\ &= 16.1\text{ kips} > 8.36\text{ kips} \quad \textbf{o.k.} \end{aligned}$$

Physically, note that t_c is the plate thickness required to develop the bolt strength, B, with no prying action, i.e., $q = 0$. It is, therefore, the maximum effective plate thickness. A thicker plate would not increase the connection's strength. From this we can see that the 1-in.-thick end plate is unnecessarily thick. A thinner plate will allow some prying force to develop, resulting in double curvature in the plate. Try a ⅝-in.-thick end plate.

From AISC *Manual* Part 9, the available bolt tensile strength, including the effects of prying action, is:

LRFD	ASD
$$\begin{aligned} \alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\ &= \frac{1}{0.688(1+0.673)}\left[\left(\frac{0.999\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right] \\ &= 1.35 \end{aligned}$$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34 $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta) \\ &= \left(\frac{5/8\text{ in.}}{0.999\text{ in.}}\right)^2 (1+0.688) \\ &= 0.661 \\ T_{avail} &= BQ \\ &= (24.2\text{ kips})(0.661) \\ &= 16.0\text{ kips} > 12.6\text{ kips} \quad \textbf{o.k.} \end{aligned}$$	$$\begin{aligned} \alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\ &= \frac{1}{0.688(1+0.673)}\left[\left(\frac{0.999\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right] \\ &= 1.35 \end{aligned}$$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34 $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta) \\ &= \left(\frac{5/8\text{ in.}}{0.999\text{ in.}}\right)^2 (1+0.688) \\ &= 0.661 \\ T_{avail} &= BQ \\ &= (16.1\text{ kips})(0.661) \\ &= 10.6\text{ kips} > 8.36\text{ kips} \quad \textbf{o.k.} \end{aligned}$$

Use a ⅝-in.-thick end plate. Now that an end-plate thickness is known, check the end plate for the remaining limit states. Note that the beam-to-column portion of the end-plate connection may require a thicker end plate.

Check bolt bearing at bolt holes on end plate

Check the bearing strength at the top bolts, assuming the optional cut on the gusset plate occurs and the end plate edge distance correspondingly decreases to 1¾ in., as the available bearing strength will have the lowest value at that location. This edge distance is reflected in Figure 5-4b. The clear distance, based on this edge distance is:

$$\begin{aligned} l_c &= 1\tfrac{3}{4} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.}) \\ &= 1.28 \text{ in.} \end{aligned}$$

Assuming that deformation at the bolt hole is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(}Spec.\text{ Eq. J3-6a)}$$

The available strength is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.28 \text{ in.})(\tfrac{5}{8} \text{ in.})(65 \text{ ksi})$ $= 46.8$ kips/bolt $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(65 \text{ ksi})$ $= 64.0$ kips/bolt Therefore, $\phi r_n = 46.8$ kips/bolt	$1.2 l_c t F_u/\Omega = 1.2(1.28 \text{ in.})(\tfrac{5}{8} \text{ in.})(65 \text{ ksi})/2.00$ $= 31.2$ kips/bolt $2.4 d t F_u/\Omega = 2.4(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(65 \text{ ksi})/2.00$ $= 42.7$ kips/bolt Therefore, $r_n/\Omega = 31.2$ kips/bolt

Because 46.8 kips/bolt > 30.7 kips/bolt, LRFD (31.2 kips/bolt > 20.4 kips/bolt, ASD), bolt shear controls over bearing strength at bolt holes, and this check will not govern.

Check block shear rupture of the end plate

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(}Spec.\text{ Eq. J4-5)}$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= (18.0 \text{ in.} + 1\tfrac{3}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 12.3 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_y A_{gv} &= 0.60(50 \text{ ksi})\left(12.3 \text{ in.}^2\right) \\ &= 369 \text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 12.3 \text{ in.}^2 - 6.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 8.24 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_u A_{nv} &= 0.60(65 \text{ ksi})\left(8.24 \text{ in.}^2\right) \\ &= 321 \text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{nt} &= \left[2.25 \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.}) \\ &= 1.09 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs} F_u A_{nt} &= 1(65 \text{ ksi})\left(1.09 \text{ in.}^2\right) \\ &= 70.9 \text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned}0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 321 \text{ kips} + 70.9 \text{ kips}\\ &= 392 \text{ kips}\end{aligned}$$

$$\begin{aligned}0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 369 \text{ kips} + 70.9 \text{ kips}\\ &= 440 \text{ kips}\end{aligned}$$

Therefore, $R_n = 392$ kips.

LRFD	ASD
$\begin{aligned}\phi R_n &= 0.75(392 \text{ kips})(2)\\ &= 588 \text{ kips} > 302 \text{ kips} \quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= \frac{(392 \text{ kips})(2)}{2.00}\\ &= 392 \text{ kips} > 201 \text{ kips} \quad \textbf{o.k.}\end{aligned}$

Check prying action on column flange

From AISC *Manual* Part 9:

$$\begin{aligned}b &= \frac{5\tfrac{1}{2} \text{ in.} - 0.440 \text{ in.}}{2}\\ &= 2.53 \text{ in.}\end{aligned}$$

$$\begin{aligned}b' &= b - \frac{d_b}{2} &&\text{(\textit{Manual} Eq. 9-21)}\\ &= 2.53 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}\\ &= 2.09 \text{ in.}\end{aligned}$$

$$\begin{aligned}a &= \frac{14.5 \text{ in.} - 5.50 \text{ in.}}{2}\\ &= 4.50 \text{ in.}\end{aligned}$$

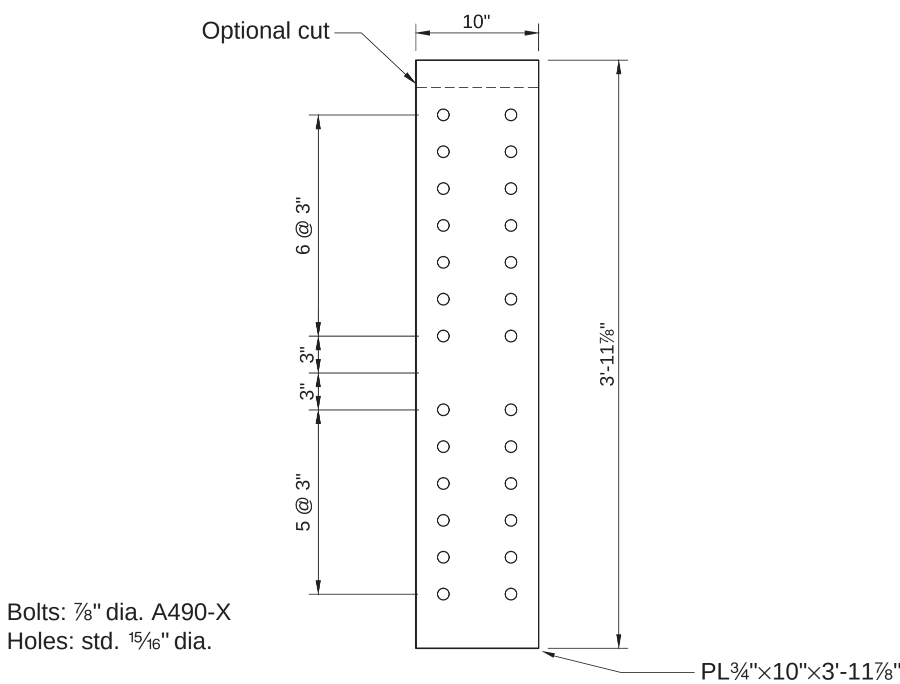


Fig. 5-4b. End-plate geometry.

The fulcrum point for prying is controlled by the end plate with a = 2.25 in., so use this value for a.

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual}\ \text{Eq. 9-27})$$

$$= 2.25\text{ in.} + \frac{\tfrac{7}{8}\text{ in.}}{2} \le \left[1.25(2.53\text{ in.}) + \frac{\tfrac{7}{8}\text{ in.}}{2}\right]$$

$$= 2.69\text{ in.} < 3.60\text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad (\textit{Manual}\ \text{Eq. 9-26})$$

$$= \frac{2.09\text{ in.}}{2.69\text{ in.}}$$

$$= 0.777$$

$$p = \text{pitch of bolts}$$

$$= 3.00\text{ in.}$$

Using p = 3.00 in. assumes that the column flange is cut above and below the bolt group, which is conservative.

$$\delta = 1 - \frac{d'}{p} \qquad (\textit{Manual}\ \text{Eq. 9-24})$$

$$= 1 - \frac{\tfrac{15}{16}\text{ in.}}{3.00\text{ in.}}$$

$$= 0.688$$

From AISC *Manual* Part 9, the available tensile strength including prying action effects is determined as follows:

LRFD	ASD
From AISC *Manual* Equation 9-30a: $t_c = \sqrt{\dfrac{4Bb'}{\phi p F_u}}$ $= \sqrt{\dfrac{4(24.2\text{ kips})(2.09\text{ in.})}{0.90(3.00\text{ in.})(65\text{ ksi})}}$ $= 1.07\text{ in.}$ $\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.688(1+0.777)}\left[\left(\dfrac{1.07\text{ in.}}{0.710\text{ in.}}\right)^2 - 1\right]$ $= 1.04$	From AISC *Manual* Equation 9-30b: $t_c = \sqrt{\dfrac{\Omega 4Bb'}{pF_u}}$ $= \sqrt{\dfrac{1.67(4)(16.1\text{ kips})(2.09\text{ in.})}{(3.00\text{ in.})(65\text{ ksi})}}$ $= 1.07\text{ in.}$ $\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.688(1+0.777)}\left[\left(\dfrac{1.07\text{ in.}}{0.710\text{ in.}}\right)^2 - 1\right]$ $= 1.04$

LRFD	ASD
Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta) \\ &= \left(\frac{0.710\text{ in.}}{1.07\text{ in.}}\right)^2 (1+0.688) \\ &= 0.743 \end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned} T_{avail} &= BQ \\ &= (24.2\text{ kips})(0.743) \\ &= 18.0\text{ kips} > 12.6\text{ kips} \quad \textbf{o.k.} \end{aligned}$$	Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $$\begin{aligned} Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta) \\ &= \left(\frac{0.710\text{ in.}}{1.07\text{ in.}}\right)^2 (1+0.688) \\ &= 0.743 \end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned} T_{avail} &= BQ \\ &= (16.1\text{ kips})(0.743) \\ &= 12.0\text{ kips} > 8.36\text{ kips} \quad \textbf{o.k.} \end{aligned}$$

If this method were to fail, a more realistic model is given in Tamboli (2010). The effective tributary length of column flange for a continuous column is:

$$p_{eff} = \frac{(n-1)p + \pi\bar{b} + 2\bar{a}}{n}$$

$$\begin{aligned}
n &= \text{number of rows of bolts} \\
&= 7 \\
p &= \text{bolt pitch} \\
&= 3.00\text{ in.} \\
\bar{b} &= b \\
&= 2.53\text{ in.} \\
\bar{a} &= \frac{b_f - gage}{2} \\
&= \frac{14.5\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2} \\
&= 4.50\text{ in.} \\
p_{eff} &= \frac{(7-1)(3.00\text{ in.}) + \pi(2.53\text{ in.}) + 2(4.50\text{ in.})}{7} \\
&= 4.99\text{ in.}
\end{aligned}$$

Using p_{eff} in place of p in the AISC *Manual* prying equations and following the procedure from AISC *Manual* Part 9:

$b' = 2.09$ in.

$\bar{a} = 4.50\text{ in.} > a = 2.25\text{ in.}$

$a' = 2.69$ in.

Use the smaller of $\bar{a}$ and a.

$\rho = 0.777$

$$\begin{aligned}
\delta &= 1 - \frac{d_h}{p} \\
&= 1 - \frac{\tfrac{15}{16}\text{ in.}}{4.99\text{ in.}} \\
&= 0.812
\end{aligned}$$

LRFD	ASD
$t_c = \sqrt{\dfrac{4Bb'}{\phi p F_u}}$ $= \sqrt{\dfrac{4(24.2\text{ kips})(2.09\text{ in.})}{0.90(4.99\text{ in.})(65\text{ ksi})}}$ $= 0.832\text{ in.}$ $\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.812(1+0.777)}\left[\left(\dfrac{0.832\text{ in.}}{0.710\text{ in.}}\right)^2 - 1\right]$ $= 0.259$ Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33 $Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$ $= \left(\dfrac{0.710}{0.832}\right)^2 \left[1+(0.812)(0.259)\right]$ $= 0.881$ $T_{avail} = BQ$ $= (24.2\text{ kips})(0.881)$ $= 21.3\text{ kips} > 12.6\text{ kips}$ **o.k.**	$t_c = \sqrt{\dfrac{4\Omega Bb'}{pF_u}}$ $= \sqrt{\dfrac{4(1.67)(16.1\text{ kips})(2.09\text{ in.})}{(4.99\text{ in.})(65\text{ ksi})}}$ $= 0.832\text{ in.}$ $\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.812(1+0.777)}\left[\left(\dfrac{0.832\text{ in.}}{0.710\text{ in.}}\right)^2 - 1\right]$ $= 0.259$ Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33 $Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$ $= \left(\dfrac{0.710\text{ in.}}{0.832\text{ in.}}\right)^2 \left[1+(0.812)(0.259)\right]$ $= 0.881$ $T_{avail} = BQ$ $= (16.1\text{ kips})(0.881)$ $= 14.2\text{ kips} > 8.36\text{ kips}$ **o.k.**

By this method the calculated available strength has increased by:

LRFD	ASD
$\dfrac{(21.3\text{ kips} - 18.0\text{ kips})}{18.0\text{ kips}} \times 100\% = 18.3\%$	$\dfrac{(14.2\text{ kips} - 12.0\text{ kips})}{12.0\text{ kips}} \times 100\% = 18.3\%$

Thus, this method is considerably less conservative than the “cut column” model.

Check bearing on column flange

Because $t_f = 0.710$ in. > $t_p = ⅝$ in., bolt bearing on the column flange will not control the design, by inspection.

Beam-to-Column Connection

This section addresses the design of the portion of the end plate that connects the beam to the column. The required shear strength is equal to V_{ub} (LRFD) or V_{ab} (ASD), based on the brace force, plus the beam reaction from the Given section of 50 kips (LRFD) or 33.3 kips (ASD). The V_{ub} and V_{ab} force is reversible as the brace force goes from tension to compression, but the gravity beam shear always remains in the same direction. Therefore, V_{ub} (or V_{ab}) and the reaction should always be added even if shown in opposite directions as in Figure 5-3.

The axial force in the beam shown in Figure 5-3 is affected by the transfer force, A_{ub}, of 100 kips (LRFD) or A_{ab} of 66.7 kips (ASD). When the braces are in tension, the transfer force is compression as shown in Figure 5-1, and will increase the required brace tensile strength of the next lower brace to:

LRFD	ASD
$\dfrac{716\text{ kips}}{\sin 47.2°} = 976\text{ kips}$	$\dfrac{477\text{ kips}}{\sin 47.2°} = 650\text{ kips}$

Figure 5-3 shows that the axial force between the beam and column from equilibrium would be $A_{ub} + H_{uc}$ for LRFD and $A_{ab} + H_{ac}$ for ASD, but it has been shown that frame action, as discussed in Section 4.2.6, will reduce the force H_{uc} for LRFD or H_{ac} for ASD. Section 4.2.6 provides the following formula to estimate the moment in the beam due to the effect of the admissible distortional forces from Tamboli (2010):

$$M_D = 6\left(\frac{P}{Abc}\right)\left(\frac{I_b I_c}{\dfrac{I_b}{b} + \dfrac{2I_c}{c}}\right)\left(\frac{b^2 + c^2}{bc}\right) \tag{4-12}$$

This formula is valid for bracing arrangements such as those shown in Figure 2-1, i.e., those involving one beam and two columns. Other arrangements would involve a different formula. For cantilever situations, gravity forces rather than lateral forces will probably dominate and $A_{ub} + H_{uc}$ for LRFD or $A_{ab} + H_{ac}$ for ASD should be used.

As given, the required strength of the brace is:

LRFD	ASD
$P_u = 840\text{ kips}$	$P_a = 560\text{ kips}$

The values of b and c to be used in the Tamboli equation are:

$$\begin{aligned} b &= \frac{l_b}{2} \\ &= \frac{(25.0\text{ ft})(12.0\text{ in./1.00 ft})}{2} \\ &= 150\text{ in.} \end{aligned}$$

$$\begin{aligned} c &= \frac{l_c}{2} \\ &= \frac{(25.0\text{ ft})\left(\dfrac{11\tfrac{1}{8}\text{ in.}}{12.0\text{ in.}}\right)\left(\dfrac{12.0\text{ in.}}{1.00\text{ ft}}\right)}{2} \\ &= 139\text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{I_b}{b} &= \frac{1{,}830\text{ in.}^4}{150\text{ in.}} \\ &= 12.2\text{ in.}^3 \end{aligned}$$

$$\begin{aligned} \frac{2I_c}{c} &= \frac{2\left(999\text{ in.}^4\right)}{139\text{ in.}} \\ &= 14.4\text{ in.}^3 \end{aligned}$$

The required flexural strength of the beam is:

LRFD	ASD
$M_{uD} = 6\left[\dfrac{840\text{ kips}}{(26.2\text{ in.}^2)(150\text{ in.})(139\text{ in.})}\right] \times \left[\dfrac{(1{,}830\text{ in.}^4)(999\text{ in.}^4)}{12.2\text{ in.}^3 + 14.4\text{ in.}^3}\right]\left[\dfrac{(150\text{ in.})^2 + (139\text{ in.})^2}{(150\text{ in.})(139\text{ in.})}\right]$ $= 1{,}270\text{ kip-in.}$ Using previously determined variables, $\bar{\beta}$ and e_b: $H_{uD} = \dfrac{M_{uD}}{\bar{\beta} + e_b}$ $= \dfrac{1{,}270\text{ kip-in.}}{12.0\text{ in.} + 10.7\text{ in.}}$ $= 55.9\text{ kips}$	$M_{aD} = 6\left(\dfrac{560\text{ kips}}{(26.2\text{ in.}^2)(150\text{ in.})(139\text{ in.})}\right) \times \left[\dfrac{(1{,}830\text{ in.}^4)(999\text{ in.}^4)}{12.2\text{ in.}^3 + 14.4\text{ in.}^3}\right]\left[\dfrac{(150\text{ in.})^2 + (139\text{ in.})^2}{(150\text{ in.})(139\text{ in.})}\right]$ $= 848\text{ kip-in.}$ Using previously determined variables, $\bar{\beta}$ and e_b: $H_{aD} = \dfrac{M_{aD}}{\bar{\beta} + e_b}$ $= \dfrac{848\text{ kip-in.}}{12.0\text{ in.} + 10.7\text{ in.}}$ $= 37.4\text{ kips}$

If this value of H_D is used, the axial force between the beam and column will be:

LRFD	ASD
$H_u = 176\text{ kips} - 55.9\text{ kips} + 100\text{ kips}$ $= 220\text{ kips}$	$H_a = 117\text{ kips} - 37.4\text{ kips} + 66.7\text{ kips}$ $= 146\text{ kips}$

A recent study (Fortney and Thornton, 2014) has shown that the average ratio of $(H_c - H_d + A)/(H_c + A)$ is about 70%, but the standard deviation of about 0.33 was too large to make a recommendation for design. This study also shows that using max (H_c, A) as the axial force may not be justified. This Design Guide will use the justifiable axial force of $H_c - H_d + A$ for the following calculations:

LRFD	ASD
Required Shear Strength $V_u = V_{ub} + \text{Reaction}$ $= 269\text{ kips} + 50\text{ kips}$ $= 319\text{ kips}$ Required Axial Strength $T_u = 176\text{ kips} - 55.9\text{ kips} + 100\text{ kips}$ $= 220\text{ kips}$	Required Shear Strength $V_a = V_{ab} + \text{Reaction}$ $= 179\text{ kips} + 33.3\text{ kips}$ $= 212\text{ kips}$ Required Axial Strength $T_a = 117\text{ kips} - 37.3\text{ kips} + 66.7\text{ kips}$ $= 146\text{ kips}$

Design bolts at beam-to-column connection

For the connection geometry, see Figure 5-1. Try (12) ⅞-in.-diameter ASTM A325-X bolts in the end-plate connection of the beam to the column.

As determined previously, the available shear strength of a ⅞-in.-diameter ASTM A325-X bolt is 30.7 kips for LRFD and 20.4 kips for ASD. The required shear and tensile bolt strengths are:

LRFD	ASD
$r_{uv} = \dfrac{319\text{ kips}}{12\text{ bolts}}$ $= 26.6\text{ kips/bolt} < 30.7\text{ kips}$ **o.k.** $r_{ut} = \dfrac{220\text{ kips}}{12\text{ bolts}}$ $= 18.3\text{ kips/bolt}$	$r_{av} = \dfrac{212\text{ kips}}{12\text{ bolts}}$ $= 17.7\text{ kips/bolt} < 20.4\text{ kips}$ **o.k.** $r_{at} = \dfrac{146\text{ kips}}{12\text{ bolts}}$ $= 12.2\text{ kips/bolt}$

The available tensile strength considers the effects of combined tension and shear. From AISC *Specification* Section J3.7, the available tensile strength of the bolts subject to combined tension and shear is:

LRFD	ASD
Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3a: $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ From AISC *Specification* Table J3.2: $F_{nt} = 90\text{ ksi}$ $F_{nv} = 68\text{ ksi}$ $F'_{nt} = 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(68\text{ ksi})}\left(\dfrac{26.6\text{ kips}}{0.601\text{ in.}^2}\right)$ $= 38.9\text{ ksi} < 90\text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\phi r_{nt} = \phi F'_{nt} A_b$ $= 0.75(38.9\text{ kips})(0.601\text{ in.}^2)$ $= 17.5\text{ kips} < 18.3\text{ kips}$ **n.g.**	Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3b: $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ From AISC *Specification* Table J3.2: $F_{nt} = 90\text{ ksi}$ $F_{nv} = 68\text{ ksi}$ $F'_{nt} = 1.3(90\text{ ksi}) - \dfrac{2.00(90\text{ ksi})}{68\text{ ksi}}\left(\dfrac{17.7\text{ kips}}{0.601\text{ in.}^2}\right)$ $= 39.0\text{ ksi} < 90\text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\dfrac{r_{nt}}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ $= \dfrac{(38.9\text{ kips})(0.601\text{ in.}^2)}{2.00}$ $= 11.7\text{ kips} < 12.2\text{ kips}$ **n.g.**

Try ⅞-in.-diameter ASTM A490-X bolts.

LRFD	ASD
From AISC *Specification* Table J3.2: $F_{nt} = 113\text{ ksi}$ $F_{nv} = 84\text{ ksi}$ $F'_{nt} = 1.3(113\text{ ksi}) - \dfrac{113\text{ ksi}}{0.75(84\text{ ksi})}\left(\dfrac{26.6\text{ kips}}{0.601\text{ in.}^2}\right)$ $= 67.5\text{ ksi} < 113\text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\phi r_{nt} = \phi F'_{nt} A_b$ $= 0.75(67.5\text{ kips})(0.601\text{ in.}^2)$ $= 30.4\text{ kips} > 18.3\text{ kips}$ **o.k.**	From AISC *Specification* Table J3.2: $F_{nt} = 113\text{ ksi}$ $F_{nv} = 84\text{ ksi}$ $F'_{nt} = 1.3(113\text{ ksi}) - \dfrac{2.00(113\text{ ksi})}{84\text{ ksi}}\left(\dfrac{17.7\text{ kips}}{0.601\text{ in.}^2}\right)$ $= 67.7\text{ ksi} < 113\text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\dfrac{r_{nt}}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ $= \dfrac{(67.7\text{ kips})(0.601\text{ in.}^2)}{2.00}$ $= 20.3\text{ kips} > 12.2\text{ kips}$ **o.k.**

Because A490-X bolts are required for the beam-to-column connection, they will be used everywhere in the connection. Using A325 and A490 bolts of the same diameter in any one connection, or even within the same project, is not recommended.

Design beam web-to-end plate weld

Using V and T, which were previously determined, the resultant load, R, and its angle with the vertical, θ, are:

LRFD	ASD
$R_u = \sqrt{V_u^2 + T_u^2}$ $= \sqrt{(220\text{ kips})^2 + (319\text{ kips})^2}$ $= 388\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{220\text{ kips}}{319\text{ kips}}\right)$ $= 34.6°$	$R_a = \sqrt{V_a^2 + T_a^2}$ $= \sqrt{(146\text{ kips})^2 + (212\text{ kips})^2}$ $= 257\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{146\text{ kips}}{212\text{ kips}}\right)$ $= 34.6°$

The weld size is determined from AISC *Manual* Equation 8-2, including the increase in strength allowed by AISC *Specification* Section J2.4 when the angle of loading is not along the weld longitudinal axis. In the following, only the weld length tributary to the group of bolts is used; therefore, $l = 6(3.00\text{ in.}) = 18.0\text{ in.}$

LRFD	ASD
Rearranging AISC *Manual* Equation 8-2a, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $D_{req'd} = \dfrac{388\text{ kips}}{2(1.392\text{ kip/in.})(18.0\text{ in.})(1.0 + 0.50\sin^{1.5} 34.6°)}$ $= 6.38$	Rearranging AISC *Manual* Equation 8-2b, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $D_{req'd} = \dfrac{257\text{ kips}}{2(0.928\text{ kip/in.})(18.0\text{ in.})(1.0 + 0.50\sin^{1.5} 34.6°)}$ $= 6.34$

Therefore, use a two-sided 7⁄16-in. fillet weld to connect the beam web to the end plate. Note that this exceeds the minimum size fillet weld of ¼ in. required by AISC *Specification* Table J2.4.

Check the 5½-in. gage with 7⁄16-in. fillet welds

From AISC *Manual* Table 7-15, the required clearance for ⅞-in.-diameter bolts with circular washers is:

$$C_{3,req'd} = \tfrac{7}{8} \text{ in.}$$

$$\text{Clearance} = \frac{5\tfrac{1}{2} \text{ in.}}{2} - \frac{0.515 \text{ in.}}{2} - \tfrac{7}{16} \text{ in.}$$
$$= 2\tfrac{1}{16} \text{ in.} > \tfrac{7}{8} \text{ in.} \quad \textbf{o.k.}$$

Therefore, the 5½-in. gage is acceptable with 7⁄16-in. fillet welds.

Check prying action on bolts and end plate

Using AISC *Manual* Part 9, determine the effect of prying action on the end-plate connection. The end plate is assumed to be ¾ in. thick × 10 in. wide:

$$b = \frac{5\tfrac{1}{2} \text{ in.} - 0.515 \text{ in.}}{2}$$
$$= 2.49 \text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$
$$= 2.49 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.05 \text{ in.}$$

$$a = \frac{10.0 \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2}$$
$$= 2.25 \text{ in.} < 1.25b \quad \textbf{o.k.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= \left(2.25 + \frac{\tfrac{7}{8} \text{ in.}}{2}\right) \le \left[1.25(2.49) + \frac{\tfrac{7}{8} \text{ in.}}{2}\right]$$
$$= 2.69 \text{ in.} < 3.55 \text{ in.}$$

From AISC *Specification* Table J3.3 for a ⅞-in.-diameter bolt, the standard hole dimension is 15⁄16 in.

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{2.05 \text{ in.}}{2.69 \text{ in.}}$$
$$= 0.762$$

$$p = \text{pitch of bolts}$$
$$= 3.00 \text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$
$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.688$$

From the AISC *Manual* Part 9, the available tensile strength including prying action effects is:

LRFD	ASD
From AISC *Manual* Equation 9-30a:	From AISC *Manual* Equation 9-30b:
$t_c = \sqrt{\dfrac{4Bb'}{\phi pF_u}}$	$t_c = \sqrt{\dfrac{\Omega 4Bb'}{pF_u}}$
$= \sqrt{\dfrac{4(30.4\text{ kips})(2.05\text{ in.})}{0.90(3.00\text{ in.})(65\text{ ksi})}}$	$= \sqrt{\dfrac{1.67(4)(20.3\text{ kips})(2.05\text{ in.})}{(3.00\text{ in.})(65\text{ ksi})}}$
$= 1.19\text{ in.}$	$= 1.19\text{ in.}$
From AISC *Manual* Equation 9-35:	From AISC *Manual* Equation 9-35:
$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$	$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$
$= \dfrac{1}{0.688(1+0.762)}\left[\left(\dfrac{1.19\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right]$	$= \dfrac{1}{0.688(1+0.762)}\left[\left(\dfrac{1.19\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right]$
$= 2.17$	$= 2.17$
Because $\alpha' > 1$, $\alpha' = 1.00$	Because $\alpha' > 1$, $\alpha' = 1.00$
Determine Q from AISC *Manual* Equation 9-33:	Determine Q from AISC *Manual* Equation 9-33:
$Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$	$Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$
$= \left(\dfrac{5/8\text{ in.}}{1.19\text{ in.}}\right)^2 [1+(0.688)(1.00)]$	$= \left(\dfrac{5/8\text{ in.}}{1.19}\right)^2 [1+(0.688)(1.00)]$
$= 0.466$	$= 0.466$
From AISC *Manual* Equation 9-31:	From AISC *Manual* Equation 9-31:
$T_{avail} = BQ$	$T_{avail} = BQ$
$= (30.4\text{ kips})(0.466)$	$= (20.3\text{ kips})(0.466)$
$= 14.2\text{ kips} < 18.3\text{ kips}$ **n.g.**	$= 9.46\text{ kips} < 12.2\text{ kips}$ **n.g.**

The connection has failed. Since $\alpha' > 1$, it is known that the ⅝-in.-thick end plate contributes to the failure.

Try an end plate of increased thickness, t_p = ¾ in.

LRFD	ASD
$$\alpha' = \frac{1}{0.688(1+0.762)}\left[\left(\frac{1.19\text{ in.}}{\tfrac{3}{4}\text{ in.}}\right)^2 - 1\right]$$ $$= 1.25$$ Because $\alpha' > 1$, use $\alpha' = 1.00$ Determine Q from AISC *Manual* Equation 9-33: $$Q = \left(\frac{\tfrac{3}{4}\text{ in.}}{1.19\text{ in.}}\right)^2 \left[1+(0.688)(1.00)\right]$$ $$= 0.671$$ $$T_{avail} = 0.671(30.4\text{ kips})$$ $$= 20.4\text{ kips} > 18.3\text{ kips} \quad \textbf{o.k.}$$	$$\alpha' = \frac{1}{0.688(1+0.762)}\left[\left(\frac{1.19\text{ in.}}{\tfrac{3}{4}\text{ in.}}\right)^2 - 1\right]$$ $$= 1.25$$ Because $\alpha' > 1$, use $\alpha' = 1.00$ Determine Q from AISC *Manual* Equation 9-33: $$Q = \left(\frac{\tfrac{3}{4}\text{ in.}}{1.19\text{ in.}}\right)^2 \left[1+(0.688)(1.00)\right]$$ $$= 0.671$$ $$T_{avail} = 0.671(20.3\text{ kips})$$ $$= 13.6\text{ kips} > 12.2\text{ kips} \quad \textbf{o.k.}$$

Use the ¾-in.-thick end plate.

Check prying action on column flange

From the calculations for the gusset-to-column connection with ⅞-in.-diameter bolts:

$b' = 2.09$ in.
$a' = 2.69$ in.
$\rho = 0.777$
$p = 3.00$ in.
$\delta = 0.688$

LRFD	ASD
$$t_c = \sqrt{\frac{4(30.4\text{ kips})(2.09\text{ in.})}{0.90(3.00\text{ in.})(65\text{ ksi})}}$$ $$= 1.20\text{ in.}$$ $$\alpha' = \frac{1}{0.688(1+0.777)}\left[\left(\frac{1.20}{0.710}\right)^2 - 1\right]$$ $$= 1.52,\ \text{use } \alpha' = 1.00$$ $$Q = \left(\frac{0.710}{1.20}\right)^2 \left[1+0.688(1.00)\right]$$ $$= 0.591$$ $$T_{avail} = 0.591(30.4\text{ kips})$$ $$= 18.0\text{ kips} < 18.3\text{ kips} \quad \textbf{n.g.}$$	$$t_c = \sqrt{\frac{4(1.67)(20.3\text{ kips})(2.09\text{ in.})}{(3.00\text{ in.})(65\text{ ksi})}}$$ $$= 1.21\text{ in.}$$ $$\alpha' = \frac{1}{0.688(1+0.777)}\left[\left(\frac{1.21}{0.710}\right)^2 - 1\right]$$ $$= 1.56,\ \text{use } \alpha' = 1.00$$ $$Q = \left(\frac{0.710}{1.20}\right)^2 \left[1+0.688(1.00)\right]$$ $$= 0.591$$ $$T_{avail} = 0.591(20.3\text{ kips})$$ $$= 12.0\text{ kips} < 12.2\text{ kips} \quad \textbf{n.g.}$$

The check indicates that the column flange is deficient. However, the effective length of the connection need not be limited to the 18 in. assumed. An increase of less than 1⁄16 in. per bolt, or slightly more than ¼ in. total, in effective length is all that is required to make the flange sufficient. This slight increase is okay by inspection. A yield-line analysis could be used to determine a better estimate of the effective length.

Check bolt bearing at end plate

By the calculations for the gusset-to-column connection, bearing strength at the bolt holes in the beam-to-column connection will not control. The calculations for the bearing strength at bolt holes at this location would be similar to those presented for the gusset-to-column connection, except that the edge distance will be greater than the 1¼ in. used there.

Check block shear rupture on end plate

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(}Spec\text{. Eq. J4-5)}$$

The edge distance to the bottom of the end plate will be 21.4 in.– 18.0 in.+ 1.00 in. = 4.40 in., with a 1.00 in. bottom projection. The controlling block shear failure path cuts through each line of bolts and then to the outer edge of the end plate on each side.

Shear yielding component:

$$\begin{aligned} A_{gv} &= (15.0\text{ in.} + 4.40\text{ in.})(\tfrac{3}{4}\text{ in.}) \\ &= 14.6\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.6F_yA_{gv} &= 0.6(50\text{ ksi})\left(14.6\text{ in.}^2\right) \\ &= 438\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 14.6\text{ in.}^2 - 5.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{4}\text{ in.}) \\ &= 10.5\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.6F_uA_{nv} &= 0.6(65\text{ ksi})\left(10.5\text{ in.}^2\right) \\ &= 410\text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{nt} &= (\tfrac{3}{4}\text{ in.})\left[2.25\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right] \\ &= 1.31\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(65\text{ ksi})\left(1.31\text{ in.}^2\right) \\ &= 85.2\text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 410\text{ kips} + 85.2\text{ kips} \\ &= 495\text{ kips} \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 438\text{ kips} + 85.2\text{ kips} \\ &= 523\text{ kips} \end{aligned}$$

Therefore, $R_n = 495$ kips.

LRFD	ASD
$\begin{aligned} \phi R_n &= 0.75(495\text{ kips})(2) \\ &= 743\text{ kips} > 319\text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{(495\text{ kips})(2)}{2.00} \\ &= 495\text{ kips} > 212\text{ kips} \quad \textbf{o.k.} \end{aligned}$

Check beam shear strength

The available beam shear strength is determined from AISC *Specification* Section J4.2, assuming shear yielding controls:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(21.4 \text{ in.})(0.515 \text{ in.})$ $= 331 \text{ kips} > 319 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(21.4 \text{ in.})(0.515 \text{ in.})}{1.50}$ $= 220 \text{ kips} > 212 \text{ kips}$ **o.k.**

Check column shear strength

The available column shear strength is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(14.0 \text{ in.})(0.440 \text{ in.})$ $= 185 \text{ kips} > 176 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(14.0 \text{ in.})(0.440 \text{ in.})}{1.50}$ $= 123 \text{ kips} > 117 \text{ kips}$ **o.k.**

Discussion

A final comment on the end-plate thickness is in order. To calculate l_h and $\bar{\alpha}$, t_p = 1.00 in. was assumed. The final plate thickness was t_p = ¾ in. This will cause α and $\bar{\alpha}$ to be slightly different and will cause a very small couple to exist on the gusset-to-beam interface. This couple can be ignored in the design of this connection.

The final completed design is shown in Figure 5-1. Note that no weld is specified for the flanges of the W21×83 beam to the end plate other than those out to approximately the k_1 dimension (the 1-in. returns). These welds are not required for the uniform force transfer assumed in the calculations for the forces shown in Figure 5-3. If the flanges are fully welded to the end plate, the bolts close to the flanges will see more load than those further away. This will affect the force distribution, which was assumed to be supplied uniformly to the column flange as well, and column flange stiffeners may be required adjacent to these more highly loaded bolts. It is important to recognize that a certain load path has been assumed by the use of the forces in Figure 5-3 and that the connection parts have been developed from that assumed load path. Thus, it is important to arrange the connection to allow the assumed load path to be realized.

If the flanges are welded to the end plate, this fact should be considered in the design from the outset. It sometimes happens that a shop will weld the flanges to the end plate with AISC minimum fillet welds because it "looks better" and "more is better." But in this case, more may not be better. Fracture of the bolts and/or weld close to the flange may occur. However, if no column stiffeners are provided adjacent to the beam flanges, the bolts near the flanges will probably not be overloaded because of column flange flexibility.

The gusset "clip" of Figure 5-1 is used to separate the welds of the gusset to the beam and the gusset to the end plate. No weld passes through this clip.

Example 5.2—Corner Connection-to-Column Flange: Uniform Force Method Special Case 1

Given:

It is sometimes necessary that the brace line of action has a work point other than the intersection of the beam and column axes. This is referred to as a nonconcentric connection and is treated as Special Case 1 of the uniform force method. The situation is discussed in Section 4.2.2 and is shown in Figure 4-13, where the work point is at an arbitrary point located by the coordinates

x and y from the intersection of the beam and column flanges for a strong-axis connection (for a weak-axis connection the origin would be at the intersection of the beam flange and the face of the column web). Thus the origin ($x = 0$, $y = 0$) is at the gusset corner as shown in Figure 4-13, or close to this point when end plates, clip angles or shear plates are used. Consider the connection of Example 5.1, as shown in Figure 5-1, but choose the gusset line of action to pass through the (theoretical) gusset corner at ($x = 0$, $y = 0$). In this example, the admissible force distribution proposed in Section 4.2.2 will be compared with that presented in the AISC *Manual* Part 13 for Special Case 1, Modified Working Point Location.

Solution:

From AISC *Manual* Table 1-1:

Beam
W21×83
$Z_x = 196$ in.^{4}

Column
W14×90
$Z_x = 157$ in.^{4}

From the geometry of Figure 5-2 and the terminology of Figure 4-13, and assuming that the brace bevel does not change (it will, but assume it does not for the sake of illustration) the eccentricity from the theoretical gusset corner to the gravity working point (the intersection of the beam and column centerlines) is:

$$\begin{aligned} e &= (e_b - y)\sin\theta - (e_c - x)\cos\theta \\ &= (10.7\text{ in.} - 0\text{ in.})\sin 47.2° - (7.00\text{ in.} - 0\text{ in.})\cos 47.2° \\ &= 3.09\text{ in.} \end{aligned}$$

Then from Section 4.2.2, Equations 4-5 through 4-8:

LRFD	ASD
$\eta = \dfrac{Z_{beam}}{Z_{beam}+2Z_{column}}$	$\eta = \dfrac{Z_{beam}}{Z_{beam}+2Z_{column}}$
$= \dfrac{196\text{ in.}^3}{196\text{ in.}^3+2\left(157\text{ in.}^3\right)}$	$= \dfrac{196\text{ in.}^3}{196\text{ in.}^3+2\left(157\text{ in.}^3\right)}$
$= 0.384$	$= 0.384$
$M = Pe$	$M = Pe$
$= (840\text{ kips})(3.09\text{ in.})$	$= (560\text{ kips})(3.09\text{ in.})$
$= 2{,}600$ kip-in.	$= 1{,}730$ kip-in.
$H' = \dfrac{(1-\eta)M}{\bar{\beta}+e_b}$	$H' = \dfrac{(1-\eta)M}{\bar{\beta}+e_b}$
$= \dfrac{(1-0.384)(2{,}600\text{ kip-in.})}{12.0\text{ in.}+10.7\text{ in.}}$	$= \dfrac{(1-0.384)(1{,}730\text{ kip-in.})}{12.0\text{ in.}+10.7\text{ in.}}$
$= 70.6$ kips	$= 46.9$ kips
$V' = \dfrac{M-H'\bar{\beta}}{\bar{\alpha}}$	$V' = \dfrac{M-H'\bar{\beta}}{\bar{\alpha}}$
$= \dfrac{2{,}600\text{ kip-in.}-(70.6\text{ kips})(12.0\text{ in.})}{17.5\text{ in.}}$	$= \dfrac{1{,}730\text{ kip-in.}-(46.9\text{ kips})(12.0\text{ in.})}{17.5\text{ in.}}$
$= 100$ kips	$= 66.7$ kips

These quantities are in the directions shown in Figure 4-14 when the brace force, P, is tension and e is positive. They are superimposed on the UFM forces, which are calculated as for a concentric case (with the correct bevel). The result of this superposition is shown in Figures 5-5a and 5-5b by showing the forces separately, and the summarized forces (summing the forces together) are shown in Figures 5-5c and 5-5d. It can be seen that the interface forces increase in some places and decrease in others. The connection must be designed with these forces. The brace-to-gusset connection will not change. The couple, M = 2,600 kip-in. (LRFD) and M = 1,730 kip-in. (ASD) is applied to the connection by the offset brace work line, but the brace itself is still an axial-force only member. This is why M is not shown on Figure 5-5a and 5-5b or Figure 5-5c and 5-5d, but its results on the beam and column are shown. The beam must be checked for an extra couple of 998 kip-in. (LRFD) and 668 kip-in. (ASD); the column must be checked for an extra couple of 801 kip-in. (LRFD) and 534 kip-in. (ASD), respectively.

Comparison with AISC Manual *Part 13, Special Case 1 method*

As noted in the problem statement, this treatment of Special Case 1, nonconcentric connections, does not follow the AISC *Manual* procedure provided. The *Manual* method is an admissible method, but Richard (1986) has reported that the gusset interface forces were not greatly affected by moving the work point from a concentric to a nonconcentric location. The *Manual* method would have only shear on the gusset edges, but the moments in the beam and column would be determined from the free body diagram given in Figure 5-6 as follows:

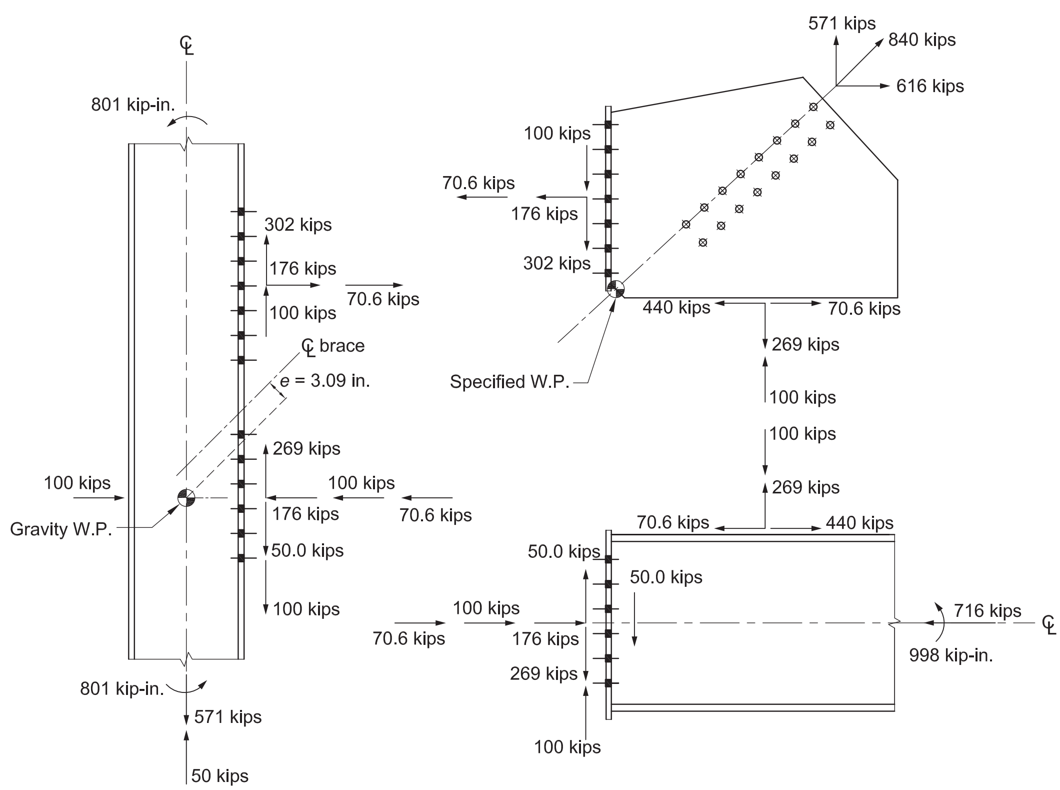


Fig. 5-5a. Admissible force fields for nonconcentric Example 5.2—LRFD.

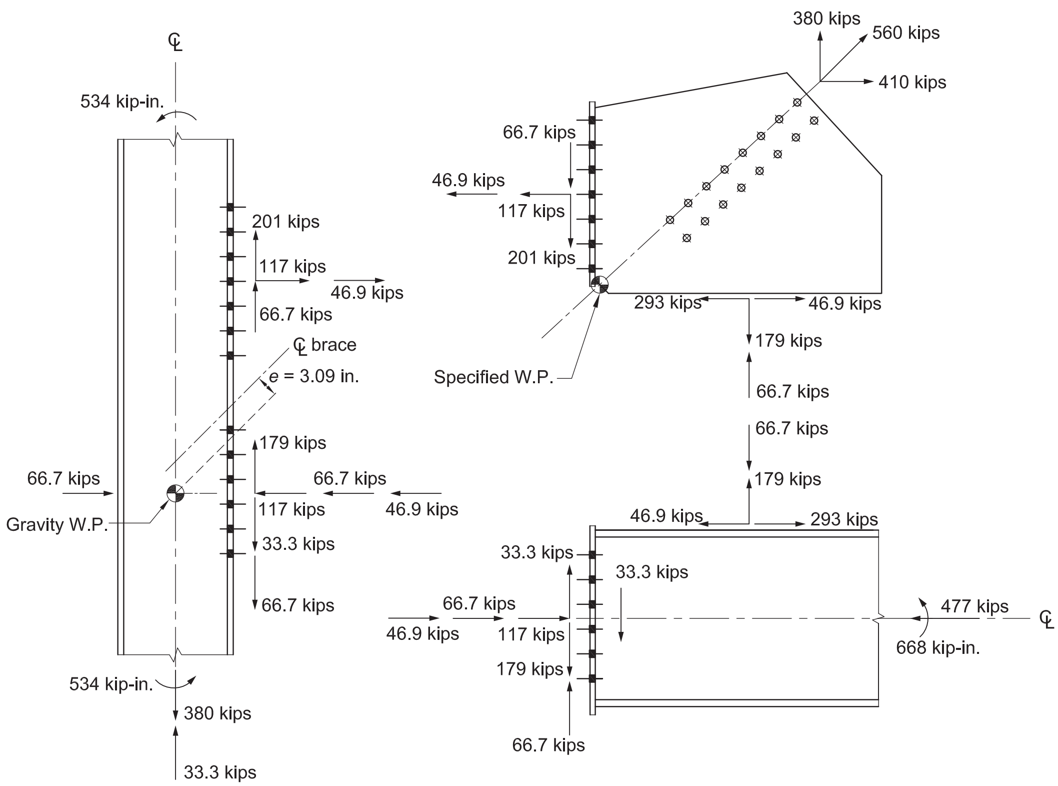


Fig. 5-5b. Admissible force fields for nonconcentric Example 5.2—ASD.

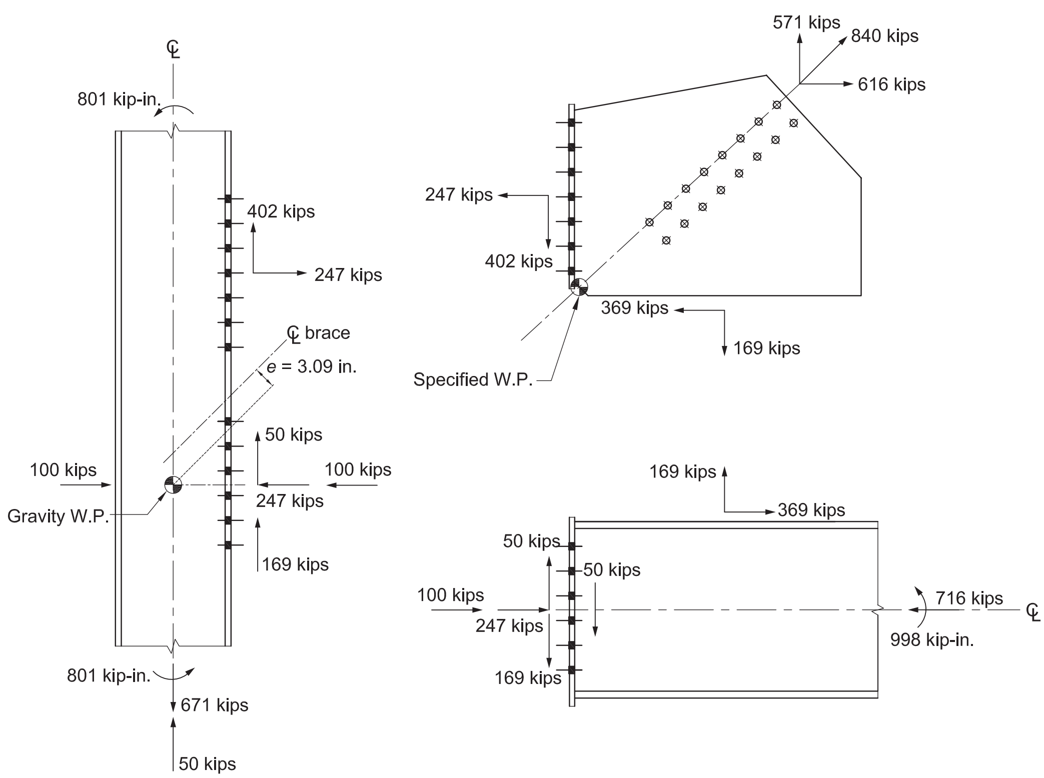


Fig. 5-5c. Admissible forces for Example 5.2 summarized—LRFD.

LRFD	ASD
In the beam: $$M_{ub} = (616\text{ kips})(21.4\text{ in.}/2) = 6{,}590\text{ kip-in.}$$ In the column: $$M_{uc} = \frac{(571\text{ kips})(14.0\text{ in.}/2)}{2} = 2{,}000\text{ kip-in.}$$	In the beam: $$M_{ab} = (410\text{ kips})(21.4\text{ in.}/2) = 4{,}390\text{ kip-in.}$$ In the column: $$M_{ac} = \frac{(380\text{ kips})(14.0\text{ in.}/2)}{2} = 1{,}330\text{ kip-in.}$$

This admissible force distribution is shown in Figure 5-6. This distribution, while admissible, is possible only if the gusset-to-column and gusset-to-beam connections can carry only shear. If very thin end plates or shear tabs with slots perpendicular to the shear loads were used in both locations, it is possible to achieve this distribution, but the authors have never seen such connections. Also, if these types of connections were achieved, the beam and column would be subjected to significant moments.

Using AISC *Manual* Table 3-2, determine the available flexural strength for the beam and column. Assume the beam is fully braced and the column unbraced length is less than $L_p = 15.1$ ft:

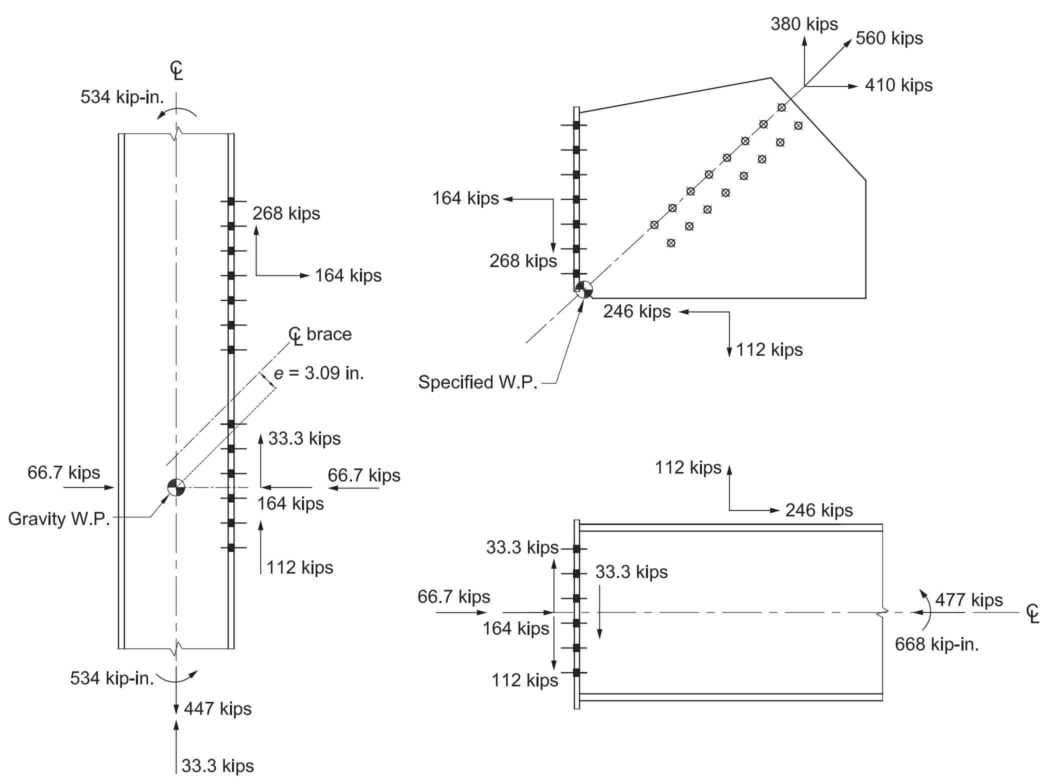


Fig. 5-5d. Admissible forces for Example 5.2 summarized—ASD.

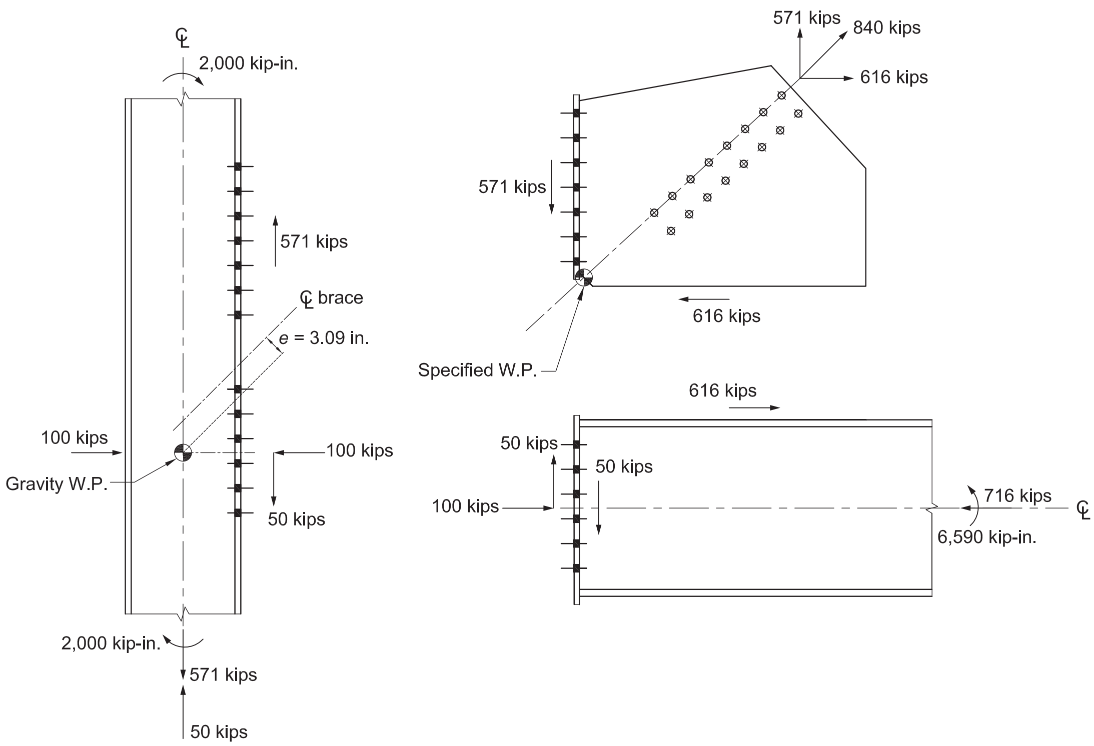


Fig. 5-6a. Admissible forces—Special Case 1 approach of the AISC Manual—*LRFD.*

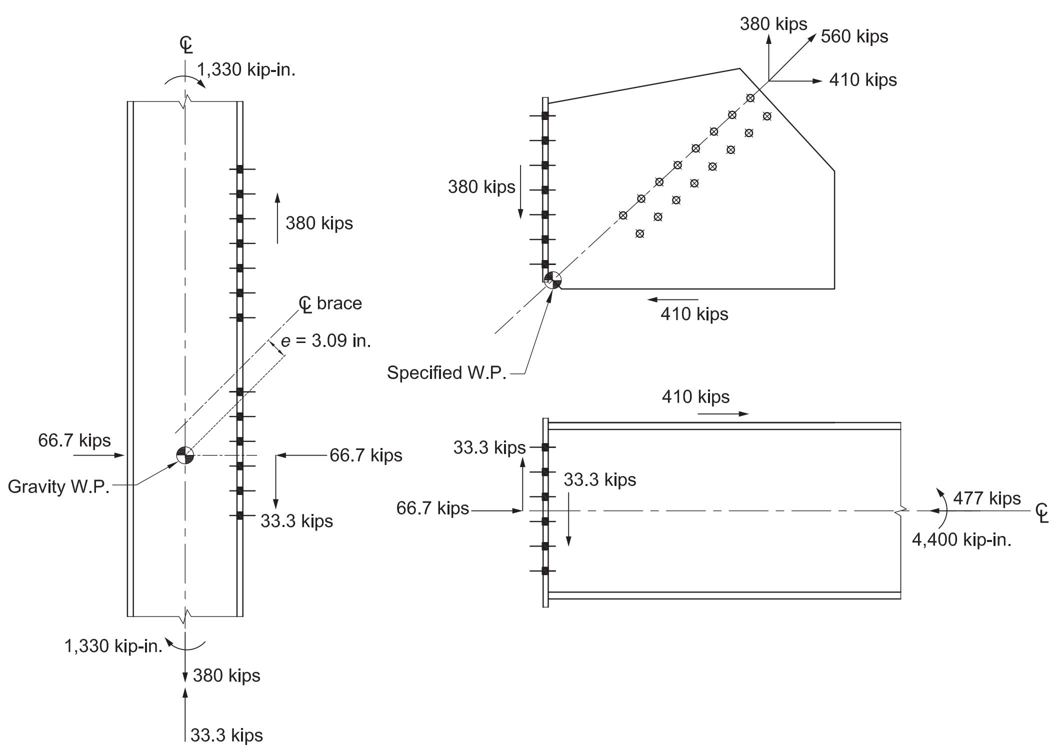


Fig. 5-6b. Admissible forces—Special Case 1 approach of the AISC Manual—*ASD.*

LRFD	ASD
For the beam: $\phi_b M_p = 735$ kip-ft As calculated previously: $M_{ub} = 6{,}590$ kip-in. /12 in./ft $= 549$ kip-ft M_{ub} is 75% of the beam's flexural strength, $\phi_b M_p$. $M_{uc} = 2{,}000$ kip-in./12 in./ft $= 167$ kip-ft For the column: $\phi_b M_p = 574$ kip-ft M_{uc} is 29% of the column's flexural strength, $\phi_b M_p$. These moments are too large to ignore. Note that the difference between the beam and column moments, $M_{ub} - M_{uc} = 6{,}590$ kip-in.– 2(2,000 kip-in.) $= 2{,}590$ kip-in. is equal to the actual applied moment of 2,600 kip-in. (with round off).	For the beam: $\dfrac{M_p}{\Omega_b} = 489$ kip-ft As calculated previously: $M_{ub} = 4{,}400$ kip-in. /12 in./ft $= 367$ kip-ft M_{ab} is 75% of the beam's flexural strength M_p/Ω_b. $M_{ac} = 1{,}330$ kip-in. /12 in./ft $= 111$ kip-ft For the column: $\dfrac{M_p}{\Omega_b} = 382$ kip-ft M_{ac} is 29% of the column's flexural strength, M_p/Ω_b. These moments are too large to ignore. Note that the difference between the beam and column moments, $M_{ab} - M_{ac} = 4{,}390$ kip-in.– 2(1,330 kip-in.) $= 1{,}730$ kip-in. is equal to the actual applied moment of 1,730 kip-in.

Therefore, the authors recommend that the distribution discussed in Section 4 and illustrated in the first part of this example be used instead of Special Case 1 presented in the AISC *Manual*, for most applications.

Example 5.3—Corner Connection-to-Column Flange: Uniform Force Method Special Case 2

Given:

Verify the connection in Figure 5-1, using a W21×44 beam and the uniform force method Special Case 2. Special Case 2 minimizes the shear in the beam-to-column connection as discussed in Section 4.2.3. The other given properties are the same as those given in Example 5.1.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are:

Beam
W21×44
$d = 20.7$ in. $t_w = 0.350$ in. $t_f = 0.450$ in. $k_1 = 13/16$ in. $k_{des} = 0.950$ in.

From Figure 5-3 and the AISC *Manual* Table 3-2, the required and available shear strengths of the beam adjacent to the end plate are:

LRFD	ASD
$V_u = 269 \text{ kips} + 50 \text{ kips}$ $= 319 \text{ kips}$ $\phi V_n = 217 \text{ kips} < 319 \text{ kips}$ **n.g.** The beam web fails in shear and a web doubler plate will be necessary to support a required shear strength of: $V_u = 319 \text{ kips} - 217 \text{ kips}$ $= 102 \text{ kips}$	$V_a = 179 \text{ kips} + 33.3 \text{ kips}$ $= 212 \text{ kips}$ $\dfrac{V_n}{\Omega} = 145 \text{ kips} < 212 \text{ kips}$ **n.g.** The beam web fails in shear and a web doubler plate will be necessary to support a required shear strength of: $V_a = 212 \text{ kips} - 145 \text{ kips}$ $= 67.0 \text{ kips}$

Rather than using a doubler plate, it is possible to carry less shear in the beam-to-column connection and more in the gusset-to-column connection. This redistribution of shear is called Special Case 2 of the uniform force method. This is discussed in the AISC *Manual* Part 13 and in Section 4.2.3 of this Design Guide. For the configuration shown in Figure 5-3, set $\Delta V_{ub} = 102$ kips (LRFD) and $\Delta V_{ab} = 68$ kips (ASD). (The value of 67 kips calculated previously is increased to 68 kips due to rounding considerations.) This means that the gusset-to-column connection must support a required shear strength equal to the vertical interface force, V_{uc} (LRFD) or V_{ac} (ASD), calculated in Example 5.1 in addition to the extra shear force, ΔV_{ub} (LRFD) or ΔV_{ab} (ASD).

LRFD	ASD
$V_u = 102 \text{ kips} + 302 \text{ kips}$ $= 404 \text{ kips}$ in addition to a required tensile strength of: $H_{uc} = 176$ kips (see Figure 5-3a)	$V_a = 68 \text{ kips} + 201 \text{ kips}$ $= 269 \text{ kips}$ in addition to a required tensile strength of: $H_{ac} = 117$ kips (see Figure 5-3b)

Preliminary Gusset-to-Column Connection Check

Check bolts at gusset-to-column connection

With the increased required shear strength of 404 kips (LRFD) and 269 kips (ASD) and required tensile strength of 176 kips (LRFD) and 117 kips (ASD), previous calculations for the gusset-to-column connection with 14 bolts can be reviewed to see if more bolts are required.

Bolts are ⅞-in.-diameter ASTM A490-X in single shear.

The available shear and tensile strengths per bolt, from AISC *Manual* Tables 7-1 and 7-2, and the required shear and tensile strengths per bolt are:

LRFD	ASD
$\phi r_{nv} = 37.9 \text{ kips}$ $\phi r_{nt} = 51.0 \text{ kips}$ Required shear strength per bolt: $r_{uv} = \dfrac{404 \text{ kips}}{14}$ $= 28.9 \text{ kips} < 37.9 \text{ kips}$ **o.k.** Required tensile strength per bolt: $r_{ut} = \dfrac{176 \text{ kips}}{14}$ $= 12.6 \text{ kips} < 51.0 \text{ kips}$ **o.k.**	$\dfrac{r_{nv}}{\Omega} = 25.2 \text{ kips}$ $\dfrac{r_{nt}}{\Omega} = 34.0 \text{ kips}$ Required shear strength per bolt: $r_{av} = \dfrac{269 \text{ kips}}{14}$ $= 19.2 \text{ kips} < 25.2 \text{ kips}$ **o.k.** Required tensile strength per bolt: $r_{at} = \dfrac{117 \text{ kips}}{14}$ $= 8.36 \text{ kips} < 34.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J3.7, the available tensile strength of the bolts subject to combined tension and shear is:

LRFD	ASD
Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3a: $$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$$ From AISC Specification Table J3.2: $$F_{nt} = 113 \text{ ksi}$$ $$F_{nv} = 84 \text{ ksi}$$ $$F'_{nt} = 1.3(113 \text{ ksi}) - \frac{113 \text{ ksi}}{0.75(84 \text{ ksi})}\left(\frac{28.9 \text{ kips}}{0.601 \text{ in.}^2}\right)$$ $$= 60.6 \text{ ksi} < 113 \text{ ksi} \quad \textbf{o.k.}$$ The reduced available tensile force per bolt due to combined forces is: $$\phi r_{nt} = \phi F'_{nt} A_b$$ $$= 0.75(60.6 \text{ kips})(0.601 \text{ in.}^2)$$ $$= 27.3 \text{ kips} > 12.6 \text{ kips} \quad \textbf{o.k.}$$	Interaction of shear and tension on bolts is given by AISC *Specification* Equation J3-3b: $$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$$ From AISC Specification Table J3.2: $$F_{nt} = 113 \text{ ksi}$$ $$F_{nv} = 84 \text{ ksi}$$ $$F'_{nt} = 1.3(113 \text{ ksi}) - \frac{(2.00)(113 \text{ ksi})}{84 \text{ ksi}}\left(\frac{19.2 \text{ kips}}{0.601 \text{ in.}^2}\right)$$ $$= 60.9 \text{ ksi} < 113 \text{ ksi} \quad \textbf{o.k.}$$ The reduced available tensile force per bolt due to combined forces is: $$\frac{r_{nt}}{\Omega} = \frac{F'_{nt} A_b}{\Omega}$$ $$= \frac{(60.9 \text{ kips})(0.601 \text{ in.}^2)}{2.00}$$ $$= 18.3 \text{ kips} > 8.36 \text{ kips}$$

It would appear from these preliminary calculations that 14 bolts will carry the force. This means that the geometry of Figure 5-1 can be used, except that the e_b = ½(20.7 in.) = 10.4 in. must be used rather than 10.7 in., which corresponds to the W21×83 beam. Because this change is small, the UFM geometry and force distribution need not be changed. The force distribution for this case is derived from Figure 5-7 and shown in Figure 5-8.

Interface Forces

The moment between the gusset and beam is:

LRFD	ASD
$$M_{ub} = \Delta V_{ub}\bar{\alpha}$$ $$= (102 \text{ kips})(17.5 \text{ in.})$$ $$= 1{,}790 \text{ kip-in.}$$	$$M_{ab} = \Delta V_{ab}\bar{\alpha}$$ $$= (68.0 \text{ kips})(17.5 \text{ in.})$$ $$= 1{,}190 \text{ kip-in.}$$

Table 5-1a compares the interface forces for the general UFM from Figure 5-3 to those of Special Case 2 of the UFM from Figure 5-8.

From Table 5-1, it can be seen that the portion of the connection that likely requires adjustment from the Figure 5-1 (general case) design is the gusset-to-column connection. This will be checked first, and then the gusset-to-beam connection will be checked. The brace-to-gusset connection and beam-to-column connection used in the general case will be satisfactory for Special Case 2.

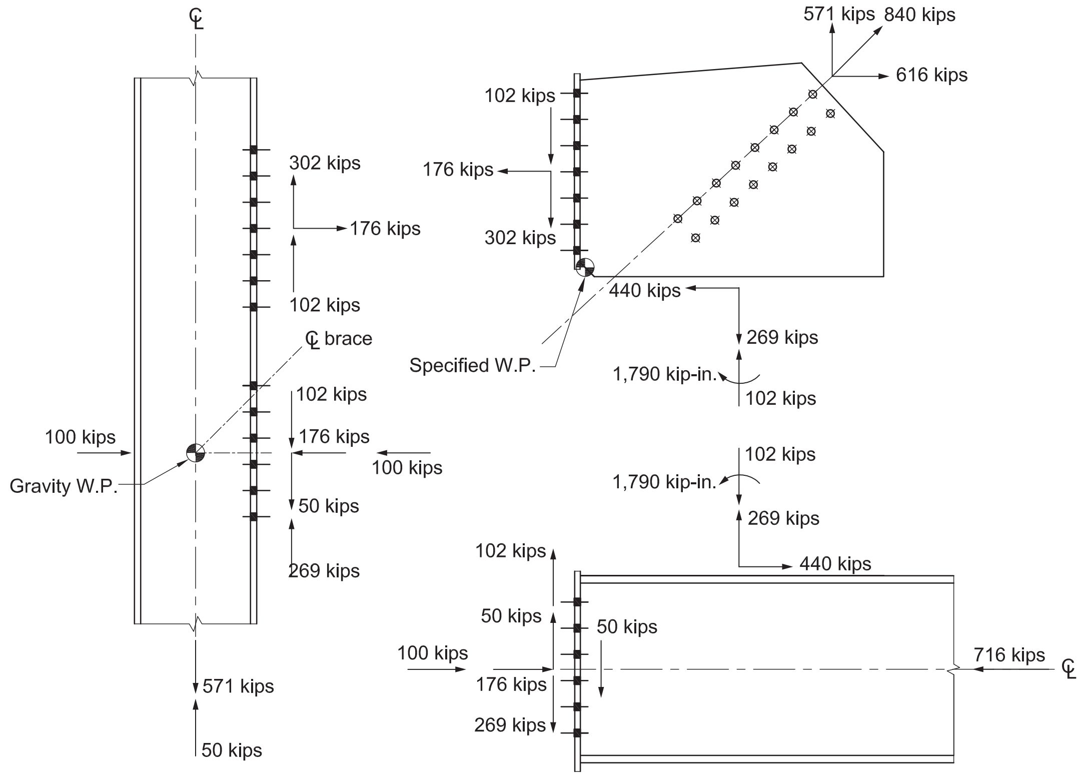


Fig. 5-7a. Special Case 2 superposition of forces—LRFD.

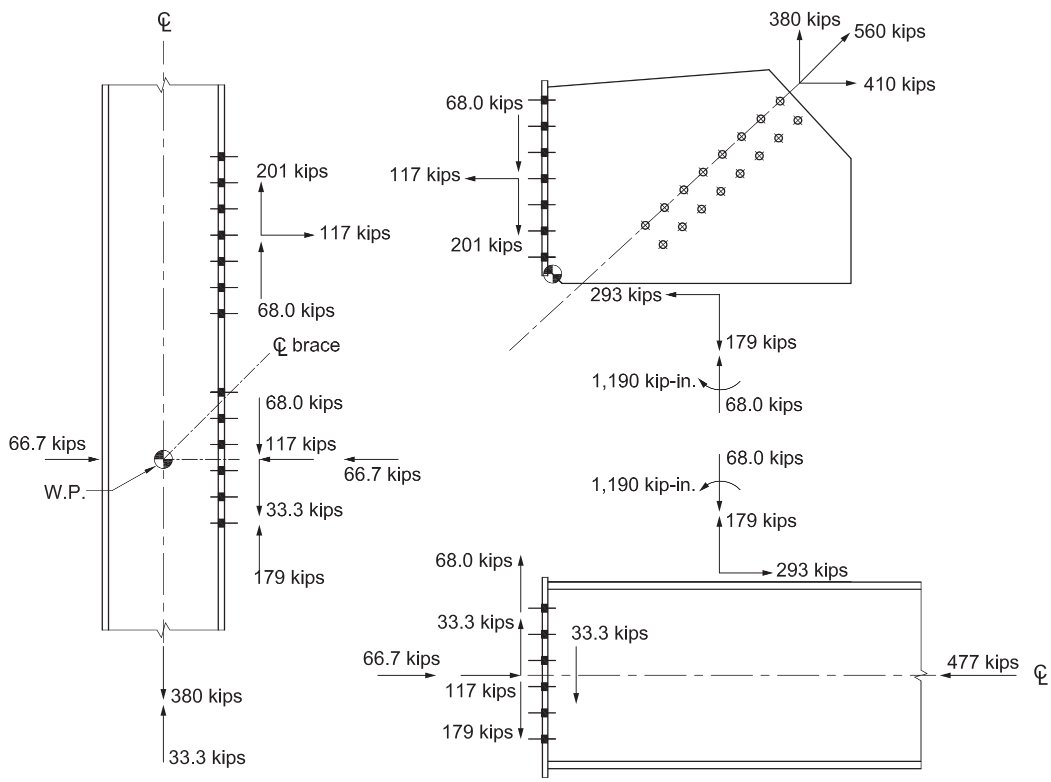


Fig. 5-7b. Special Case 2 superposition of forces—ASD.

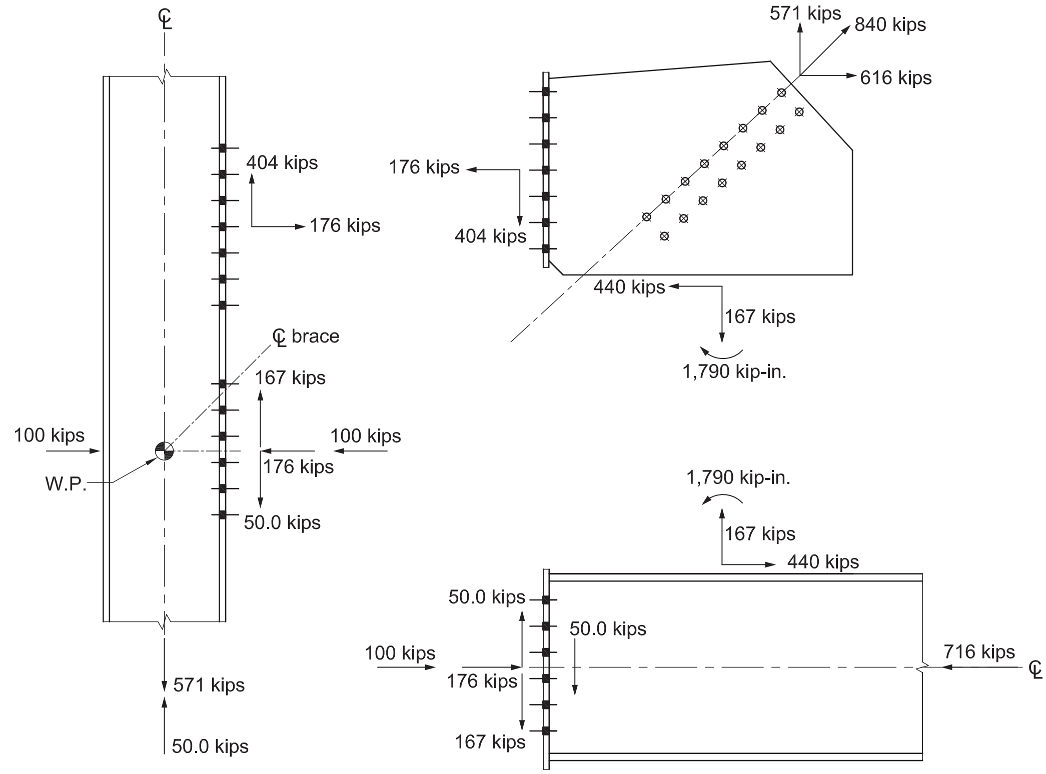


Fig. 5-8a. Admissible force fields—Special Case 2—LRFD.

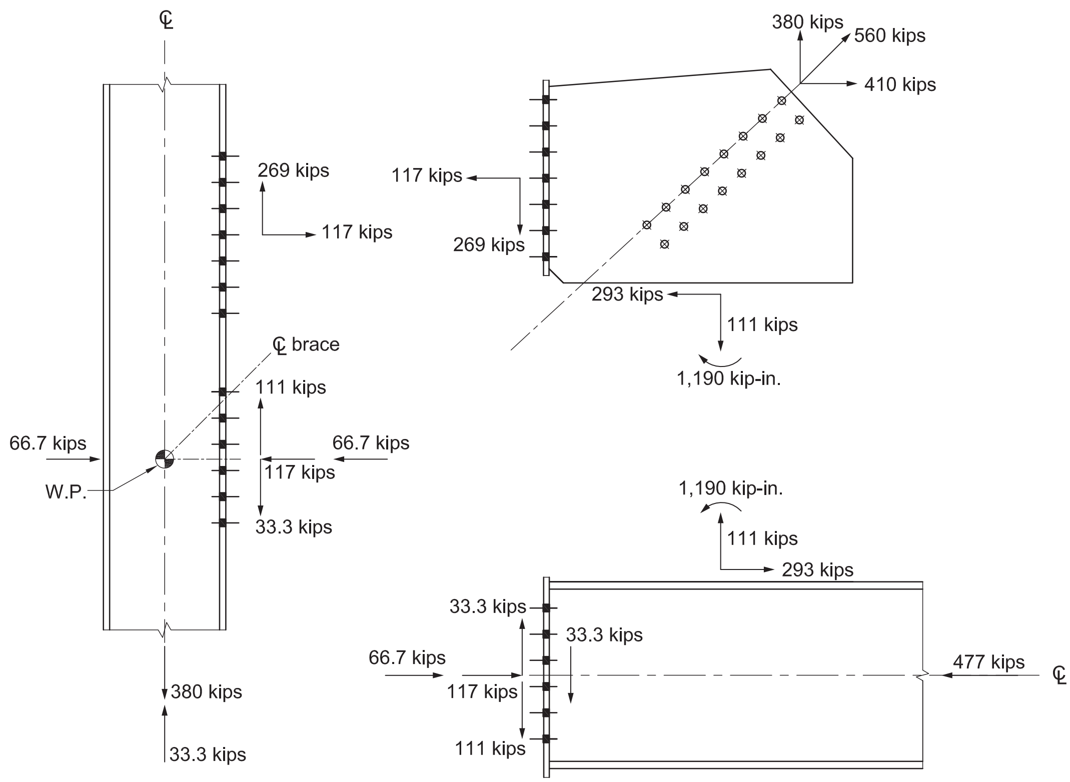


Fig. 5-8b. Admissible force fields—Special Case 2—ASD.

Table 5-1a. Comparison of Interface Forces (LRFD)						
Interface	General Case			Special Case 2		
	Shear	Normal	Moment	Shear	Normal	Moment
	kips	kips	kip-in.	kips	kips	kip-in.
Brace-to-Gusset	0	840	0	0	840	0
Gusset-to-Beam	440	269	0	440	167	1790
Gusset-to-Column	302	176	0	404	176	0
Beam-to-Column	319	176	0	217	176	0
Note: The distortional force H_D is not shown in the figures, but it is included in the table.						

Table 5-1b. Comparison of Interface Forces (ASD)						
Interface	General Case			Special Case 2		
	Shear	Normal	Moment	Shear	Normal	Moment
	kips	kips	kip-in.	kips	kips	kip-in.
Brace-to-Gusset	0	560	0	0	560	0
Gusset-to-Beam	293	179	0	293	111	1190
Gusset-to-Column	201	117	0	269	117	0
Beam-to-Column	212	117	0	144	117	0
Note: The distortional force H_D is not shown in the figures, but it is included in the table.						

Gusset-to-Column Connection (continued)

The required shear and axial strengths are (Figure 5-8):

LRFD	ASD
Required shear strength, $V_u = 404$ kips Required normal strength, $P_u = 176$ kips	Required shear strength, $V_a = 269$ kips Required normal strength, $P_a = 117$ kips

Design gusset-to-end plate weld

The resultant load, R, and its angle with the vertical, θ, are:

LRFD	ASD
$R_u = \sqrt{(404\text{ kips})^2 + (176\text{ kips})^2}$ $= 441\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{176\text{ kips}}{404\text{ kips}}\right)$ $= 23.5°$	$R_a = \sqrt{(269\text{ kips})^2 + (117\text{ kips})^2}$ $= 293\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{117\text{ kips}}{269\text{ kips}}\right)$ $= 23.5°$

The weld size is determined from AISC *Manual* Equation 8-2, including the increase in strength allowed by AISC *Specification* Section J2.4 when the angle of loading is not along the weld longitudinal axis. In the following, only the weld length tributary to the group of bolts is used, therefore use $l = 7(3.00\text{ in.}) = 21.0\text{ in.}$:

LRFD	ASD
Rearranging AISC *Manual* Equation 8-2a, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $$D_{req'd} = \frac{441\text{ kips}}{2\left(1.392\text{ kip/in.}\right)\left(21.0\text{ in.}\right)\left(1.0+0.50\sin^{1.5}23.5°\right)} = 6.70$$	Rearranging AISC *Manual* Equation 8-2b, and incorporating AISC *Specification* Equation J2-5, the weld size required is: $$D_{req'd} = \frac{293\text{ kips}}{2\left(0.928\text{ kip/in.}\right)\left(21.0\text{ in.}\right)\left(1.0+0.50\sin^{1.5}23.5°\right)} = 6.68$$

Therefore, use a two-sided 7⁄16-in. fillet weld at the gusset-to-end plate connection. Note that this exceeds the minimum size fillet weld of 5⁄16-in. required by AISC *Specification* Table J2.4. From previous calculations, this will be adequate for the 5½-in. gage and 1-in.-thick gusset plate. As noted before, a ductility factor is not required here because of the end plate flexibility.

Check bolts at gusset-to-column connection

Use ⅞-in.-diameter ASTM A490-X bolts with standard holes. Preliminary calculations have established that (14) A490-X ⅞-in.-diameter bolts are satisfactory.

Check prying action on bolts at the end plate

Using AISC *Manual* Part 9, determine the effect of prying action on the end-plate connection with a ¾-in.-thick end plate and the geometry shown in Figure 5-4b.

From Example 5.1, the parameters are:

b = 2.25 in.
a = 2.25 in. < $1.25b$ **o.k.**
b' = 1.81 in.
a' = 2.69 in.
ρ = 0.673
δ = 0.688
p = 3.00 in.

From the AISC *Manual* Part 9, the available tensile strength, including prying action effects, is:

LRFD	ASD
From AISC *Manual* Equation 9-30a, with $B = \phi r_{nt}$ = 27.3 kips, previously determined: $$t_c = \sqrt{\frac{4Bb'}{\phi p F_u}} = \sqrt{\frac{4\left(27.3\text{ kips}\right)\left(1.81\text{ in.}\right)}{0.90\left(3.00\text{ in.}\right)\left(65\text{ ksi}\right)}} = 1.06\text{ in.}$$	From AISC *Manual* Equation 9-30b, with $B = r_{nt}/\Omega$ = 18.3 kips, previously determined: $$t_c = \sqrt{\frac{\Omega 4Bb'}{pF_u}} = \sqrt{\frac{1.67\left(4\right)\left(18.3\text{ kips}\right)\left(1.81\text{ in.}\right)}{\left(3.00\text{ in.}\right)\left(65\text{ ksi}\right)}} = 1.07\text{ in.}$$

LRFD	ASD
$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.688(1+0.673)}\left[\left(\dfrac{1.06\text{ in.}}{\frac{3}{4}\text{ in.}}\right)^2 - 1\right]$ $= 0.867$ Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33: $Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$ $= \left(\dfrac{\frac{3}{4}\text{ in.}}{1.06}\right)^2 [1+0.688(0.867)]$ $= 0.799$ From AISC *Manual* Equation 9-31: $T_{avail} = BQ$ $= (27.3\text{ kips})(0.800)$ $= 21.8\text{ kips} > 12.6\text{ kips}$ **o.k.**	$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.688(1+0.673)}\left[\left(\dfrac{1.07\text{ in.}}{\frac{3}{4}\text{ in.}}\right)^2 - 1\right]$ $= 0.900$ Because $0 \le \alpha' \le 1$, determine Q from AISC *Manual* Equation 9-33: $Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta\alpha')$ $= \left(\dfrac{\frac{3}{4}\text{ in.}}{1.07}\right)^2 [1+0.688(0.900)]$ $= 0.796$ From AISC *Manual* Equation 9-31: $T_{avail} = BQ$ $= (18.5\text{ kips})(0.796)$ $= 14.7\text{ kips} > 8.36\text{ kips}$ **o.k.**

Check bolt bearing on end plate

From calculations to determine the available bearing strength for a ⅝-in.-thick end plate in Example 5.1, the available bearing strength per bolt is:

LRFD	ASD
$\phi r_n = 46.8\text{ kips} > 28.9\text{ kips}$ **o.k.**	$\dfrac{r_n}{\Omega} = 31.2\text{ kips} > 19.2\text{ kips}$ **o.k.**

Therefore, the thicker ¾-in.-thick end plate ultimately chosen will have adequate bearing strength.

Check block shear rupture of the end plate

From the similar calculations to determine block shear rupture strength for a ⅝-in.-thick end plate in Example 5.1, the available strength is:

LRFD	ASD
$\phi R_n = 588\text{ kips} > 404\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 392\text{ kips} > 269\text{ kips}$ **o.k.**

Therefore, the thicker ¾-in.-thick end plate ultimately chosen will have adequate bearing strength.

Check prying action on column flange

From the calculations for Example 5.1:

$b = 2.53$ in.
$a = 2.25$ in. $< 1.25b$ **o.k.**
$b' = 2.09$ in.
$a' = 2.69$ in.
$\rho = 0.777$
$p = 3.00$ in.
$\delta = 0.688$

From AISC *Manual* Part 9, the available tensile strength including prying action effects is determined as follows:

LRFD	ASD
$t_c = \sqrt{\dfrac{4(27.3 \text{ kips})(2.09 \text{ in.})}{0.90(3.00 \text{ in.})(65 \text{ ksi})}}$ $= 1.14$ in. $\alpha' = \dfrac{1}{0.688(1+0.777)}\left[\left(\dfrac{1.14}{0.710}\right)^2 - 1\right]$ $= 1.29$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $Q = \left(\dfrac{0.710}{1.14}\right)^2 (1+0.688)$ $= 0.655$ $T_{avail} = (27.3 \text{ kips})(0.655)$ $= 17.9 \text{ kips} > 12.6 \text{ kips}$ **o.k.**	$t_c = \sqrt{\dfrac{1.67(4)(18.5 \text{ kips})(2.09 \text{ in.})}{(3.00 \text{ in.})(65 \text{ ksi})}}$ $= 1.15$ in. $\alpha' = \dfrac{1}{0.688(1+0.777)}\left[\left(\dfrac{1.15}{0.710}\right)^2 - 1\right]$ $= 1.33$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $Q = \left(\dfrac{0.710}{1.15}\right)^2 (1+0.688)$ $= 0.643$ $T_{avail} = (18.5 \text{ kips})(0.643)$ $= 11.9 \text{ kips} > 8.36 \text{ kips}$ **o.k.**

Gusset-to-Beam Connection

From Figure 5-8, the required strengths at the gusset-to-beam connection are:

LRFD	ASD
Required shear strength, $H_{ub} = 440$ kips Required normal strength, $V_{ub} = 167$ kips Required flexural strength, $M_{ub} = 1{,}790$ kip-in.	Required shear strength, $H_{ab} = 293$ kips Required normal strength, $V_{ab} = 111$ kips Required flexural strength, $M_{ab} = 1{,}190$ kip-in.

Check gusset plate for tensile yielding and shear yielding along the beam flange

As shown in Figure 5-1, the gusset-to-beam interface length is the length of the gusset less the clip, or 31.5 in. In this example, to allow combining the required normal strength and flexural strength, stresses will be used to check the tensile and shear yielding limit states. Tensile yielding is checked using AISC *Specification* Section J4.1, and shear yielding is checked using Section J4.2:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{A_g}$ $= \dfrac{167\text{ kips}}{(1.00\text{ in.})(31.5\text{ in.})}$ $= 5.30\text{ ksi}$ $f_{ub} = \dfrac{M_{ub}}{Z}$ $= \dfrac{1{,}790\text{ kip-in.}}{(1.00\text{ in.})(31.5\text{ in.})^2/4}$ $= 7.22\text{ ksi}$ Total normal stress = f_{un} Tensile yielding strength: $f_{un} = f_{ua} + f_{ub}$ $= 5.30\text{ ksi} + 7.22\text{ ksi}$ $= 12.5\text{ ksi} < 0.90(50\text{ ksi}) = 45.0\text{ ksi}$ **o.k.** Shear yielding strength: $f_{uv} = \dfrac{440\text{ kips}}{(1.00\text{ in.})(31.5\text{ in.})}$ $= 14.0\text{ ksi} < 1.00(0.60)(50\text{ ksi}) = 30.0\text{ ksi}$ **o.k.**	$f_{aa} = \dfrac{V_{ab}}{A_g}$ $= \dfrac{111\text{ kips}}{(1.00\text{ in.})(31.5\text{ in.})}$ $= 3.52\text{ ksi}$ $f_{ab} = \dfrac{M_{ab}}{Z}$ $= \dfrac{(1{,}190\text{ kip-in.})}{(1.00\text{ in.})(31.5\text{ in.})^2/4}$ $= 4.80\text{ ksi}$ Total normal stress = f_{an} Tensile yielding strength: $f_{an} = f_{aa} + f_{ab}$ $= 3.52\text{ ksi} + 4.80\text{ ksi}$ $= 8.32\text{ ksi} < \dfrac{50\text{ ksi}}{1.67} = 29.9\text{ ksi}$ **o.k.** Shear yielding strength: $f_{av} = \dfrac{293\text{ kips}}{(1.00\text{ in.})(31.5\text{ in.})}$ $= 9.30\text{ ksi} < \dfrac{0.60(50\text{ ksi})}{1.50} = 20.0\text{ ksi}$ **o.k.**

Design weld at gusset-to-beam flange connection

As determined previously, the stresses on this interface are:

LRFD	ASD
$f_{ua} = 5.30\text{ ksi}$ $f_{ub} = 7.22\text{ ksi}$ $f_{un} = f_{ua} + f_{ub}$ $= 12.5\text{ ksi}$ $f_{uv} = 14.0\text{ ksi}$	$f_{aa} = 3.52\text{ ksi}$ $f_{ab} = 4.80\text{ ksi}$ $f_{an} = f_{aa} + f_{ab}$ $= 8.32\text{ ksi}$ $f_{av} = 9.30\text{ ksi}$

Following the method of Hewitt and Thornton (2004) and AISC *Manual* Part 13, where it is explained that if the gusset plate is directly welded to the beam, the weld is designed for the larger of the peak stress and 1.25 times the average stress:

LRFD	ASD
$f_{u\,peak} = \sqrt{(f_{ua}+f_{ub})^2 + f_{uv}^2}$ $= \sqrt{(5.30\text{ ksi}+7.22\text{ ksi})^2+(14.0\text{ ksi})^2}$ $= 18.8\text{ ksi}$ $f_{u\,avg} = \frac{1}{2}\left[\sqrt{(f_{ua}-f_{ub})^2+f_{uv}^2}+\sqrt{(f_{ua}+f_{ub})^2+f_{uv}^2}\right]$ $= \frac{1}{2}\left[\sqrt{(5.30\text{ ksi}-7.22\text{ ksi})^2+(14.0\text{ ksi})^2} + \sqrt{(5.30\text{ ksi}+7.22\text{ ksi})^2+(14.0\text{ ksi})^2}\right]$ $= 16.5\text{ ksi}$ $f_{u\,weld} = \max\left(1.25 f_{u\,avg}, f_{u\,peak}\right)$ $= \max\left[1.25(16.5\text{ ksi}),\ 18.8\text{ ksi}\right]$ $= 20.6\text{ ksi}$	$f_{a\,peak} = \sqrt{(f_{aa}+f_{ab})^2 + f_{av}^2}$ $= \sqrt{(3.52\text{ ksi}+4.80\text{ ksi})^2+(9.30\text{ ksi})^2}$ $= 12.5\text{ ksi}$ $f_{a\,avg} = \frac{1}{2}\left[\sqrt{(f_{aa}-f_{ab})^2+f_{av}^2}+\sqrt{(f_{aa}+f_{ab})^2+f_{av}^2}\right]$ $= \frac{1}{2}\left[\sqrt{(3.52\text{ ksi}-4.80\text{ ksi})^2+(9.30\text{ ksi})^2} + \sqrt{(3.52\text{ ksi}+4.80\text{ ksi})^2+(9.30\text{ ksi})^2}\right]$ $= 10.9\text{ ksi}$ $f_{a\,weld} = \max\left(1.25 f_{a\,avg}, f_{a\,peak}\right)$ $= \max\left[1.25(10.9\text{ ksi}),\ 12.5\text{ ksi}\right]$ $= 13.6\text{ ksi}$

To account for the directional strength increase of the fillet weld, the total normal stress and the shear stress can be used to determine the load angle. Then, from AISC *Manual* Part 8, Equations 8-2a and 8-2b:

LRFD	ASD
$\theta = \tan^{-1}\left(\frac{f_{un}}{f_{uv}}\right)$ $= \tan^{-1}\left(\frac{12.5\text{ ksi}}{14.0\text{ ksi}}\right)$ $= 41.8°$ The required number of sixteenths of weld is: $D = \frac{f_{u\,weld}\,t}{2(1.392\text{ kip/in.})\left(1+0.50\sin^{1.5}\theta\right)}$ $= \frac{(20.6\text{ ksi})(1.00\text{ in.})}{2(1.392\text{ kip/in.})\left(1+0.50\sin^{1.5}41.8°\right)}$ $= 5.82\text{ sixteenths}$	$\theta = \tan^{-1}\left(\frac{f_{an}}{f_{av}}\right)$ $= \tan^{-1}\left(\frac{8.32\text{ ksi}}{9.30\text{ ksi}}\right)$ $= 41.8°$ The required number of sixteenths of weld is: $D = \frac{f_{a\,weld}\,t}{2(0.928\text{ kip/in.})\left(1+0.50\sin^{1.5}\theta\right)}$ $= \frac{(13.6\text{ ksi})(1.00\text{ in.})}{2(0.928\text{ kip/in.})\left(1+0.50\sin^{1.5}41.8°\right)}$ $= 5.76\text{ sixteenths}$

From AISC *Specification* Table J2.4, the minimum fillet weld required is 5⁄16 in. which does not control in this case. Use a two-sided 3⁄8-in. fillet weld to connect the gusset plate to the beam.

Check beam web local yielding

The total equivalent normal force (including both the force and the moment resolved into a force couple) at the gusset-to-beam interface is:

LRFD	ASD
$N_{u\,equiv} = V_{ub} + \dfrac{2M_{ub}}{\left(\dfrac{L}{2}\right)}$ $= 167 \text{ kips} + \dfrac{(4)(1{,}790 \text{ kip-in.})}{31.5 \text{ in.}}$ $= 394 \text{ kips}$	$N_{a\,equiv} = V_{ab} + \dfrac{2M_{ab}}{\left(\dfrac{L}{2}\right)}$ $= 111 \text{ kips} + \dfrac{(4)(1{,}190 \text{ kip-in.})}{31.5 \text{ in.}}$ $= 262 \text{ kips}$

See Appendix B, Figure B-1 for an explanation of N_{equiv}.

From AISC *Specification* Section J10.2, because the resultant force is applied at a distance, α = 17.5 in. (determined in Example 5.1) from the member end that is less than or equal to the depth of the W21×44 beam, the available strength is determined from Equation J10-3, with $k = k_{des}$. Alternatively, AISC *Manual* Table 9-4 could be used.

LRFD	ASD
$\phi R_n = \phi F_{yw} t_w (2.5k_{des} + l_b)$ $= 1.00(50 \text{ ksi})(0.350 \text{ in.})[2.5(0.950 \text{ in.}) + 31.5 \text{ in.}]$ $= 593 \text{ kips} > 394 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w (2.5k_{des} + l_b)}{\Omega}$ $= \dfrac{(50 \text{ ksi})(0.350 \text{ in.})[2.5(0.950 \text{ in.}) + 31.5 \text{ in.}]}{1.50}$ $= 395 \text{ kips} > 262 \text{ kips}$ **o.k.**

Check beam web local crippling

From AISC *Specification* Section J10.3, because the resultant force is applied at a distance greater than $d/2$ from the member end, use Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $\phi R_n = (0.75)(0.80)(0.350 \text{ in.})^2 \left[1 + 3\left(\dfrac{31.5 \text{ in.}}{20.7 \text{ in.}}\right)\left(\dfrac{0.350 \text{ in.}}{0.450 \text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.450 \text{ in.})}{0.350 \text{ in.}}}$ $= 415 \text{ kips} > 394 \text{ kips}$	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}}{\Omega}$ $\dfrac{R_n}{\Omega} = \dfrac{(0.80)(0.350 \text{ in.})^2 \left[1 + 3\left(\dfrac{31.5 \text{ in.}}{20.7 \text{ in.}}\right)\left(\dfrac{0.350 \text{ in.}}{0.450 \text{ in.}}\right)^{1.5}\right] \times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.450 \text{ in.})}{0.350 \text{ in.}}}}{2.00}$ $= 276 \text{ kips} > 262 \text{ kips}$ **o.k.**

Beam-to-Column Connection

The required shear strength of this connection has been reduced from V_u = 319 kips to V_u = 217 kips (LRFD) or from V_a = 212 kips to V_a = 144 kips (ASD) (see Table 5-1), but the beam also has been made lighter and must be checked for strength.

The required strengths for this connection are:

LRFD	ASD
$V_u = 217$ kips $A_u = \max(A_{ub}, H_{uc})$ $= \max(100\text{ kips}, 176\text{ kips})$ $= 176$ kips	$V_a = 144$ kips $A_a = \max(A_{ab}, H_{ac})$ $= \max(66.7\text{ kips}, 117\text{ kips})$ $= 117$ kips

For a discussion on this, see Section 4.2.6 and the beam-to-column connection analysis for Example 5.1.

Check shear yielding on beam web

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(0.350\text{ in.})(20.7\text{ in.})$ $= 217\text{ kips} = 217\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})(0.350\text{ in.})(20.7\text{ in.})}{1.50}$ $= 145\text{ kips} > 144\text{ kips}$ **o.k.**

Design beam web-to-end plate weld

The resultant load, R, and its angle with the vertical, θ, are:

LRFD	ASD
$R = \sqrt{(217\text{ kips})^2 + (176\text{ kips})^2}$ $= 279$ kips $\theta = \tan^{-1}\left(\dfrac{176\text{ kips}}{217\text{ kips}}\right)$ $= 39.0°$	$R = \sqrt{(144\text{ kips})^2 + (117\text{ kips})^2}$ $= 186$ kips $\theta = \tan^{-1}\left(\dfrac{117\text{ kips}}{144\text{ kips}}\right)$ $= 39.1°$

The number of sixteenths of weld required is determined from AISC *Manual* Equation 8-2, including the increase in strength allowed by AISC *Specification* Section J2.4 when the angle of loading is not along the weld longitudinal axis. In the following, only the weld length tributary to the group of bolts is used; therefore, use $l = 6(3.00\text{ in.}) = 18.0$ in:

LRFD	ASD
$D_{req'd} = \dfrac{279\text{ kips}}{2(1.392\text{ kip/in.})(18.0\text{ in.})\left(1.0 + 0.50\sin^{1.5} 39.0°\right)}$ $= 4.46$ sixteenths	$D_{req'd} = \dfrac{186\text{ kips}}{2(0.928\text{ kip/in.})(18.0\text{ in.})\left(1.0 + 0.50\sin^{1.5} 39.1°\right)}$ $= 4.45$ sixteenths

Therefore, use a two-sided 5⁄16-in. fillet weld to connect the beam web to the end plate. Note that this exceeds the minimum size fillet weld of ¼ in. required by AISC *Specification* Table J2.4.

Check W21×44 *web weld to deliver load to bolts*

In the preceding weld calculation, only the weld adjacent to the bolts, 18.0 in., is used to size the weld. The actual effective weld length to the k_1 distance inside the flange is:

$$
\begin{aligned}
l_{eff} &= d - 2t_f + 2(k_1 - t_w/2) \\
&= 20.7 \text{ in.} - 2(0.450 \text{ in.}) + 2(^{13}\!/_{16} \text{ in.} - 0.350 \text{ in.}/2) \\
&= 21.1 \text{ in.}
\end{aligned}
$$

Since 21.1 in. > 20.7 in., the weld and web are fully effective.

Check tensile yielding in the web adjacent to the weld

For the 18-in. length of weld assumed to be tributary to the load on the bolts, the available tensile yielding strength of the web from AISC *Specification* Equation J4-1 is:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(0.350 \text{ in.})(18.0 \text{ in.})$ $= 284 \text{ ksi} > 176 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50 \text{ ksi})(0.350 \text{ in.})(18.0 \text{ in.})}{1.67}$ $= 189 \text{ kips} > 117 \text{ kips}$ **o.k.**

Summary

Figure 5-9 shows the final design of this connection. The final connection is similar to that of Example 5.1 for the general UFM shown in Figure 5-1. As a result of the decision to carry less shear in the beam-to-column connection and more in the

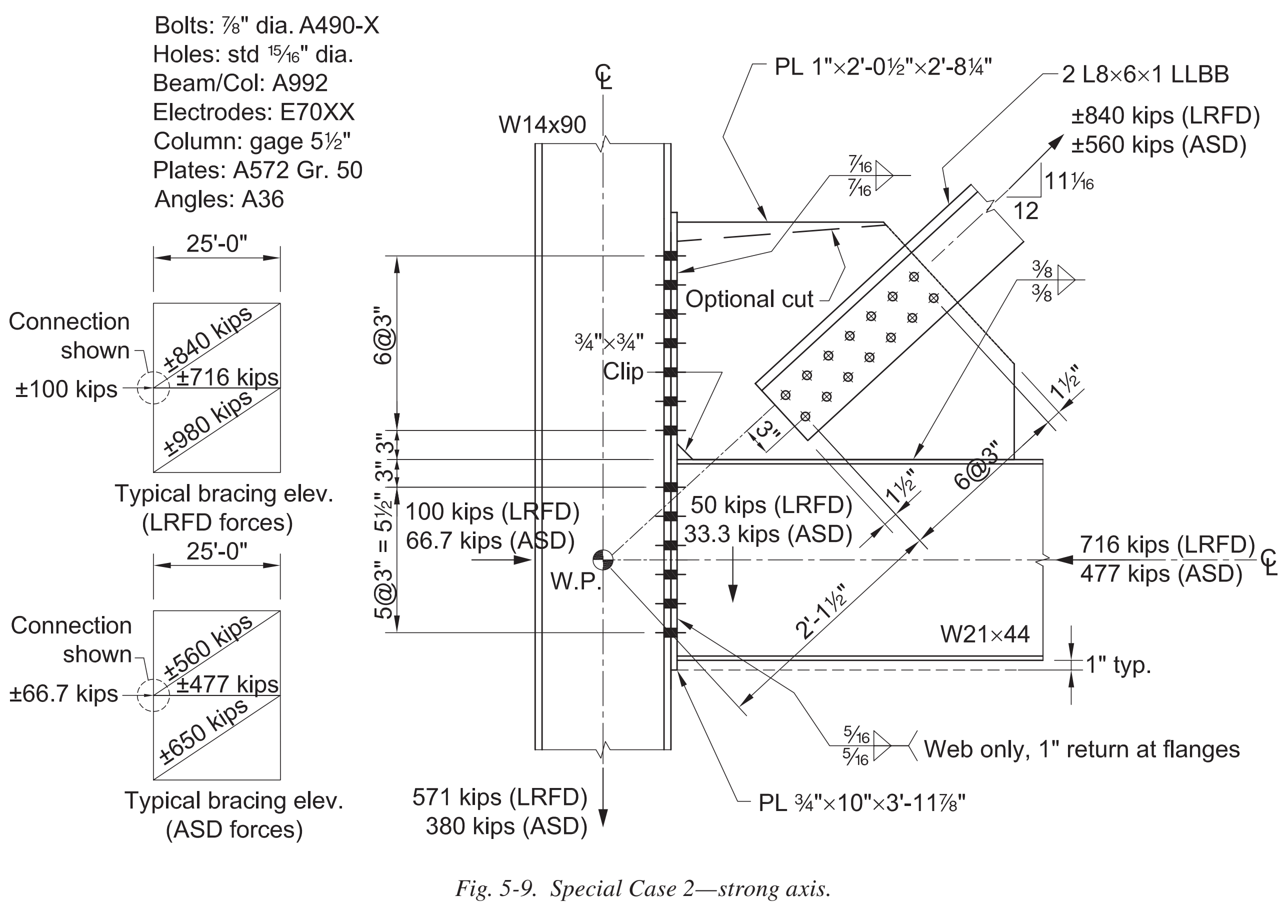


Fig. 5-9. Special Case 2—strong axis.

gusset-to-column connection as discussed in the beginning of the example, the gusset-to-end-plate weld has increased from ⅜ in. to 7⁄16 in., the gusset-to-beam-flange weld has changed to ⅜ in., and the beam-web to end-plate weld has decreased from 7⁄16 in. to 5⁄16 in.

Example 5.4—Corner Connection-to-Column Flange with Gusset Connected to Beam Only: Uniform Force Method Special Case 3

Given:

Special Case 3 of the uniform force method was discussed in Section 4.2.4, where it was noted that a moment $M_{bc} = Ve_c$ will exist on the beam-to-column connection, in addition to the vertical brace component, V (and the beam shear, R, if any). A design example of the connection shown in Figure 5-10 will illustrate this case. The bracing arrangement is as shown in Figure 2-1(a). The loading, member sizes, material strengths, bolt size and type, hole size, and connection element sizes are as given in Figure 5-10. Use 70-ksi electrodes.

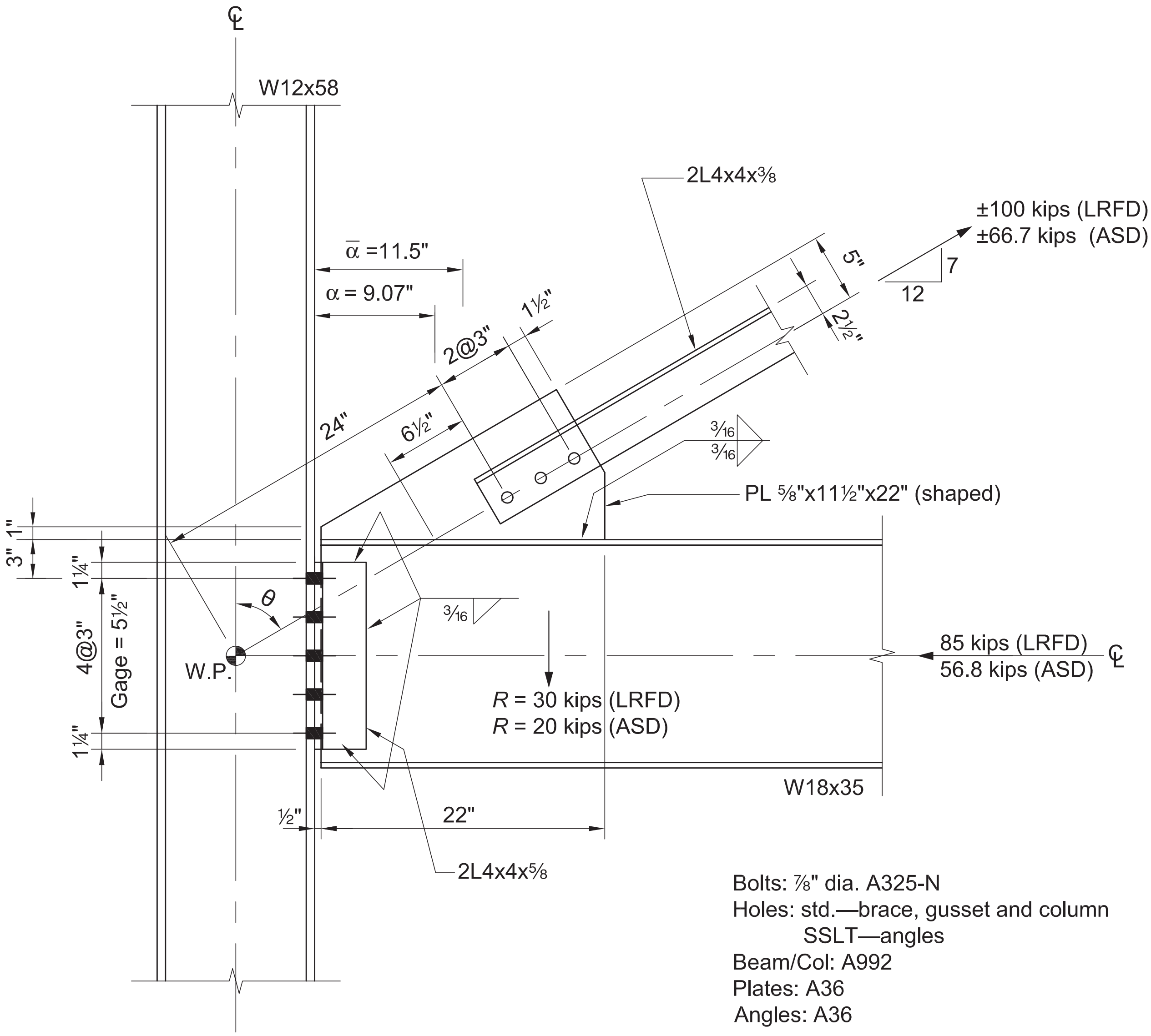


Fig. 5-10. Special Case 3—strong axis.

The required strengths are:

LRFD	ASD
Brace required strength, $P_u = \pm\, 100$ kips Beam required shear strength, $V_u = 30.0$ kips	Brace required strength, $P_a = \pm\, 66.7$ kips Beam required shear strength, $V_a = 20.0$ kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W18×35
$t_w = 0.300$ in. $\quad d = 17.7$ in. $\quad b_f = 6.00$ in. $\quad t_f = 0.425$ in. $\quad k_{des} = 0.827$ in.

Column
W12×58
$t_w = 0.360$ in. $\quad d = 12.2$ in. $\quad b_f = 10.0$ in. $\quad t_f = 0.640$ in.

Brace
2L4×4×⅜ in.
$A_g = 5.72$ in.^{2} $\quad \bar{x} = 1.13$ in. (single angle)

From AISC *Specification* Table J3.3, the hole dimensions for ⅞-in.-diameter bolts are as follows:

Brace, gusset plate and column
Standard: 15⁄16 in. diameter

Angles
Short slots: 15⁄16 in. × 1⅛ in. (short-slotted holes transverse to the line of force)

From AISC *Manual* Tables 7-1 and 7-2, the available shear and tensile strengths of a ⅞-in.-diameter ASTM A325-N bolt are:

LRFD	ASD
$\phi r_{nv} = 48.7$ kips double shear at gusset-to-brace connection $\phi r_{nv} = 24.3$ kips single shear at beam-to-column connection $\phi r_{nt} = 40.6$ kips	$\dfrac{r_{nv}}{\Omega} = 32.5$ kips double shear at gusset-to-brace connection $\dfrac{r_{nv}}{\Omega} = 16.2$ kips single shear at beam-to-column connection $\dfrac{r_{nt}}{\Omega} = 27.1$ kips

Brace-to-Gusset Connection

Check tension yielding on the brace

The available tensile yielding strength is determined from AISC *Specification* Equation J4-1. Alternatively, AISC *Manual* Table 5-8 may be used.

LRFD	ASD
$\phi_t R_n = \phi_t F_y A_g$ $= 0.90(36 \text{ ksi})(5.72 \text{ in.}^2)$ $= 185 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_t} = \dfrac{F_y A_g}{\Omega_t}$ $= \dfrac{(36 \text{ ksi})(5.72 \text{ in.}^2)}{1.67}$ $= 123 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check tensile rupture on the brace

The net area of the double-angle brace is determined in accordance with AISC *Specification* Section B4.3:

$$A_n = A_g - 2t(d_h + 1/16 \text{ in.})$$
$$= 5.72 \text{ in.}^2 - 2(3/8 \text{ in.})(15/16 \text{ in.} + 1/16 \text{ in.})$$
$$= 4.97 \text{ in.}^2$$

Because the outstanding legs of the double angle are not connected to the gusset plate, an effective net area of the double angle needs to be determined. From AISC *Specification* Section D3 and Table D3.1, Case 2, the effective net area is:

$$A_e = A_n U \qquad \text{(\textit{Spec}. Eq. D3-1)}$$

where

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{1.13 \text{ in.}}{6.00 \text{ in.}} \quad \text{(with 6 in. connection length)}$$
$$= 0.812$$

Then:

$$A_e = (4.97 \text{ in.}^2)0.812$$
$$= 4.04 \text{ in.}^2$$

From AISC *Specification* Section J4.1, the available strength for tensile rupture on the net section is:

LRFD	ASD
$\phi_t R_n = \phi_t F_u A_e$ $= 0.75(58 \text{ ksi})(4.04 \text{ in.}^2)$ $= 176 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_t} = \dfrac{F_u A_e}{\Omega_t}$ $= \dfrac{(58 \text{ ksi})(4.04 \text{ in.}^2)}{2.00}$ $= 117 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check bolt shear strength

From AISC *Manual* Table 7-1, the available shear strength of the three ⅞-in.-diameter ASTM A325-N bolts in double shear in the brace-to-gusset connection is:

LRFD	ASD
$\phi R_n = n\phi r_{nv}$ $= 3(48.7 \text{ kips})$ $= 146 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\phi R_n = n\left(\dfrac{r_{nv}}{\Omega}\right)$ $= 3(32.5 \text{ kips})$ $= 97.5 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check bolt bearing on the gusset plate and on the brace

Try a ½-in.-thick gusset plate with standard bolt holes. This thickness will be increased later, as shown in Figure 5-10. For the brace, short-slotted holes transverse to the line of force are used and the hole width is the same as for a standard hole.

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength on the gusset plate or brace. Bearing strength is limited by bearing deformation of the hole or by tearout of the material. From AISC *Specification* Section J3.10, when deformation at the bolt hole is a design consideration, the available strength for the limit state of bearing at bolt holes on the gusset is determined as follows:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \quad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

where, for the end bolt:

$$\begin{aligned} l_c &= 1\tfrac{1}{2} \text{ in.} - 0.5 d_h \\ &= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.}) \\ &= 1.03 \text{ in.} \end{aligned}$$

For the end bolt, using AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.03 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 26.9 \text{ kips/bolt}$	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{(1.2)(1.03 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 17.9 \text{ kips/bolt}$
$\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 45.7 \text{ kips/bolt}$	$\dfrac{2.4 d t F_u}{\Omega} = \dfrac{(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 30.5 \text{ kips/bolt}$
Therefore, $\phi R_n = 26.9$ kips/bolt.	Therefore $R_n/\Omega = 17.9$ kips/bolt.

$\phi r_{nv} = 48.7$ kips/bolt > 26.9 kips/bolt for LRFD and $r_{nv}/\Omega = 32.5$ kips/bolt > 17.9 kips/bolt = 17.9 kips/bolt for ASD, indicates that bearing at bolt holes on the end bolt is less than the shear strength of the bolt in double shear and bearing strength controls the design. The edge distance can be increased, the gusset made thicker, or the end bolt can be used with the reduced strength. The latter approach will be attempted in this example.

Bearing strength at bolt holes on the gusset for the two inner bolts is calculated as follows:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - 1.0 d_h \\ &= 3.00 \text{ in.} - 1.0(\tfrac{15}{16} \text{ in.}) \\ &= 2.06 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 53.8 \text{ kips} > 48.7 \text{ kips}$ Therefore, available bolt shear strength controls the strength of the inner bolt. For the end bolt on the brace: $l_c = 1.03 \text{ in.}$ $\phi r_n = 0.75(1.2)(1.03 \text{ in.})(\tfrac{3}{8} \text{ in.})(2)(58 \text{ ksi})$ $= 40.3 \text{ kips}$ *controls* $< 0.75(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{3}{8} \text{ in.})(2)(58 \text{ ksi})$ $= 68.5 \text{ kips}$ The bearing strength of the end bolt on the brace controls over the interior bolt strength on the gusset plate. The effective strength of the bolts, including bolt shear and bearing at bolt holes on both the brace and the gusset is: $\phi R_n = 26.9 \text{ kips} + 48.7 \text{ kips} + 40.3 \text{ kips}$ $= 116 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(2.06 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 35.8 \text{ kips} > 32.5 \text{ kips}$ Therefore, available bolt shear strength controls the strength of the inner bolt. For the end bolt on the brace: $l_c = 1.03 \text{ in.}$ $\dfrac{r_n}{\Omega} = \dfrac{1.2(1.03 \text{ in.})(\tfrac{3}{8} \text{ in.})(2)(58 \text{ ksi})}{2.00}$ $= 26.9 \text{ kips}$ *controls* $< \dfrac{2.4(\tfrac{7}{8} \text{ in.})(\tfrac{3}{8} \text{ in.})(2)(58 \text{ ksi})}{2.00}$ $= 45.7 \text{ kips}$ The bearing strength of the end bolt on the brace controls over the interior bolt strength on the gusset plate. The effective strength of the connection including bolt shear and bearing at bolt holes on both the brace and the gusset is: $\dfrac{R_n}{\Omega} = 17.9 \text{ kips} + 32.5 \text{ kips} + 26.9 \text{ kips}$ $= 77.3 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Note that bearing on both the brace and gusset should be considered in these checks.

Check block shear rupture on the gusset plate

The block shear rupture failure path on the gusset is assumed to run down the line of bolts and then perpendicular to the free edge of the gusset plate. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = \left[1\tfrac{1}{2} \text{ in.} + 2(3.00 \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 3.75 \text{ in.}^2$$

$$0.60 F_y A_{gv} = 0.60(36 \text{ ksi})(3.75 \text{ in.}^2)$$
$$= 81.0 \text{ kips}$$

Shear rupture component:

$$A_{nv} = 3.75 \text{ in.}^2 - \left[2.50(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})\right]$$
$$= 2.50 \text{ in.}^2$$

$$0.60 F_u A_{nv} = 0.60(58 \text{ ksi})(2.50 \text{ in.}^2)$$
$$= 87.0 \text{ kips}$$

Tension rupture component:

U_{bs} = 1 from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = [5.00 \text{ in.} - 0.5(1.00 \text{ in.})](\tfrac{1}{2} \text{ in.})$$
$$= 2.25 \text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(58 \text{ ksi})(2.25 \text{ in.}^2)$$
$$= 131 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 87.0 \text{ kips} + 131 \text{ kips}$$
$$= 218 \text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 81.0 \text{ kips} + 131 \text{ kips}$$
$$= 212 \text{ kips}$$

Therefore, because 218 kips > 212 kips, R_n = 212 kips.

LRFD	ASD
$\phi R_n = 0.75(212 \text{ kips})$ $= 159 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{212 \text{ kips}}{2.00}$ $= 106 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check the gusset plate for tensile yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section, using a ½-in.-thick gusset plate, is:

$$l_w = 2(6.00 \text{ in.})\tan 30°$$
$$= 6.93 \text{ in.}$$

Therefore, by geometry, the Whitmore section stays within the gusset plate. The effective area is:

$$A_w = (\tfrac{1}{2} \text{ in.})(6.93 \text{ in.})$$
$$= 3.47 \text{ in.}^2$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_yA_w$ $= 0.90(36 \text{ ksi})(3.47 \text{ in.}^2)$ $= 112 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_yA_w}{\Omega}$ $= \dfrac{(36 \text{ ksi})(3.47 \text{ in.}^2)}{1.67}$ $= 74.8 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check the gusset plate for compression buckling on the Whitmore section

The available compressive strength of the gusset plate based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4. Use an effective length factor of K = 1.2 due to the possibility of sidesway buckling (see AISC *Specification* Commentary Table C-A-7.1). From Figure 5-10, the unbraced length of the gusset plate is shown as 6½ in.

$$\frac{KL}{r} = \frac{1.2(6\tfrac{1}{2} \text{ in.})}{\tfrac{1}{2} \text{ in.}/\sqrt{12}}$$
$$= 54.0$$

Because $KL/r > 25$, use AISC *Manual* Table 4-22 to determine the available critical stress for the 36-ksi gusset plate. Then, the available compressive strength can be determined from AISC *Specification* Sections E1 and E3:

LRFD	ASD
$\phi_c F_{cr} = 27.8\text{ ksi}$ $\phi_c P_n = \phi_c F_{cr} A_w$ $= (27.8\text{ ksi})(3.47\text{ in.}^2)$ $= 96.5\text{ kips} < 100\text{ kips}$ **n.g.**	$\dfrac{F_{cr}}{\Omega_c} = 18.5\text{ ksi}$ $\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= (18.5\text{ ksi})(3.47\text{ in.}^2)$ $= 64.2\text{ kips} < 66.7\text{ kips}$ **n.g.**

Because the gusset fails in buckling, a thicker gusset could be used, or the material could be changed to ASTM A572 Grade 50. The thicker gusset will be used here.

Try a ⅝-in.-thick gusset plate:

$$\frac{KL}{r} = \frac{1.2(6\tfrac{1}{2}\text{ in.})}{\tfrac{5}{8}\text{ in.}/\sqrt{12}} = 43.2$$

Use AISC *Manual* Table 4-22 to find the available critical stress, and determine the available compressive strength from AISC *Specification* Sections E1 and E3:

LRFD	ASD
$\phi_c F_{cr} = 29.4\text{ ksi}$ $\phi_c P_n = \phi_c F_{cr} A_g$ $= (29.4\text{ ksi})(\tfrac{5}{8}\text{ in.})(6.93\text{ in.})$ $= 127\text{ kips} > 100\text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 19.6\text{ ksi}$ $\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}}{\Omega_c} A_g$ $= (19.6\text{ ksi})(\tfrac{5}{8}\text{ in.})(6.93\text{ in.})$ $= 84.9\text{ kips} > 66.7\text{ kips}$ **o.k.**

Therefore, use a gusset plate thickness of ⅝ in.

Check block shear rupture on the brace

The available strength for the limit state of block shear rupture on the brace is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = 2(7.50\text{ in.})(\tfrac{3}{8}\text{ in.}) = 5.63\text{ in.}^2$$

$$0.60F_y A_{gv} = 0.60(36\text{ ksi})(5.63\text{ in.}^2) = 122\text{ kips}$$

Shear rupture component:

$$A_{nv} = 5.63\text{ in.}^2 - 2.5(2)(\tfrac{3}{8}\text{ in.})(1.00\text{ in.}) = 3.76\text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(58\text{ ksi})(3.76\text{ in.}^2) = 131\text{ kips}$$

Tension rupture component:

U_{bs} = 1 from AISC *Specifications* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = [1.50 \text{ in.} - 0.5(1.00 \text{ in.})](3/8 \text{ in.})(2) = 0.750 \text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(58 \text{ ksi})(0.750 \text{ in.}^2) = 43.5 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 131 \text{ kips} + 43.5 \text{ kips} = 175 \text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 122 \text{ kips} + 43.5 \text{ kips} = 166 \text{ kips}$$

Therefore, because 175 kips > 166 kips, R_n = 166 kips.

LRFD	ASD
$\phi R_n = 0.75(166 \text{ kips}) = 125 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{166 \text{ kips}}{2.00} = 83.0 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Connection Interface Forces

The forces at the gusset-to-beam and gusset-to-column interfaces are determined using Special Case 3 of the UFM as discussed in Section 4.2.4 and AISC *Manual* Part 13.

From Figure 5-10 and the given beam and column geometry:

$$e_b = \frac{d_b}{2} = \frac{17.7 \text{ in.}}{2} = 8.85 \text{ in.}$$

$$e_c = \frac{d_c}{2} = \frac{12.2 \text{ in.}}{2} = 6.10 \text{ in.}$$

$$\tan\theta = \frac{12}{7}$$

$$\theta = \tan^{-1}\left(\frac{12}{7}\right) = 59.7°$$

Because there is no connection of the gusset plate to the column, $\beta = 0$, and

$$\alpha = e_b \tan\theta - e_c \qquad \text{(\textit{Manual} Eq. 13-1)}$$

$$= (8.85 \text{ in.})\left(\frac{12}{7}\right) - 6.10 \text{ in.} = 9.07 \text{ in.}$$

From Figure 5-10, the horizontal length of the gusset, l_g, is 22.0 in., therefore:

$$\begin{aligned}\bar{\alpha} &= \frac{l_g}{2} + \tfrac{1}{2} \text{ in.} \\ &= \frac{22.0 \text{ in.}}{2} + \tfrac{1}{2} \text{ in.} \\ &= 11.5 \text{ in.}\end{aligned}$$

From Figure 4-16 and Section 4.2.4, the forces on the gusset-to-beam interface are:

LRFD	ASD
$H_{ub} = H$ $= P_u \sin\theta$ $= (100 \text{ kips}) \sin 59.7°$ $= 86.3 \text{ kips}$ $V_{ub} = V$ $= P_u \cos\theta$ $= (100 \text{ kips}) \cos 59.7°$ $= 50.5 \text{ kips}$ $M_{ub} = V_{ub}(\bar{\alpha} - \alpha)$ $= (50.5 \text{ kips})(11.5 \text{ in.} - 9.07 \text{ in.})$ $= 123 \text{ kip-in.}$	$H_{ab} = H$ $= P_a \sin\theta$ $= (66.7 \text{ kips}) \sin 59.7°$ $= 57.6 \text{ kips}$ $V_{ab} = V$ $= P_a \cos\theta$ $= (66.7 \text{ kips}) \cos 59.7°$ $= 33.7 \text{ kips}$ $M_{ab} = V_{ab}(\bar{\alpha} - \alpha)$ $= (33.7 \text{ kips})(11.5 \text{ in.} - 9.07 \text{ in.})$ $= 81.9 \text{ kip-in.}$

The forces on the beam-to-column interface are:

LRFD	ASD
$M_{bc} = V_{ub}e_c$ $= (50.5 \text{ kips})(6.10 \text{ in.})$ $= 308 \text{ kip-in.}$ $V_{bc} = V_{ub} + R$ $= 50.5 \text{ kips} + 30 \text{ kips}$ $= 80.5 \text{ kips}$	$M_{bc} = V_{ab}e_c$ $= (33.7 \text{ kips})(6.10 \text{ in.})$ $= 206 \text{ kip-in.}$ $V_{bc} = V_{ab} + R$ $= 33.7 \text{ kips} + 20 \text{ kips}$ $= 53.7 \text{ kips}$

Gusset-to-Beam Connection

Check gusset plate for shear yielding and tensile yielding along the beam flange

In this example, to allow combining the required normal strength and flexural strength, stresses will be used to check the tensile and shear yielding limit states. The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{A}$ $= \dfrac{50.5 \text{ kips}}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})}$ $= 3.67 \text{ ksi}$ $f_{ub} = \dfrac{M_{ub}}{Z}$ $= \dfrac{123 \text{ kip-in.}}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})^2/4}$ $= 1.63 \text{ ksi}$ Total normal stress $= f_{un}$ Check tensile yielding: $f_{un} = f_{ua} + f_{ub}$ $= 3.67 \text{ ksi} + 1.63 \text{ ksi}$ $= 5.30 \text{ ksi} < 0.90(36 \text{ ksi}) = 32.4 \text{ ksi}$ **o.k.** Check shear yielding: $f_{uv} = \dfrac{H_{ub}}{A}$ $= \dfrac{86.3 \text{ kips}}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})}$ $= 6.28 \text{ ksi} < 1.00(0.60)(36 \text{ ksi}) = 21.6 \text{ ksi}$ **o.k.**	$f_{aa} = \dfrac{V_{ab}}{A}$ $= \dfrac{33.7 \text{ kips}}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})}$ $= 2.45 \text{ ksi}$ $f_{ab} = \dfrac{M_{ab}}{Z}$ $= \dfrac{(81.9 \text{ kip-in.})}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})^2/4}$ $= 1.08 \text{ ksi}$ Total normal stress $= f_{an}$ Check tensile yielding: $f_{an} = f_{aa} + f_{ab}$ $= 2.45 \text{ ksi} + 1.08 \text{ ksi}$ $= 3.53 \text{ ksi} < \dfrac{36 \text{ ksi}}{1.67} = 21.6 \text{ ksi}$ **o.k.** Check shear yielding: $f_{av} = \dfrac{H_{ab}}{A}$ $= \dfrac{57.6 \text{ kips}}{(\frac{5}{8} \text{ in.})(22.0 \text{ in.})}$ $= 4.19 \text{ ksi} < \dfrac{0.60(36 \text{ ksi})}{1.50} = 14.4 \text{ ksi}$ **o.k.**

Design weld at gusset-to-beam flange connection

Following the method of Hewitt and Thornton (2004) and AISC *Manual* Part 13, the weld is designed for the larger of the peak stress and 1.25 times the average stress:

LRFD	ASD
$f_{peak} = \sqrt{(5.30 \text{ ksi})^2 + (6.28 \text{ ksi})^2}$ $= 8.22 \text{ ksi}$ $f_{ave} = \dfrac{1}{2}\left(\sqrt{(3.67 \text{ ksi} - 1.63 \text{ ksi})^2 + (6.28 \text{ ksi})^2} + \sqrt{(5.30 \text{ ksi})^2 + (6.28 \text{ ksi})^2}\right)$ $= 7.41 \text{ ksi}$ $1.25 f_{ave} = 1.25(7.41 \text{ ksi})$ $= 9.26 \text{ ksi}$	$f_{peak} = \sqrt{(3.53 \text{ ksi})^2 + (4.19 \text{ ksi})^2}$ $= 5.48 \text{ ksi}$ $f_{ave} = \dfrac{1}{2}\left(\sqrt{(2.45 \text{ ksi} - 1.08 \text{ ksi})^2 + (4.19 \text{ ksi})^2} + \sqrt{(3.53 \text{ ksi})^2 + (4.19 \text{ ksi})^2}\right)$ $= 4.94 \text{ ksi}$ $1.25 f_{ave} = 1.25(4.94 \text{ ksi})$ $= 6.18 \text{ ksi}$

The direction of the load angle is calculated and then AISC *Manual* Equations 8-2a and 8-2b are used to determine the number of sixteenths of fillet weld required:

LRFD	ASD
$\theta = \tan^{-1}\dfrac{f_{un}}{f_{uv}}$ $= \tan^{-1}\left(\dfrac{5.30\text{ ksi}}{6.28\text{ ksi}}\right)$ $= 40.2°$ $D_{req'd} = \dfrac{f_{u\,weld}t}{2(1.392\text{ kip/in.})(1.0+0.50\sin^{1.5}\theta)}$ $= \dfrac{(9.26\text{ ksi})(5/8\text{ in.})}{2(1.392\text{ kip/in.})(1.0+0.50\sin^{1.5}40.2°)}$ $= 1.65$ sixteenths	$\theta = \tan^{-1}\dfrac{f_{an}}{f_{av}}$ $= \tan^{-1}\left(\dfrac{3.53\text{ ksi}}{4.19\text{ ksi}}\right)$ $= 40.1°$ $D_{req'd} = \dfrac{f_{a\,weld}t}{2(0.928\text{ kip/in.})(1.0+0.50\sin^{1.5}\theta)}$ $= \dfrac{(6.18\text{ ksi})(5/8\text{ in.})}{2(0.928\text{ kip/in.})(1.0+0.50\sin^{1.5}40.1°)}$ $= 1.65$ sixteenths

From AISC *Specification* Table J2.4, the minimum fillet weld required is 3/16 in. for the 0.425-in. beam flange. Therefore, use a two-sided 3/16-in. fillet weld.

Check beam web local yielding

The total equivalent normal force at the gusset-to-beam interface is (see Appendix B, Figure B-1):

LRFD	ASD
$N_{u\,equiv} = V_{ub} + \dfrac{2M_{ub}}{\left(\dfrac{L}{2}\right)}$ $= 50.5\text{ kips} + \dfrac{4(123\text{ kip-in.})}{22.0\text{ in.}}$ $= 72.9$ kips	$N_{a\,equiv} = V_{ab} + \dfrac{2M_{ab}}{\left(\dfrac{L}{2}\right)}$ $= 33.7\text{ kips} + \dfrac{4(81.9\text{ kip-in.})}{22.0\text{ in.}}$ $= 48.6$ kips

From AISC *Specification* Equation J10-3, because the resultant force is applied at a distance, $\bar{\alpha} = 11.5$ in., from the member end that is less than or equal to the depth of the W18×35 beam, the available strength for web local yielding is:

LRFD	ASD
$\phi R_n = \phi F_{yw}t_w(2.5k_{des} + l_b)$ $= 1.00(50\text{ ksi})(0.300\text{ in.})[2.5(0.827\text{ in.}) + 22.0\text{ in.}]$ $= 361\text{ kips} > 72.9\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_{yw}t_w(2.5k_{des} + l_b)}{\Omega}$ $= \dfrac{(50\text{ ksi})(0.300\text{ in.})[2.5(0.827\text{ in.}) + 22.0\text{ in.}]}{1.50}$ $= 241\text{ kips} > 48.6\text{ kips}$ **o.k.**

Check beam web local crippling

Using AISC *Specification* Equation J10-4 because the force to be resisted is applied a distance from the member end, $\bar{\alpha} = 11.5$ in., that is greater than or equal to $d/2$, the available strength for web local crippling is:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$ $= 0.75(0.80)(0.300\text{ in.})^2\left[1+3\left(\frac{22.0\text{ in.}}{17.7\text{ in.}}\right)\left(\frac{0.300\text{ in.}}{0.425\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\frac{(29{,}000\text{ ksi})(50\text{ ksi})(0.425\text{ in.})}{0.300\text{ in.}}}$ $= 249\text{ kips} > 72.9\text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = \frac{0.80 t_w^2 \left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}}{\Omega}$ $= \frac{(0.80)(0.300\text{ in.})^2\left[1+3\left(\frac{22.0\text{ in.}}{17.7\text{ in.}}\right)\left(\frac{0.300\text{ in.}}{0.425\text{ in.}}\right)^{1.5}\right]\times\sqrt{\frac{(29{,}000\text{ ksi})(50\text{ ksi})(0.425\text{ in.})}{0.300\text{ in.}}}}{2.00}$ $= 166\text{ kips} > 48.6\text{ kips}$ **o.k.**

Beam-to-Column Connection

LRFD	ASD
Required shear strength, $V_u = 80.5$ kips Required flexural strength, $M_u = 308$ kip-in.	Required shear strength, $V_a = 53.7$ kips Required flexural strength, $M_a = 206$ kip-in.

For convenience of calculation, the moment, M_u or M_a, can be converted into an equivalent axial force, N_u or N_a (see Appendix B):

$$N = \frac{4M}{L}$$

where L is the length of the angles.

Design bolts at beam-to-column connection

The available tensile and shear strength are determined from AISC *Manual* Tables 7-1 and 7-2 for ASTM A325-N bolts. The equivalent axial force is:

LRFD	ASD
$N_u = \frac{4(308\text{ kip-in.})}{14.5\text{ in.}}$ $= 85.0\text{ kips}$ The required tensile strength per bolt is: $r_{ut} = \frac{85.0\text{ kips}}{10\text{ bolts}}$ $= 8.50\text{ kips} < \phi r_{nt} = 40.6\text{ kips}$ **o.k.** The required shear strength per bolt is: $r_{uv} = \frac{80.5\text{ kips}}{10\text{ bolts}}$ $= 8.05\text{ kips} < \phi r_{nv} = 24.3\text{ kips}$ **o.k.**	$N_a = \frac{4(206\text{ kip-in.})}{14.5\text{ in.}}$ $= 56.8\text{ kips}$ The required tensile strength per bolt is: $r_{at} = \frac{56.8\text{ kips}}{10\text{ bolts}}$ $= 5.68\text{ kips} < r_{nt}/\Omega = 27.1\text{ kips}$ **o.k.** The required shear strength per bolt is: $r_{av} = \frac{53.7\text{ kips}}{10\text{ bolts}}$ $= 5.37\text{ kips} < r_{nv}/\Omega = 16.2\text{ kips}$ **o.k.**

The available tensile strength considers the effects of combined tension and shear. From AISC *Specification* Section J3.7 and Table J3.2, the available tensile strengths are:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}\left(\dfrac{8.05 \text{ kips}}{0.601 \text{ in.}^2}\right)$ $= 87.2 \text{ ksi} < 90 \text{ ksi}$ **o.k.** The reduced available tensile strength per bolt due to combined forces is: $\phi r_{nt} = \phi F'_{nt} A_b$ $= 0.75(87.2 \text{ kips})(0.601 \text{ in.}^2)$ $= 39.3 \text{ kips} > 8.50 \text{ kips}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}\left(\dfrac{5.37 \text{ kips}}{0.601 \text{ in.}^2}\right)$ $= 87.2 \text{ ksi} < 90 \text{ ksi}$ **o.k.** The reduced available tensile strength per bolt due to combined forces is: $\dfrac{r_{nt}}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ $= \dfrac{(87.2 \text{ kips})(0.601 \text{ in.}^2)}{2.00}$ $= 26.2 \text{ kips} > 5.68 \text{ kips}$ **o.k.**

Check prying action on bolts and angles

Following the notation of the AISC *Manual* Part 9:

$$
\begin{aligned}
a &= leg\ width + \frac{t_w}{2} - \frac{g}{2} \\
&= 4 \text{ in.} + \frac{0.300 \text{ in.}}{2} - \frac{5\tfrac{1}{2} \text{ in.}}{2} \\
&= 1.40 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
b &= \frac{g - t_w - t}{2} \\
&= \frac{5\tfrac{1}{2} \text{ in.} - 0.300 \text{ in.} - \tfrac{5}{8} \text{ in.}}{2} \\
&= 2.29 \text{ in.}
\end{aligned}
$$

$$
a' = a + \frac{d_b}{2} \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual} \text{ Eq. 9-27})
$$

$$
\begin{aligned}
&= 1.40 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \\
&= 1.84 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
1.25b + \frac{d_b}{2} &= 1.25(2.29 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2} \\
&= 3.30 \text{ in.}
\end{aligned}
$$

1.84 in. < 3.30 in. **o.k.**

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$

$$= 2.29 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$

$$= 1.85 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$

$$= \frac{1.85 \text{ in.}}{1.84 \text{ in.}}$$

$$= 1.01$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{14.5 \text{ in.}}{5}$$

$$= 2.90 \text{ in.}$$

From AISC *Specification* Table J3.3, for a ⅞-in.-diameter bolt, the standard hole dimension is 15⁄16 in.

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-24)}$$

$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{2.90 \text{ in.}}$$

$$= 0.677$$

Note that t_c is the angle thickness required to eliminate prying action. It is, therefore, an upper bound on angle thickness. Any angle thicker than t_c would provide no additional strength.

LRFD	ASD
From AISC *Manual* Equation 9-30a: $$t_c = \sqrt{\frac{4Bb'}{\phi p F_u}}$$ $$= \sqrt{\frac{4(39.3 \text{ kips})(1.85 \text{ in.})}{0.90(2.90 \text{ in.})(58 \text{ ksi})}}$$ $$= 1.39 \text{ in.}$$ From AISC *Specification* Equation 9-35: $$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ $$= \frac{1}{0.677(1+1.01)}\left[\left(\frac{1.39 \text{ in.}}{\tfrac{5}{8} \text{ in.}}\right)^2 - 1\right]$$ $$= 2.90$$	From AISC *Manual* Equation 9-30b: $$t_c = \sqrt{\frac{4\Omega Bb'}{pF_u}}$$ $$= \sqrt{\frac{4(1.67)(26.2 \text{ kips})(1.85 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$$ $$= 1.39 \text{ in.}$$ From AISC *Specification* Equation 9-35: $$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ $$= \frac{1}{0.677(1+1.01)}\left[\left(\frac{1.39 \text{ in.}}{\tfrac{5}{8} \text{ in.}}\right)^2 - 1\right]$$ $$= 2.90$$

LRFD	ASD
Because $\alpha' > 1$, the angles, not the bolts, will govern the available strength of the connection. Use AISC *Manual* Equation 9-34: $$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$ $$= \left(\frac{5/8 \text{ in.}}{1.39 \text{ in.}}\right)^2 (1+0.677)$$ $$= 0.339$$ $$T_{avail} = BQ$$ $$= (39.3 \text{ kips})(0.339)$$ $$= 13.3 \text{ kips} > 8.50 \text{ kips} \quad \textbf{o.k.}$$	Because $\alpha' > 1$, the angles, not the bolts, will govern the available strength of the connection. Use AISC *Manual* Equation 9-34: $$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$ $$= \left(\frac{5/8 \text{ in.}}{1.39 \text{ in.}}\right)^2 (1+0.677)$$ $$= 0.339$$ $$T_{avail} = BQ$$ $$= (26.2 \text{ kips})(0.339)$$ $$= 8.88 \text{ kips} > 5.68 \text{ kips} \quad \textbf{o.k.}$$

Check prying action on column flange

The fulcrum point for the column is governed by that of the angles. Follow the notation of Part 9 of the AISC *Manual*. The variables a and a' are as calculated previously for the angle.

$$\begin{aligned} b &= \frac{g - t_w}{2} \\ &= \frac{5\tfrac{1}{2} \text{ in.} - 0.360 \text{ in.}}{2} \\ &= 2.57 \text{ in.} \end{aligned}$$

$$\begin{aligned} a' &\leq 1.25b + \frac{d_b}{2} \\ &= 1.25(2.57 \text{ in.}) + \frac{7/8 \text{ in.}}{2} \\ &= 3.65 \text{ in.} \end{aligned}$$

$$1.84 \text{ in.} < 3.65 \text{ in.} \quad \textbf{o.k.}$$

$$\begin{aligned} b' &= b - \frac{d_b}{2} \\ &= 2.57 \text{ in.} - \frac{7/8 \text{ in.}}{2} \\ &= 2.13 \text{ in.} \end{aligned}$$ (*Manual* Eq. 9-21)

$$\begin{aligned} \rho &= \frac{b'}{a'} \\ &= \frac{2.13 \text{ in.}}{1.84 \text{ in.}} \\ &= 1.16 \end{aligned}$$ (*Manual* Eq. 9-26)

$$p = 3.00 \text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$
$$= 1 - \frac{15/16 \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.688$$

From AISC *Manual* Part 9, the available tensile strength including prying action effects, T_{avail}, is determined as follows:

LRFD	ASD
From AISC *Manual* Equation 9-30a:	From AISC *Manual* Equation 9-30b:
$t_c = \sqrt{\dfrac{4Bb'}{\phi pF_u}}$	$t_c = \sqrt{\dfrac{\Omega 4Bb'}{pF_u}}$
$= \sqrt{\dfrac{4(39.3 \text{ kips})(2.13 \text{ in.})}{0.90(3.00 \text{ in.})(65 \text{ ksi})}}$	$= \sqrt{\dfrac{1.67(4)(26.2 \text{ kips})(2.13 \text{ in.})}{(3.00 \text{ in.})(65 \text{ ksi})}}$
$= 1.38 \text{ in.}$	$= 1.38 \text{ in.}$
From AISC *Manual* Equation 9-35:	From AISC *Manual* Equation 9-35:
$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$	$\alpha' = \dfrac{1}{\delta(1+\rho)}\left[\left(\dfrac{t_c}{t}\right)^2 - 1\right]$
$= \dfrac{1}{0.688(1+1.16)}\left[\left(\dfrac{1.38 \text{ in.}}{0.640 \text{ in.}}\right)^2 - 1\right]$	$= \dfrac{1}{0.688(1+1.16)}\left[\left(\dfrac{1.38 \text{ in.}}{0.640 \text{ in.}}\right)^2 - 1\right]$
$= 2.46$	$= 2.46$
Because $\alpha' > 1$, use Equation 9-34.	Because $\alpha' > 1$, use Equation 9-34.
$Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta)$	$Q = \left(\dfrac{t}{t_c}\right)^2 (1+\delta)$
$= \left(\dfrac{0.640 \text{ in.}}{1.38 \text{ in.}}\right)^2 (1+0.688)$	$= \left(\dfrac{0.640 \text{ in.}}{1.38 \text{ in.}}\right)^2 (1+0.688)$
$= 0.363$	$= 0.363$
From AISC *Manual* Equation 9-31:	From AISC *Manual* Equation 9-31:
$T_{avail} = BQ$	$T_{avail} = BQ$
$= (39.3 \text{ kips})(0.363)$	$= (26.2 \text{ kips})(0.363)$
$= 14.3 \text{ kips} > 8.50 \text{ kips}$ **o.k.**	$= 9.51 \text{ kips} > 5.68 \text{ kips}$ **o.k.**

Design angle-to-beam web welds

The resultant force on the welds and resultant load angle are determined as follows:

LRFD	ASD
Required shear strength: $V_u = 50.5 \text{ kips} + 30 \text{ kips} = 80.5 \text{ kips}$ Required equivalent axial strength: $N_{u\,equiv} = 85.0 \text{ kips}$ $R = \sqrt{(80.5 \text{ kips})^2 + (85.0 \text{ kips})^2} = 117 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{85.0 \text{ kips}}{80.5 \text{ kips}}\right) = 46.6°$	Required shear strength: $V_a = 33.7 \text{ kips} + 20 \text{ kips} = 53.7 \text{ kips}$ Required equivalent axial strength: $N_{a\,equiv} = 56.8 \text{ kips}$ $R = \sqrt{(53.7 \text{ kips})^2 + (56.8 \text{ kips})^2} = 78.2 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{56.8 \text{ kips}}{53.7 \text{ kips}}\right) = 46.6°$

Refer to AISC *Manual* Table 8-8 with $\theta = 45°$, $kl = 3.5$ in.:

$$k = \frac{3.50 \text{ in.}}{14.5 \text{ in.}} = 0.241$$

$$al = 4 - xl$$

$$x = \frac{k^2}{(1+2k)} = \frac{(0.241)^2}{[1+2(0.241)]} = 0.0392$$

$$xl = (0.0392)(14.5 \text{ in.}) = 0.568 \text{ in.}$$

$$al = 4 \text{ in.} - 0.568 \text{ in.} = 3.43 \text{ in.}$$

$$a = \frac{3.43 \text{ in.}}{14.5 \text{ in.}} = 0.237$$

By interpolation, $C = 2.98$. C_1 is taken as 1.00 for E70 electrodes from AISC *Manual* Table 8-3.

LRFD	ASD
$D_{req'd} = \dfrac{P_u}{\phi C C_1 l} = \dfrac{(117 \text{ kips})}{0.75(2.98)(1.00)(2)(14.5 \text{ in.})} = 1.81 \text{ sixteenths}$	$D_{req'd} = \dfrac{\Omega P_a}{C C_1 l} = \dfrac{2.00(78.2 \text{ kips})}{2.98(1.00)(2)(14.5 \text{ in.})} = 1.81 \text{ sixteenths}$

Therefore, use a 3⁄16-in. fillet weld (minimum size according to AISC *Specification* Table J2.4).

Since there is no simple way to check the beam web under the "C" shaped weld, use the conservative AISC *Manual* Part 9 equation for the minimum beam web thickness:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-3)}$$

For the present problem, $D = 1.81$.

$$t_{min} = \frac{6.19(1.81)}{65 \text{ ksi}} = 0.172 \text{ in.} < 0.300 \text{ in.} \quad \textbf{o.k.}$$

Note: Further discussion of the weld design presented here is given in Appendix B.

Check shear yielding on double angles

From AISC *Specification* Section J4.2, the available shear yielding strength of the double angles is determined from Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(2)(14.5 \text{ in.})(\tfrac{5}{8} \text{ in.})$ $= 392 \text{ kips} > 80.5 \text{ kips} \quad \textbf{o.k.}$	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{(0.60)(36 \text{ ksi})(2)(14.5 \text{ in.})(\tfrac{5}{8} \text{ in.})}{1.50}$ $= 261 \text{ kips} > 53.7 \text{ kips} \quad \textbf{o.k.}$

Check shear rupture on double angles

From AISC *Specification* Section J4.2, the available shear rupture strength of the double angles is determined through the line of bolts using Equation J4-4:

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(2)\left[14.5 \text{ in.} - 5(1.00 \text{ in.})\right](\tfrac{5}{8} \text{ in.})$ $= 310 \text{ kips} > 80.5 \text{ kips} \quad \textbf{o.k.}$	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(2)\left[14.5 \text{ in.} - 5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.})}{2.00}$ $= 207 \text{ kips} > 53.7 \text{ kips} \quad \textbf{o.k.}$

Check block shear rupture on the double angle at the column flange

Block shear is checked relative to the shear force assuming the controlling failure path would run through the bolt lines, and then perpendicular to the angle edge, as shown in Figure 5-10a. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(\textit{Spec.} Eq. J4-5)}$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= 2(13\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 16.6 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_y A_{gv} &= 0.60(36 \text{ ksi})(16.6 \text{ in.}^2) \\ &= 359 \text{ kips} \end{aligned}$$

Shear rupture component:

$$A_{nv} = 16.6 \text{ in.}^2 - 4.5(2)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.})$$
$$= 11.0 \text{ in.}^2$$

$$0.60F_uA_{nv} = 0.60(58 \text{ ksi})\left(11.0 \text{ in.}^2\right)$$
$$= 383 \text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specifications* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = 2\left(\tfrac{5}{8} \text{ in.}\right)\left[\frac{4 \text{ in.} + 4 \text{ in.} + 0.300 \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2} - 0.5\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]$$
$$= 1.13 \text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(58 \text{ ksi})\left(1.13 \text{ in.}^2\right)$$
$$= 65.5 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 383 \text{ kips} + 65.5 \text{ kips}$$
$$= 449 \text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 359 \text{ kips} + 65.5 \text{ kips}$$
$$= 425 \text{ kips}$$

Because 449 kips > 425 kips, use $R_n = 425$ kips:

LRFD	ASD
$\phi R_n = 0.75(425 \text{ kips})$ $= 319 \text{ kips} > 80.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{425 \text{ kips}}{2.00}$ $= 213 \text{ kips} > 53.7 \text{ kips}$ **o.k.**

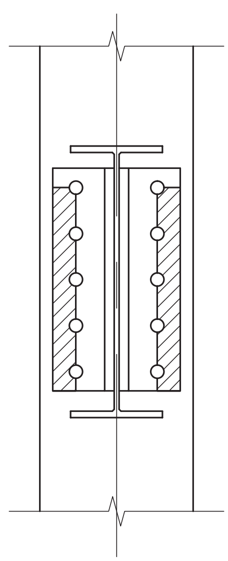


Fig. 5-10a. Block shear rupture failure path on double angle.

Check shear yielding on beam web

The available shear yielding strength of the beam web is determined using AISC *Specification* Equation J4-3 of Section J4.2:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(17.7\text{ in.})(0.300\text{ in.})$ $= 159\text{ kips} > 80.5\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})(17.7\text{ in.})(0.300\text{ in.})}{1.50}$ $= 106\text{ kips} > 53.7\text{ kips}$ **o.k.**

Check block shear rupture on beam web due to axial force

Block shear rupture is checked relative to the axial force on the beam with the failure path running along the welds of the double angle to the beam web. By inspection, the shear yielding component will control; therefore, AISC *Specification* Equation J4-5 reduces to:

$$R_n = 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(from } \textit{Spec}\text{. Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= 2tL_e \\ &= 2(0.300\text{ in.})(3.50\text{ in.}) \\ &= 2.10\text{ in.}^2 \\ A_{nt} &= tL \\ &= (0.300\text{ in.})(14.5\text{ in.}) \\ &= 4.35\text{ in.}^2 \end{aligned}$$

The available block shear rupture strength of the beam web is:

LRFD	ASD
$\phi R_n = \phi\left(0.60 F_y A_{gv} + U_{bs} F_u A_{nt}\right)$ $= 0.75\left[0.60(50\text{ ksi})\left(2.10\text{ in.}^2\right) + 1(65\text{ ksi})\left(4.35\text{ in.}^2\right)\right]$ $= 259\text{ kips} > 85.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv} + F_u A_{nt}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})\left(2.10\text{ in.}^2\right) + (65\text{ ksi})\left(4.35\text{ in.}^2\right)}{2.00}$ $= 173\text{ kips} > 56.8\text{ kips}$ **o.k.**

Check bolt bearing on the column flange

Because $t_f = 0.640$ in. is greater than the ⅝-in. angle thickness, bolt bearing on the column flange will not control the design.

Check bolt bearing on the double angles

Check bearing strength on the double angle at the top bolts, as the available bearing strength will have the lowest value at that location. Assuming that deformation at the bolt holes is a design consideration, use AISC *Specification* Equation J3-6a to determine the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad (\textit{Spec}\text{. Eq. J3-6a})$$

where l_c is based on the angle edge distance of 1¼ in.

$$\begin{aligned} l_c &= 1\tfrac{1}{4}\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 0.781\text{ in.} \end{aligned}$$

The available bearing strength per bolt is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.781\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 25.5\text{ kips/bolt}$	$1.2 l_c t F_u/\Omega = 1.2(0.781\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 17.0\text{ kips/bolt}$
$\phi 2.4 dtF_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 57.1\text{ kips/bolt}$	$2.4 dtF_u/\Omega = 2.4(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 38.1\text{ kips/bolt}$
Therefore, $\phi r_n = 25.5$ kips/bolt.	Therefore, $r_n/\Omega = 17.0$ kips/bolt.

Because 25.5 kips/bolt > 24.3 kips/bolt for LRFD and 17.0 kips/bolt >16.2 kips/bolt for ASD, bolt shear strength controls over the bearing strength for the top bolts (as well as the inner bolts, which would have a higher bearing strength due to the larger clear distance between the bolts). Therefore, the limit state of bearing does not apply.

Example 5.5—Corner Connection-to-Column Web: General Uniform Force Method

Given:

Verify the corner bracing connection design shown in Figure 5-11 using the general uniform force method. The loading, member sizes, material strengths, bolt size and type, hole size, and connection element sizes are as given in Figure 5-11. The unbraced length of the brace is 24 ft. Use 70-ksi electrodes.

The required strengths are:

LRFD	ASD
Brace: $\pm P_u = 270$ kips Beam: $V_u = 60$ kips Transfer force = ±37.5 kips	Brace: $\pm P_a = 180$ kips Beam: $V_a = 40$ kips Transfer force = ±25 kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $F_u = 65$ ksi

ASTM A36
$F_y = 36$ ksi $F_u = 58$ ksi

From AISC *Manual* Tables 1-1 and 1-8, the member geometric properties are as follows:

Beam
W18×97
$A_g = 28.5\text{ in.}^2$ $t_w = 0.535$ in. $d = 18.6$ in. $b_f = 11.1$ in. $t_f = 0.870$ in. $k_{des} = 1.27$ in.

Column
W14×233
$t_w = 1.07$ in. $d = 16.0$ in. $b_f = 15.9$ in. $t_f = 1.72$ in. $h/t_w = 10.7$

Brace
W12×58
$A_g = 17.0\text{ in.}^2$ $t_w = 0.360$ in. $d = 12.2$ in. $b_f = 10.0$ in. $t_f = 0.640$ in.

Brace-to-Gusset Connecting Element
WT5×16.5
$A_g = 4.85\text{ in.}^2$ $t_f = 0.435$ in. $\bar{y} = 0.869$ in. $b_f = 7.96$ in.

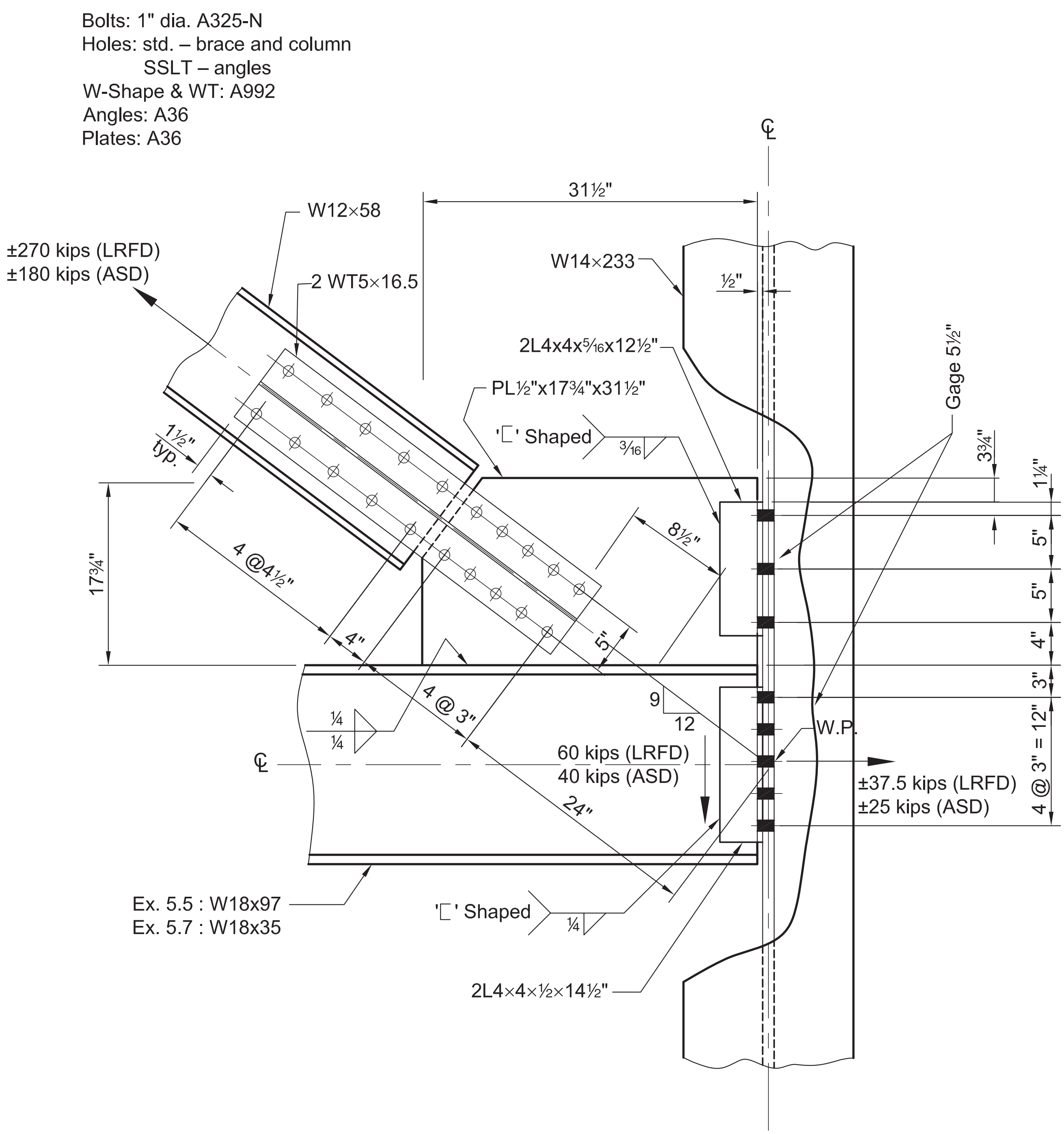


Fig. 5-11. General case—weak axis.

From AISC *Specification* Table J3.3, the hole dimensions for 1-in.-diameter bolts are as follows:

Brace and column
Standard: $d_h = 1\frac{1}{16}$ in. diameter

Angles
Short slots: $1\frac{1}{16}$ in. × $1\frac{5}{16}$ in.

From AISC *Manual* Tables 7-1 and 7-2, the available shear and tensile strengths of the bolts are:

LRFD	ASD
$\phi r_{nv} = 31.8$ kips (single shear at beam-to-column connection) $\phi r_{nv} = 63.6$ kips (double shear at brace-to-gusset connection) $\phi r_{nt} = 53.0$ kips	$\dfrac{r_{nv}}{\Omega} = 21.2$ kips (single shear at beam-to-column connection) $\dfrac{r_{nv}}{\Omega} = 42.4$ kips (double shear at brace-to-gusset connection) $\dfrac{r_{nt}}{\Omega} = 35.3$ kips

The brace-to-gusset connection should be designed first to determine an approximate gusset size and then the remaining parts of the connection can be designed. The distribution of forces by the UFM is simpler in this case than it was for the connection to the column's strong axis, as will be seen.

Brace-to-Gusset Connection

Determine the number of bolts required

The number of bolts required is:

LRFD	ASD
$n = \dfrac{P_u}{\phi r_{nv}}$ $= \dfrac{270 \text{ kips}}{63.6 \text{ kips/bolt}}$ $= 4.25$ bolts	$n = \dfrac{P_a}{r_n/\Omega}$ $= \dfrac{180 \text{ kips}}{42.4 \text{ kips/bolt}}$ $= 4.25$ bolts

This calculation indicates that two lines of three bolts will be sufficient to carry the brace force; however, Figure 5-11 shows five bolts per line. The reason for this difference will become apparent as the calculations proceed. This example will continue with the two lines of five bolts, as shown in Figure 5-11.

Check tensile yielding on the brace

From AISC *Specification* Section J4.1(a), Equation J4-1, the available tensile yielding strength is:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(17.0 \text{ in.}^2)$ $= 765 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50 \text{ ksi})(17.0 \text{ in.}^2)}{1.67}$ $= 509 \text{ kips} > 180 \text{ kips}$ **o.k.**

Check tensile rupture on the brace

The net area of the brace is determined according to AISC *Specification* Section B4.3, with the bolt hole diameter, $d_h = 1\frac{1}{16}$ in.

$$A_n = A_g - 2t(d_h + \tfrac{1}{16}\text{ in.})$$
$$= 17.0\text{ in.}^2 - 2(0.360\text{ in.})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})$$
$$= 16.2\text{ in.}^2$$

Because the flanges of the brace are not connected, an effective net area of the brace needs to be determined. From AISC *Specification* Section D3, the effective net area is:

$$A_e = UA_n$$

where

$U = 0.70$ from Table D3.1, Case 7

Alternatively, Table D3.1, Case 2 may also be used. From AISC *Specification* Commentary Figure C-D3.1, use $\bar{x} = 1.93$ in. for the W12×58:

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{1.93\text{ in.}}{4\left(4\tfrac{1}{2}\text{ in.}\right)}$$
$$= 0.893$$

The larger value of U may be used; therefore, the effective net area is:

$$A_e = UA_n$$
$$= 0.893\left(16.2\text{ in.}^2\right)$$
$$= 14.5\text{ in.}^2$$

From AISC *Specification* Section D2(b), the available strength for tensile rupture on the net section is:

LRFD	ASD
$\phi_t R_n = \phi_t F_u A_e$ $= 0.75(65\text{ ksi})\left(14.5\text{ in.}^2\right)$ $= 707\text{ kips} > 270\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_t} = \dfrac{F_u A_e}{\Omega_t}$ $= \dfrac{(65\text{ ksi})\left(14.5\text{ in.}^2\right)}{2.00}$ $= 471\text{ kips} > 180\text{ kips}$ **o.k.**

The designs of braces that are subjected to reversible loads are usually controlled by compression. The available compressive strength of a W12×58 is $\phi_c P_n = 292$ kips (LRFD) or $P_n/\Omega_c = 194$ kips (ASD) at a length $KL = 24$ ft, per AISC *Manual* Table 4-1. The choice of a W12×58 brace to support a required strength of 270 kips (LRFD) and 180 kips (ASD) is appropriate.

Check block shear rupture on the brace

The block shear rupture failure path on the brace is assumed to run down both bolt lines and then across a perpendicular line connecting the two bolt lines. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec.}\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = \left[4(4\tfrac{1}{2}\text{ in.}) + 1\tfrac{1}{2}\text{ in.}\right](0.360\text{ in.})(2)$$
$$= 14.0\text{ in.}^2$$

$$0.60F_yA_{gv} = 0.60(50\text{ ksi})(14.0\text{ in.}^2)$$
$$= 420\text{ kips}$$

Shear rupture component:

$$A_{nv} = 14.0\text{ in.}^2 - 4.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.360\text{ in.})(2)$$
$$= 10.4\text{ in.}^2$$

$$0.60F_uA_{nv} = 0.60(65\text{ ksi})(10.4\text{ in.}^2)$$
$$= 406\text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specifications* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = \left[5.00\text{ in.} - (0.5 + 0.5)(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.360\text{ in.})$$
$$= 1.40\text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(65\text{ ksi})(1.40\text{ in.}^2)$$
$$= 91.0\text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 406\text{ kips} + 91.0\text{ kips}$$
$$= 497\text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 420\text{ kips} + 91.0\text{ kips}$$
$$= 511\text{ kips}$$

Therefore, $R_n = 497$ kips.

LRFD	ASD
$\phi R_n = 0.75(497\text{ kips})$ $= 373\text{ kips} > 270\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{497\text{ kips}}{2.00}$ $= 249\text{ kips} > 180\text{ kips}$ **o.k.**

Note that the 4½-in. bolt spacing in the brace is required to provide adequate available strength due to block shear rupture. A spacing of 3 in. would provide a reduced available strength of $\phi R_n = 246$ kips for LRFD and $R_n/\Omega = 164$ kips for ASD.

Check bolt bearing on the brace

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use Equation J3-6a:

$$R_n = 1.2l_ctF_u \le 2.4dtF_u \qquad (\textit{Spec.}\text{ Eq. J3-6a})$$

For the inner bolts, the clear distance is:

$$l_c = 4\tfrac{1}{2}\text{ in.} - (0.5 + 0.5)d_h$$
$$= 4\tfrac{1}{2}\text{ in.} - 1(1\tfrac{1}{16}\text{ in.})$$
$$= 3.44\text{ in.}$$

The available bearing strength per bolt is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(3.44\text{ in.})(0.360\text{ in.})(65\text{ ksi})$ $= 72.4\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(0.360\text{ in.})(65\text{ ksi})$ $= 42.1\text{ kips/bolt}$ Therefore, $\phi R_n = 42.1$ kips/bolt.	$1.2 l_c t F_u/\Omega = 1.2(3.44\text{ in.})(0.360\text{ in.})(65\text{ ksi})/2.00$ $= 48.3\text{ kips/bolt}$ $2.4 d t F_u/\Omega = (2.4)(1.00\text{ in.})(0.360\text{ in.})(65\text{ ksi})/2.00$ $= 28.1\text{ kips/bolt}$ Therefore, $R_n/\Omega = 28.1$ kips/bolt.

Because these strengths are less than the available bolt shear strength determined previously (63.6 kips for LRFD and 42.4 kips for ASD), the limit state of bolt bearing controls the strength of the inner bolts.

For the edge bolts:

$$
\begin{aligned}
l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5d_h \\
&= 1\tfrac{1}{2}\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.}) \\
&= 0.969\text{ in.}
\end{aligned}
$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.969\text{ in.})(0.360\text{ in.})(65\text{ ksi})$ $= 20.4\text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(0.360\text{ in.})(65\text{ ksi})$ $= 42.1\text{ kips}$ Therefore, $\phi R_n = 20.4$ kips.	$1.2 l_c t F_u/\Omega = 1.2(0.969\text{ in.})(0.360\text{ in.})(65\text{ ksi})/2.00$ $= 13.6\text{ kips}$ $2.4 d t F_u/\Omega = 2.4(1.00\text{ in.})(0.360\text{ in.})(65\text{ ksi})/2.00$ $= 28.1\text{ kips}$ Therefore, $R_n/\Omega = 13.6$ kips.

Because these strengths are less than the available bolt shear strength determined previously (63.6 kips for LRFD and 42.4 kips for ASD), the limit state of bolt bearing controls the strength of the edge bolts.

To determine the available strength of the bolt group, sum the individual effective strengths for each bolt. The total available strength of the bolt group is:

LRFD	ASD
$\phi R_n = (2\text{ bolts})(20.4\text{ kips}) + (8\text{ bolts})(42.1\text{ kips})$ $= 378\text{ kips} > 270\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (2\text{ bolts})(13.6\text{ kips}) + (8\text{ bolts})(28.1\text{ kips})$ $= 252\text{ kips} > 180\text{ kips}$ **o.k.**

Note that this is significantly less than the bolt shear strength of $\phi R_n = 636$ kips for LRFD and $R_n/\Omega = 424$ kips for ASD.

Check the gusset plate for tensile yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section is:

$$
\begin{aligned}
l_w &= 5\text{ in.} + 2(12\text{ in.})\tan 30^\circ \\
&= 18.9\text{ in.}
\end{aligned}
$$

The Whitmore section enters the beam web, but $t_w = 0.535$ in. > $t_g = 0.500$ in., so the entire section is effective. For simplicity, use a thickness of ½ in. for the entire Whitmore area:

$$
\begin{aligned}
A_w &= (\tfrac{1}{2}\text{ in.})(18.9\text{ in.}) \\
&= 9.45\text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(36 \text{ ksi})(9.45 \text{ in.}^2)$ $= 306 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(36 \text{ ksi})(9.45 \text{ in.}^2)}{1.67}$ $= 204 \text{ kips} > 180 \text{ kips}$ **o.k.**

From this calculation, it can be seen that Whitmore yield requires the two lines of 5 bolts. A thicker gusset plate could also be used.

Check the gusset plate for compression buckling on the Whitmore section

From Dowswell (2006), this is a compact corner gusset and buckling is not an applicable limit state. However, if buckling is checked, the available compressive strength based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4. Use the common AISC *Manual* value for the effective length factor of $K = 0.5$ (Gross, 1990) and the unbraced length of the gusset plate shown in Figure 5-11.

$$\frac{KL}{r} = \frac{0.5(8\tfrac{1}{2} \text{ in.})}{\tfrac{1}{2} \text{ in.}/\sqrt{12}}$$
$$= 29.4$$

Because $KL/r > 25$, use AISC *Manual* Table 4-22 to determine the available critical stress for the 36-ksi gusset plate. Then, the available strength can be determined from AISC *Specification* Sections E1 and E3:

LRFD	ASD
$\phi_c F_{cr} = 31.0 \text{ ksi}$ $\phi P_n = \phi_c F_{cr} A_w$ $= (31.0 \text{ ksi})(9.45 \text{ in.}^2)$ $= 293 \text{ ksi} > 270 \text{ kips}$ **o.k.**	$F_{cr}/\Omega_c = 20.6 \text{ ksi}$ $\dfrac{P_n}{\Omega} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= (20.6 \text{ ksi})(9.45 \text{ in.}^2)$ $= 195 \text{ kips} > 180 \text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

As demonstrated previously for the brace, using AISC *Specification* Section J4.3, determine the available strength due to block shear rupture on the ½-in.-thick gusset plate.

Shear yielding component:

$$A_{gv} = 2(\tfrac{1}{2} \text{ in.})[4(3.00 \text{ in.}) + 1\tfrac{1}{2} \text{ in.}]$$
$$= 13.5 \text{ in.}^2$$

$$0.60 F_y A_{gv} = 0.60(36 \text{ ksi})(13.5 \text{ in.}^2)$$
$$= 292 \text{ kips}$$

Shear rupture component:

$$A_{nv} = 13.5 \text{ in.}^2 - 4.5(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})(2)$$
$$= 8.44 \text{ in.}^2$$

$$0.60 F_u A_{nv} = 0.60(58 \text{ ksi})(8.44 \text{ in.}^2)$$
$$= 294 \text{ kips}$$

Tension rupture component:

U_{bs} = 1 from AISC *Specifications* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = (\tfrac{1}{2}\text{ in.})\left[5.00\text{ in.} - (\tfrac{1}{2} + \tfrac{1}{2})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right]$$
$$= 1.94\text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(58\text{ ksi})\left(1.94\text{ in.}^2\right)$$
$$= 113\text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 294\text{ kips} + 113\text{ kips}$$
$$= 407\text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 292\text{ kips} + 113\text{ kips}$$
$$= 405\text{ kips}$$

Therefore, R_n = 405 kips.

LRFD	ASD
$\phi R_n = 0.75(405\text{ kips})$ $= 304\text{ kips} > 270\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{405\text{ kips}}{2.00}$ $= 203\text{ kips} > 180\text{ kips}$ **o.k.**

Check bolt bearing on the gusset plate

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use Equation J3-6a:

$$R_n = 1.2l_ctF_u \le 2.4dtF_u \qquad (\textit{Spec}.\text{ Eq. J3-6a})$$

For the inner bolts, assuming standard holes, the clear distance is:

$$l_c = 3.00\text{ in.} - (0.5 + 0.5)d_h$$
$$= 3.00\text{ in.} - 1(1\tfrac{1}{16}\text{ in.})$$
$$= 1.94\text{ in.}$$

LRFD	ASD
$\phi 1.2l_ctF_u = 0.75(1.2)(1.94\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 50.6\text{ kips}$	$1.2l_ctF_u/\Omega = 1.2(1.94\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})/2.00$ $= 33.8\text{ kips}$
$\phi 2.4dtF_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 52.2\text{ kips}$	$2.4dtF_u/\Omega = 2.4(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})/2.00$ $= 34.8\text{ kips}$
Therefore, ϕR_n = 50.6 kips.	Therefore, R_n/Ω = 33.8 kips.

Because these strengths are less than the available bolt shear strength determined previously (63.6 kips for LRFD and 42.4 kips for ASD), the limit state of bolt bearing controls the strength of the inner bolts.

For the edge bolts:

$$l_c = 1\tfrac{1}{2}\text{ in.} - 0.5d_h$$
$$= 1\tfrac{1}{2}\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.})$$
$$= 0.969\text{ in.}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.969 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 25.3 \text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 52.2 \text{ kips}$ Therefore, $\phi R_n = 25.3$ kips.	$1.2 l_c t F_u/\Omega = 1.2(0.969 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})/2.00$ $= 16.9 \text{ kips}$ $2.4 d t F_u/\Omega = 2.4(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})/2.00$ $= 34.8 \text{ kips}$ Therefore, $R_n/\Omega = 16.9$ kips.

Because these strengths are less than the available bolt shear strength determined previously (63.6 kips for LRFD and 42.4 kips for ASD), the limit state of bolt bearing controls the strength of the edge bolts.

To determine the available strength of the bolt group, sum the individual effective strengths for each bolt. The total available strength of the bolt group is:

LRFD	ASD
$\phi R_n = (2 \text{ bolts})(25.3 \text{ kips}) + (8 \text{ bolts})(50.6 \text{ kips})$ $= 455 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (2 \text{ bolts})(16.9 \text{ kips}) + (8 \text{ bolts})(33.8 \text{ kips})$ $= 304 \text{ kips} > 180 \text{ kips}$ **o.k.**

Checks on double-tee connecting elements

For these members, the critical side of the connection will be the gusset side, because the bolt spacing is 3 in. rather than 4½ in. Therefore, the double-tee to the gusset plate connection will be checked in the following.

Check tensile yielding on the double-tee connecting elements

The gross area of the 2-WT5×16.5 is:

$$\begin{aligned} A_g &= 2(4.85 \text{ in.}^2) \\ &= 9.70 \text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength is:

LRFD	ASD
$\phi P_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(9.70 \text{ in.}^2)$ $= 437 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{50 \text{ ksi}(9.70 \text{ in.}^2)}{1.67}$ $= 290 \text{ kips} > 180 \text{ kips}$ **o.k.**

Check tensile rupture on the double-tee connecting elements

The net area of the double-tee brace is determined in accordance with AISC *Specification* Section B4.3, with the bolt hole diameter, $d_h = 1\tfrac{1}{16}$ in., from AISC *Specification* Table J3.3:

$$\begin{aligned} A_n &= A_g - 2(2)t(d_h + \tfrac{1}{16} \text{ in.}) \\ &= 9.70 \text{ in.}^2 - 4(0.435 \text{ in.})(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}) \\ &= 7.74 \text{ in.}^2 \end{aligned}$$

Because the outstanding webs of the tees are not connected, an effective net area of the double tee needs to be determined. From AISC *Specification* Section D3, the effective net area is:

$$A_e = A_n U \qquad (\textit{Spec.} \text{ Eq. D3-1})$$

where

$$U = 1 - \frac{\bar{x}}{l} \text{ from AISC } \textit{Specification} \text{ Table D3.1, Case 2}$$

$$= 1 - \frac{0.869 \text{ in.}}{12.0 \text{ in.}}$$

$$= 0.928$$

$$A_e = A_n U$$

$$= \left(7.74 \text{ in.}^2\right)(0.928)$$

$$= 7.18 \text{ in.}^2$$

From AISC *Specification* Section D2(b), the available tensile rupture strength of the double-tee connecting element is:

LRFD	ASD
$\phi P_n = \phi F_u A_e$ $= 0.75(65 \text{ ksi})\left(7.18 \text{ in.}^2\right)$ $= 350 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(65 \text{ ksi})\left(7.18 \text{ in.}^2\right)}{2.00}$ $= 233 \text{ kips} > 180 \text{ kips}$ **o.k.**

Check block shear rupture on the double-tee connecting element

The block shear failure path through the double-tee section is assumed to follow along the bolt lines on each side of the web and then perpendicular to the bolt line to the edge of the tee flange. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 and determined as previously discussed.

Shear yielding component:

$$A_{gv} = 2(0.435 \text{ in.})\left[4(3.00 \text{ in.}) + 1\tfrac{1}{2} \text{ in.}\right]$$
$$= 11.7 \text{ in.}^2$$

$$0.60F_y A_{gv} = 0.60(50 \text{ ksi})(11.7 \text{ in.}^2)$$
$$= 351 \text{ kips}$$

Shear rupture component:

$$A_{nv} = 11.7 \text{ in.}^2 - 4.5(2)(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.435 \text{ in.})$$
$$= 7.30 \text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(65 \text{ ksi})\left(7.30 \text{ in.}^2\right)$$
$$= 285 \text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = 2(0.435 \text{ in.})\left[7.96 \text{ in.} - 5.00 \text{ in.} - 1(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right]$$
$$= 1.60 \text{ in.}^2$$

$$U_{bs}F_u A_{nt} = 1(65 \text{ ksi})(1.60 \text{ in.}^2)$$
$$= 104 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_u A_{nv} + U_{bs}F_u A_{nt} = 285 \text{ kips} + 104 \text{ kips}$$
$$= 389 \text{ kips}$$

$$0.60F_y A_{gv} + U_{bs}F_u A_{nt} = 351 \text{ kips} + 104 \text{ kips}$$
$$= 455 \text{ kips}$$

Therefore, $R_n = 389$ kips.

LRFD	ASD
$\phi R_n = 0.75(2)(389\text{ kips})$ $= 584\text{ kips} > 270\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\left(\dfrac{389\text{ kips}}{2.00}\right)$ $= 389\text{ kips} > 180\text{ kips}$ **o.k.**

Bolt bearing and tearout will not control for the WTs because the combined thickness of the double-tee flanges is greater than the thickness of the gusset plate, i.e., 2(0.435 in.) > 0.500 in.

This completes the design calculations for the brace-to-gusset connection. These calculations will be the same for weak and strong-axis connections and for Special Cases 1, 2 and 3 in both the weak and strong axes. However, the calculations that follow will change depending on the case being considered.

Connection Interface Forces

The forces at the gusset-to-beam and gusset-to-column interfaces are determined using the general case of the UFM as discussed in AISC *Manual* Part 13. The geometric properties of the connection configuration given in Figure 5-11 are:

$$e_c = 0 \text{ (neglecting the column web thickness)}$$

$$\begin{aligned} e_b &= \frac{d_b}{2} \\ &= \frac{18.6\text{ in.}}{2} \\ &= 9.30\text{ in.} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{12}{9}\right) \\ &= 53.1^\circ \end{aligned}$$

$$\begin{aligned} \bar{\alpha} &= \frac{31\tfrac{1}{2}\text{ in.}}{2} + \tfrac{1}{2}\text{ in.} \\ &= 16.2\text{ in.} \end{aligned}$$

$$\begin{aligned} \bar{\beta} &= 4\text{ in.} + 5\text{ in.} \\ &= 9.00\text{ in.} \end{aligned}$$

In the preceding calculation, e_c is assumed to equal zero. However, e_c is actually half the column web thickness, or 1.07 in./2 = 0.535 in. If $e_c = 0.535$ in. were used in the calculations, it would result in a force of:

LRFD	ASD
$H_{uc} = \dfrac{(270\text{ kips})(0.535\text{ in.})\cos 53.1^\circ}{(9.00\text{ in.} + 9.30\text{ in.})}$ $= 4.74\text{ kips}$	$H_{ac} = \dfrac{(180\text{ kips})(0.535\text{ in.})\cos 53.1^\circ}{(9.00\text{ in.} + 9.30\text{ in.})}$ $= 3.16\text{ kips}$

Considering the other forces on the connection, this force is negligible and will not change the resulting design in any significant way. Therefore, for ease of calculation, e_c is taken as zero, which results in $H_{uc} = 0$ and $H_{ac} = 0$.

Because $e_c = 0$, α can be taken as $\bar{\alpha}$, and β is computed from:

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \qquad \text{(\textit{Manual} Eq. 13-1)}$$

$$\beta = \frac{\alpha - (e_b \tan\theta) + e_c}{\tan\theta}$$
$$= \frac{16.2 \text{ in.} - \left[(9.30 \text{ in.}) \tan 53.1°\right] + 0}{\tan 53.1°}$$
$$= 2.86 \text{ in.}$$

This value of β is used in all the subsequent equations. The fact that $\beta \neq \bar{\beta}$ is unimportant, because H_{uc} is 0 and therefore, there is no moment $\left[M_c = H_c(\beta - \bar{\beta})\right]$.

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad \text{(\textit{Manual} Eq. 13-6)}$$
$$= \sqrt{(16.2 \text{ in.} + 0)^2 + (2.86 \text{ in.} + 9.30 \text{ in.})^2}$$
$$= 20.3 \text{ in.}$$

LRFD	ASD
$\frac{P_u}{r} = \frac{270 \text{ kips}}{20.3 \text{ in.}}$ $= 13.3 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ub} = \alpha \frac{P_u}{r}$ $= (16.2 \text{ in.})(13.3 \text{ kip/in.})$ $= 215 \text{ kips}$ From AISC *Manual* Equation 13-3: $H_{uc} = e_c \frac{P_u}{r}$ $= (0 \text{ in.})(13.3 \text{ kip/in.})$ $= 0 \text{ kips}$ From AISC *Manual* Equation 13-4: $V_{ub} = e_b \frac{P_u}{r}$ $= (9.30 \text{ in.})(13.3 \text{ kip/in.})$ $= 124 \text{ kips}$ From AISC *Manual* Equation 13-2: $V_{uc} = \beta \frac{P_u}{r}$ $= (2.86 \text{ in.})(13.3 \text{ kip/in.})$ $= 38.0 \text{ kips}$ $\sum V = 124 \text{ kips} + 38.3 \text{ kips}$ $= 162 \text{ kips}$ $(270 \text{ kips})\cos 53.1° = 162 \text{ kips}$ **o.k.**	$\frac{P_a}{r} = \frac{180 \text{ kips}}{20.3 \text{ in.}}$ $= 8.87 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ab} = \alpha \frac{P_a}{r}$ $= (16.2 \text{ in.})(8.87 \text{ kip/in.})$ $= 144 \text{ kips}$ From AISC *Manual* Equation 13-3: $H_{ac} = e_c \frac{P_a}{r}$ $= (0 \text{ in.})(8.87 \text{ kip/in.})$ $= 0 \text{ kips}$ From AISC *Manual* Equation 13-4: $V_{ab} = e_b \frac{P_a}{r}$ $= (9.30 \text{ in.})(8.87 \text{ kip/in.})$ $= 82.5 \text{ kips}$ From AISC *Manual* Equation 13-2: $V_{ac} = \beta \frac{P_a}{r}$ $= (2.86 \text{ in.})(8.87 \text{ kip/in.})$ $= 25.4 \text{ kips}$ $\sum V = 82.5 \text{ kips} + 25.4 \text{ kips}$ $= 108 \text{ kips}$ $(180 \text{ kips})\cos 53.1° = 108 \text{ kips}$ **o.k.**

These admissible forces are shown in Figure 5-12.

The figure shows that the line of action of the gusset-to-beam interface passes through the control point at the centerline of the beam and face of the column web. As discussed previously, this point and the work point differ only by the column web half thickness, which is small compared to the other dimensions and is therefore neglected in the calculations. Because the location of the control point is known and point A is at the center of the gusset-to-beam connection, the interface forces can be calculated as:

LRFD	ASD
$H_{ub} = H_u = 215 \text{ kips}$ $V_{ub} = \dfrac{e_b}{\alpha} H_u$ $= \left(\dfrac{9.30 \text{ in.}}{16.2 \text{ in.}}\right)(215 \text{ kips})$ $= 123 \text{ kips}$	$H_{ab} = H_a = 144 \text{ kips}$ $V_{ab} = \dfrac{e_b}{\alpha} H_a$ $= \left(\dfrac{9.30 \text{ in.}}{16.2 \text{ in.}}\right)(144 \text{ kips})$ $= 82.7 \text{ kips}$

These values are similar to that achieved by the UFM, V_{ub} = 124 kips for LRFD and V_{ab} = 82.5 kips for ASD (the difference is due to rounding). If more significant figures were carried, the results would be exactly the same.

Once V_{ub} (V_{ab}) and H_{ub} (H_{ab}) are known, all the other forces of Figure 5-12 can be determined by a free body diagram. Therefore, for weak-axis connections, either the simplified method given in the foregoing or the formal UFM can be used.

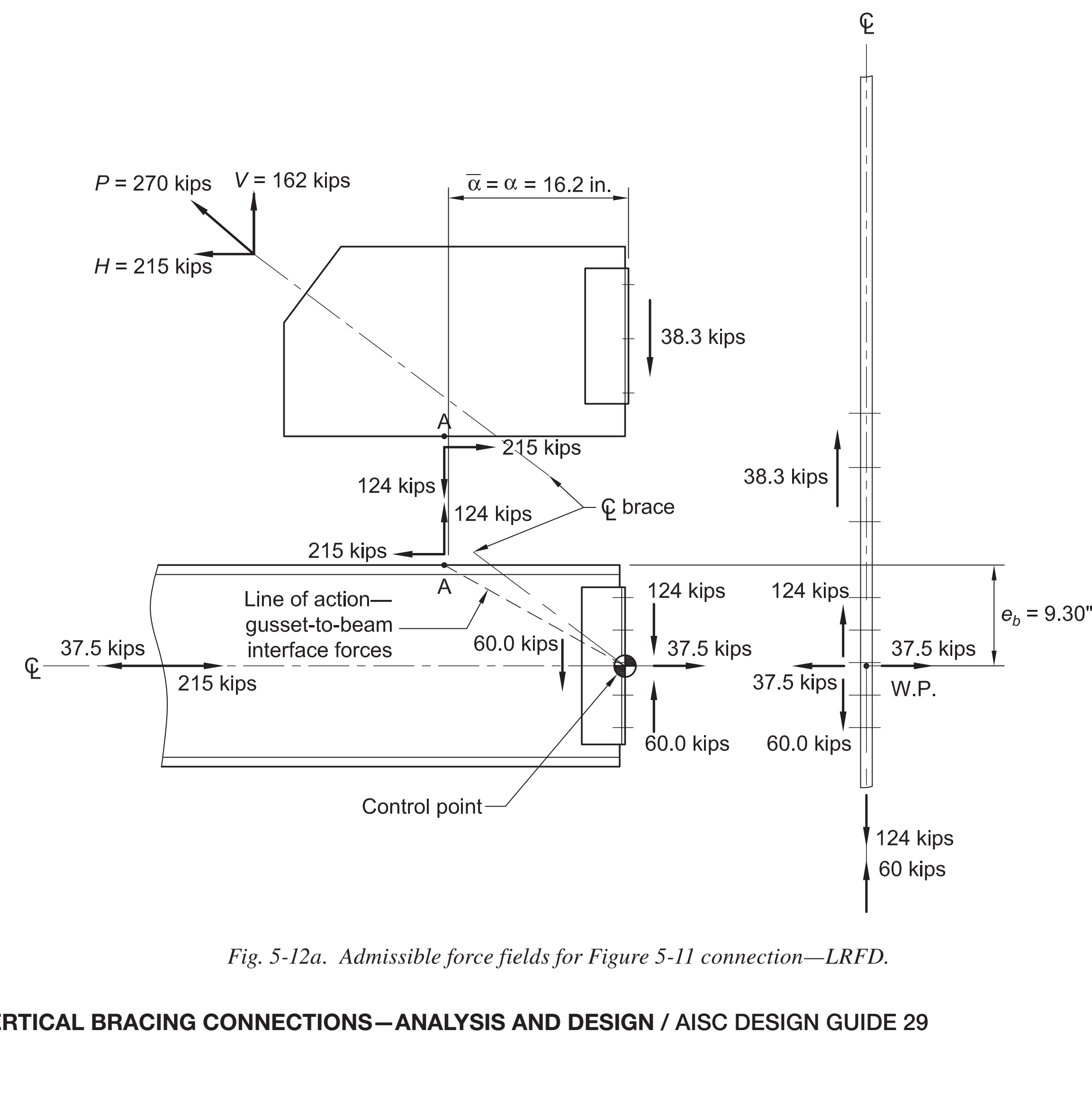


Fig. 5-12a. Admissible force fields for Figure 5-11 connection—LRFD.

Gusset-to-Beam Connection

Check the gusset plate for shear yielding along the beam flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Section J4.2, Equation J4-3, as follows:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(\tfrac{1}{2}\text{ in.})(31\tfrac{1}{2}\text{ in.})$ $= 340\text{ kips} > 215\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(\tfrac{1}{2}\text{ in.})(31\tfrac{1}{2}\text{ in.})}{1.50}$ $= 227\text{ kips} > 144\text{ kips}$ **o.k.**

Check the gusset plate for tensile yielding along the beam flange

The available tensile yielding strength of the gusset plate is determined from AISC *Specification* Section J4.1, Equation J4-1, as follows:

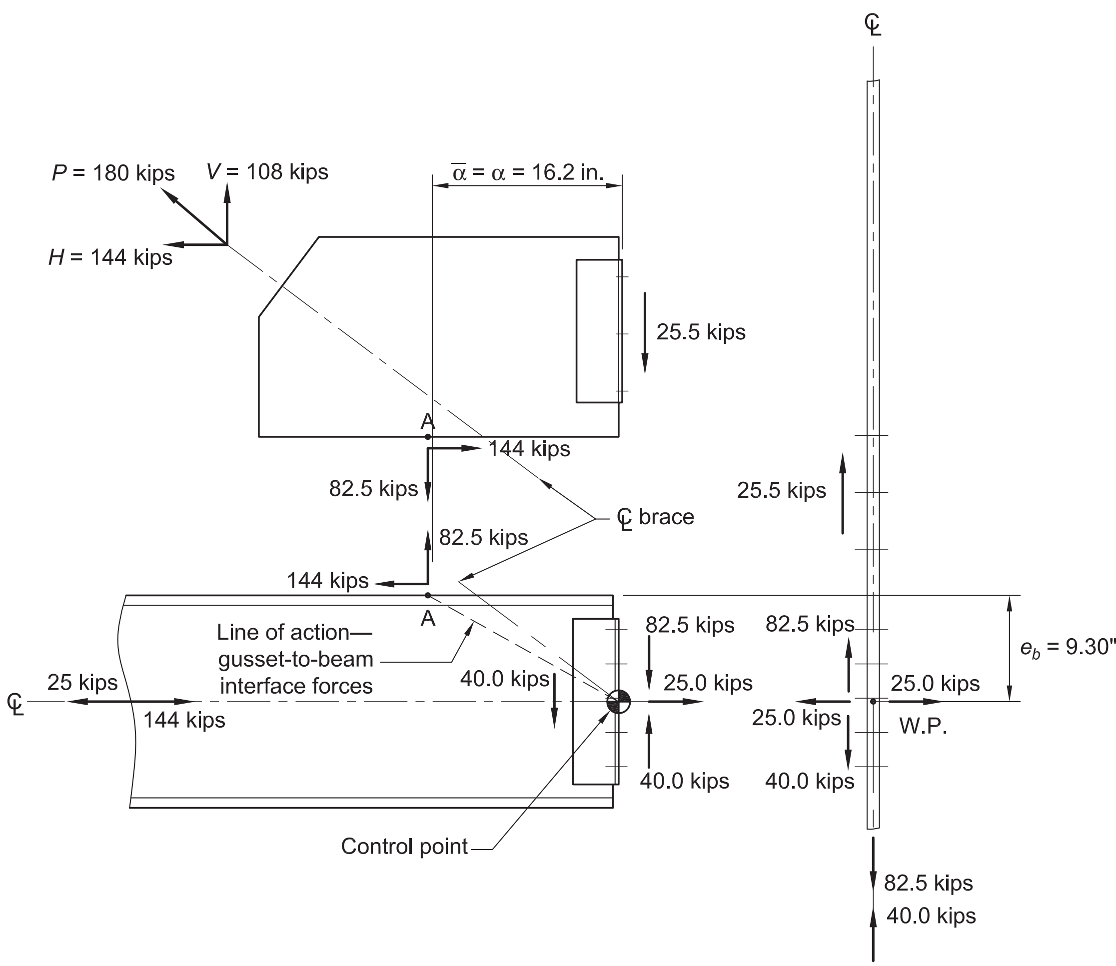


Fig. 5-12b. Admissible force fields for Figure 5-11 connection—ASD.

LRFD	ASD
$\phi N_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(½\text{ in.})(31½\text{ in.})$ $= 510\text{ kips} > 124\text{ kips}$ **o.k.**	$\dfrac{N_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(36\text{ ksi})(½\text{ in.})(31½\text{ in.})}{1.67}$ $= 340\text{ kips} > 82.5\text{ kips}$ **o.k.**

Note that as shown in a previous example, the interaction of the forces at the gusset-to-beam interface is assumed to have no impact on the design.

Design weld at gusset-to-beam flange connection

According to AISC *Manual* Part 13, because the gusset is directly welded to the beam, the weld size determination includes a ductility factor of 1.25. Therefore, the weld is designed for 1.25 times the resultant force.

The strength determination of fillet welds defined in AISC *Specification* Section J2.4 can be simplified as explained in AISC *Manual* Part 8 to AISC *Manual* Equations 8-2a and 8-2b. Also, incorporating the increased strength due to the load angle from AISC *Specification* Equation J2-5, the required weld size is determined as follows:

LRFD	ASD
Resultant load: $R_u = \sqrt{(215\text{ kips})^2 + (124\text{ kips})^2}$ $= 248\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{124\text{ kips}}{215\text{ kips}}\right)$ $= 30.0°$ $D_{req} = \dfrac{1.25R_u}{2l(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)}$ $= \dfrac{1.25(248\text{ kips})}{2(31½\text{ in.})(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5}30°)}$ $= 3.00\text{ sixteenths}$	Resultant load: $R_a = \sqrt{(144\text{ kips})^2 + (82.5\text{ kips})^2}$ $= 166\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{82.5\text{ kips}}{144\text{ kips}}\right)$ $= 29.8°$ $D_{req} = \dfrac{1.25R_a}{2l(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)}$ $= \dfrac{1.25(166\text{ kips})}{2(31½\text{ in.})(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5}29.8°)}$ $= 3.02\text{ sixteenths}$

From AISC *Specification* Table J2.4, the AISC minimum fillet weld size is 3⁄16 in., but many fabricators use ¼ in. minimum; therefore, use ¼-in. fillet welds.

Check beam web local yielding

The normal force is applied at a distance $\alpha = 16.2$ in. from the beam end, which is less than the depth of the W18×97 beam. Therefore, use AISC *Specification* Equation J10-3 to determine the available strength due to beam web local yielding:

LRFD	ASD
$\phi R_n = \phi F_y t_w(2.5k + l_b)$ $= 1.00(50\text{ ksi})(0.535\text{ in.})[2.5(1.27\text{ in.}) + 31\tfrac{1}{2}\text{ in.}]$ $= 928\text{ kips} > 124\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w(2.5k + l_b)}{\Omega}$ $= \dfrac{(50\text{ ksi})(0.535\text{ in.})[2.5(1.27\text{ in.}) + 31\tfrac{1}{2}\text{ in.}]}{1.50}$ $= 618\text{ kips} > 82.5\text{ kips}$ **o.k.**

Check beam web local crippling

The normal force is applied at a distance, $\alpha = 16.2$ in., from the beam end, which is greater than $d/2$, where d is the depth of the W18×97 beam. Therefore, determine the beam web local crippling strength from AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= 0.75(0.80)(0.535\text{ in.})^2\left[1 + 3\left(\dfrac{31\tfrac{1}{2}\text{ in.}}{18.6\text{ in.}}\right)\left(\dfrac{0.535\text{ in.}}{0.870\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(0.870\text{ in.})}{0.535\text{ in.}}}$ $= 910\text{ kips} > 124\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1}{\Omega}0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= \dfrac{1}{2.00}(0.80)(0.535\text{ in.})^2\left[1 + 3\left(\dfrac{31\tfrac{1}{2}\text{ in.}}{18.6\text{ in.}}\right)\left(\dfrac{0.535\text{ in.}}{0.870\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(0.870\text{ in.})}{0.535\text{ in.}}}$ $= 607\text{ kips} > 82.5\text{ kips}$ **o.k.**

Gusset-to-Column Connection

This connection must support a required shear strength of 38.3 kips (LRFD) or 25.5 kips (ASD).

Check bolt shear strength

The required shear strength per bolt (six bolts total) is:

LRFD	ASD
$\dfrac{38.3\text{ kips}}{6} = 6.38\text{ kips/bolt}$	$\dfrac{25.5\text{ kips}}{6} = 4.25\text{ kips/bolt}$

From AISC *Manual* Table 7-1,the available shear strength of 1-in.-diameter ASTM A325-N bolts in single shear is:

LRFD	ASD
$\phi r_{nv} = 31.8\text{ kips/bolt} > 6.38\text{ kips/bolt}$ **o.k.**	$\dfrac{r_{nv}}{\Omega} = 21.2\text{ kips/bolt} > 4.25\text{ kips/bolt}$ **o.k.**

Check bolt bearing on the angles

Check the bearing strength at the top bolts on the angles. The clear distance, based on the 1¼-in. edge distance is:

$$l_c = 1\tfrac{1}{4}\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.})$$
$$= 0.719\text{ in.}$$

Assuming that deformation at the bolt hole is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength.

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \quad \text{(\textit{Spec}. Eq. J3-6a)}$$

The available bearing strength is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.719\text{ in.})(5/16\text{ in.})(58\text{ ksi}) = 11.7\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(5/16\text{ in.})(58\text{ ksi}) = 32.6\text{ kips/bolt}$ Therefore, $\phi r_n = 11.7$ kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(0.719\text{ in.})(5/16\text{ in.})(58\text{ ksi})}{2.00} = 7.82\text{ kips}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(5/16\text{ in.})(58\text{ ksi})}{2.00} = 21.8\text{ kips}$ Therefore, $r_n/\Omega = 7.82$ kips/bolt.

For the top bolts, because the available strength due to bolt bearing on the angles is less than the bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt bearing controls the strength of the top bolts.

For the inner bolts in the angles, the clear distance is:

$$\begin{aligned} l_c &= 5.00\text{ in.} - 1(1\tfrac{1}{16}\text{ in.}) \\ &= 3.94\text{ in.} \end{aligned}$$

The available bearing strength is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(3.94\text{ in.})(5/16\text{ in.})(58\text{ ksi}) = 64.3\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(5/16\text{ in.})(58\text{ ksi}) = 32.6\text{ kips/bolt}$ Therefore, $\phi r_n = 32.6$ kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(3.94\text{ in.})(5/16\text{ in.})(58\text{ ksi})}{2.00} = 42.8\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(5/16\text{ in.})(58\text{ ksi})}{2.00} = 21.8\text{ kips/bolt}$ Therefore, $r_n/\Omega = 21.8$ kips/bolt.

For the inner bolts, because the available shear strength of the bolts is less than the bolt bearing strength, the limit state of bolt shear controls the strength of the inner bolts.

Check bolt bearing on the column web

Because $t_w/2 = 1.07\text{ in.}/2 = 0.535\text{ in.} >$ angle thickness of 5⁄16 in., bolt bearing on the column web will not control the design, by inspection. Note that in this check, one half of the column web thickness is used. The other one half is reserved for connections, if any, on the other side of the column web. If there are no connections on the other side, the entire web thickness of 1.07 in. may be used here. This is the simplest way to deal with this problem, and is correct for connections of the same size and load on both sides. It is conservative for any other case.

Considering the limit states of bolt shear and bolt bearing on the angles, the available strength of the six bolts is:

LRFD	ASD
$\phi R_n = (2\text{ bolts})(11.7\text{ kips/bolt}) + (4\text{ bolts})(31.8\text{ kips/bolt}) = 151\text{ kips} > 38.3\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (2\text{ bolts})(7.82\text{ kips/bolt}) + (4\text{ bolts})(21.2\text{ kips/bolt}) = 100\text{ kips} > 25.5\text{ kips}$ **o.k.**

Check shear yielding on the angles

The available shear yielding strength from AISC *Specification* Section J4.2(a) is:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= \phi 0.60 F_y l_a t_a (2)$ $= 1.00(0.60)(36\text{ ksi})(12\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$ $= 169\text{ kips} > 38.3\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60 F_y l_a t_a (2)}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)}{1.50}$ $= 113\text{ kips} > 25.5\text{ kips}$ **o.k.**

Check shear rupture on the angles

The net area is determined from AISC *Specification* Section B4.3:

$$A_{nv} = \left[12\tfrac{1}{2}\text{ in.} - 3(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{16}\text{ in.})(2)$$
$$= 5.70\text{ in.}^2$$

The available shear rupture strength from AISC *Specification* Section J4.2(b) is:

LRFD	ASD
$\phi V_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(5.70\text{ in.}^2)$ $= 149\text{ kips} > 38.3\text{ kips}$ **o.k.**	$\dfrac{R_{gv}}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(5.70\text{ in.}^2)}{2.00}$ $= 99.2\text{ kips} > 25.5\text{ kips}$ **o.k.**

Check block shear rupture on the angles

The controlling block shear failure path is assumed to start at the top of the angles and follows the vertical bolt lines and then turns perpendicular to the edge of each angle. For the tension area calculated in the following, the net area is based on the length of the short-slotted hole dimension given previously. For the shear area, the net area is based on the width of the short-slotted hole, which is the same as for a standard hole.

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = (12\tfrac{1}{2}\text{ in.} - 1\tfrac{1}{4}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 7.03\text{ in.}^2$$

$$0.60 F_y A_{gv} = 0.60(36\text{ ksi})(7.03\text{ in.}^2)$$
$$= 152\text{ kips}$$

Shear rupture component:

$$A_{nv} = 7.03\text{ in.}^2 - 2.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 5.27\text{ in.}^2$$

$$0.60 F_u A_{nv} = 0.60(58\text{ ksi})(5.27\text{ in.}^2)$$
$$= 183\text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$L_e = \frac{4.00 \text{ in.} + 4.00 \text{ in.} + \tfrac{1}{2} \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2} - 0.50\left(1\tfrac{5}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)$$
$$= 0.813 \text{ in.}$$

$$A_{nt} = tL_e(2)$$
$$= \left(\tfrac{5}{16} \text{ in.}\right)(0.813 \text{ in.})(2)$$
$$= 0.508 \text{ in.}^2$$

$$U_{bs}F_u A_{nt} = 1(58 \text{ ksi})\left(0.508 \text{ in.}^2\right)$$
$$= 29.5 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_u A_{nv} + U_{bs}F_u A_{nt} = 183 \text{ kips} + 29.5 \text{ kips}$$
$$= 213 \text{ kips}$$

$$0.60F_y A_{gv} + U_{bs}F_u A_{nt} = 152 \text{ kips} + 29.5 \text{ kips}$$
$$= 182 \text{ kips}$$

Therefore, $R_n = 182$ kips.

LRFD	ASD
$\phi R_n = 0.75(182 \text{ kips})$ $= 137 \text{ kips} > 38.3 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_{nt} + 0.6F_y A_{gv}}{\Omega}$ $= \dfrac{182 \text{ kips}}{2.00}$ $= 91.0 \text{ kips} > 25.5 \text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

It is possible for the gusset to fail in block shear rupture under the angles at the weld lines. This will be an L-shaped tearout. By inspection, the shear yielding component will control over the shear rupture component; therefore, AISC *Specification* Equation J4-5 reduces to:

$$R_n = 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$$A_{gv} = t_g(L_{ev} + L_{angle})$$
$$= (\tfrac{1}{2} \text{ in.})(2.50 \text{ in.} + 12\tfrac{1}{2} \text{ in.})$$
$$= 7.50 \text{ in.}^2$$

$$A_{nt} = t_g L_{eh}$$
$$= (\tfrac{1}{2} \text{ in.})(3.50 \text{ in.})$$
$$= 1.75 \text{ in.}^2$$

$$U_{bs} = 1$$

The available block shear rupture strength of the gusset plate at the angle-to-gusset plate weld is:

LRFD	ASD
$\phi R_n = \phi(0.60F_y A_{gv} + U_{bs}F_u A_{nt})$ $= 0.75\left[0.60(36 \text{ ksi})\left(7.50 \text{ in.}^2\right) + 1(58 \text{ ksi})\left(1.75 \text{ in.}^2\right)\right]$ $= 198 \text{ kips} > 38.3 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_y A_{gv} + U_{bs}F_u A_{nt}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})\left(7.50 \text{ in.}^2\right) + 1(58 \text{ ksi})\left(1.75 \text{ in.}^2\right)}{2.00}$ $= 132 \text{ kips} > 25.5 \text{ kips}$ **o.k.**

Design gusset plate-to-angles weld

From the AISC *Manual* Table 8-8:

$$
\begin{aligned}
&\text{Angle} = 0^\circ\\
kl &= 3.50 \text{ in.}\\
l &= 12\tfrac{1}{2} \text{ in.}\\
k &= \frac{3.50 \text{ in.}}{12\frac{1}{2} \text{ in.}}\\
&= 0.280
\end{aligned}
$$

The distance to the center of gravity from the vertical weld line can be determined as follows:

$$
\begin{aligned}
x &= \frac{k^2}{1+2k}\\
&= \frac{(0.280)^2}{1+2(0.280)}\\
&= 0.050\\
xl &= 0.050\left(12\tfrac{1}{2} \text{ in.}\right)\\
&= 0.625 \text{ in.}\\
al &= 4.00 \text{ in.} - xl\\
&= 4.00 \text{ in.} - 0.625 \text{ in.}\\
&= 3.38 \text{ in.}\\
a &= \frac{al}{l}\\
&= \frac{3.38 \text{ in.}}{12\frac{1}{2} \text{ in.}}\\
&= 0.270
\end{aligned}
$$

By interpolation:

$C = 2.80$ from Table 8-8

and

$C_1 = 1.0$ for E70XX electrodes

The number of sixteenths of fillet weld required is:

LRFD	ASD
$D_{req} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{38.3 \text{ kips}}{0.75(2.80)(1.0)(12\frac{1}{2} \text{ in.})(2)}$ $= 0.730$ sixteenths	$D_{req} = \dfrac{P_a \Omega}{C C_1 l}$ $= \dfrac{(25.5 \text{ kips})(2.00)}{2.80(1.0)(12\frac{1}{2} \text{ in.})(2)}$ $= 0.729$ sixteenths

Use a minimum fillet weld of 3⁄16 in. according to AISC *Specification* Table J2.4 for a thickness of 5⁄16 in. for the thinner part joined (the angle). Using the formula $t_{min} = 6.19D/F_u$ (AISC *Manual* Equation 9-3) is not required in this case, because the gusset had to be checked for the block shear rupture limit state. Generally, the t_{min} formula should only be used when it is not possible to estimate the stresses in the gusset under the weld.

Beam-to-Column Connection

The maximum forces on this interface, as shown in Figure 5-12, are:

LRFD	ASD
Required shear strength $V_u = 124 \text{ kips} + 60.0 \text{ kips}$ $= 184 \text{ kips}$ Required axial strength $A_{ub} = 37.5 \text{ kips transfer force}$	Required shear strength $V_a = 82.5 \text{ kips} + 40.0 \text{ kips}$ $= 123 \text{ kips}$ Required axial strength $A_{ab} = 25.0 \text{ kips transfer force}$

Except for the 60-kip (LRFD) and 40-kip (ASD) gravity beam shear, these loads can reverse; therefore, the worst case envelope of shear and axial forces is considered.

Design bolts at beam-to-column connection

Using the available strengths from AISC *Manual* Tables 7-1 and 7-2, check the shear and tension per bolt:

LRFD	ASD
$V_u = \dfrac{V_u}{N_b}$ $= \dfrac{184 \text{ kips}}{10 \text{ bolts}}$ $= 18.4 \text{ kips/bolt} < 31.8 \text{ kips}$ **o.k.** $T_u = \dfrac{A_{ub}}{N_b}$ $= \dfrac{37.5 \text{ kips}}{10 \text{ bolts}}$ $= 3.75 \text{ kips/bolt} < 53.0 \text{ kips}$ **o.k.**	$V_a = \dfrac{V_a}{N_b}$ $= \dfrac{123 \text{ kips}}{10 \text{ bolts}}$ $= 12.3 \text{ kips/bolt} < 21.2 \text{ kips}$ **o.k.** $T_a = \dfrac{A_{ab}}{N_b}$ $= \dfrac{25.0 \text{ kips}}{10 \text{ bolts}}$ $= 2.50 \text{ kips/bolt} < 35.3 \text{ kips}$ **o.k.**

The tensile strength requires an additional check due to the combination of tension and shear. From AISC *Specification* Section J3.7 and AISC *Specification* Table J3.2, the available tensile strength of ASTM A325-X bolts subject to tension and shear is:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}\left(\dfrac{18.4 \text{ kips}}{0.785 \text{ in.}^2}\right)$ $= 64.9 \text{ ksi} < 90 \text{ ksi}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}\left(\dfrac{12.3 \text{ kips}}{0.785 \text{ in.}^2}\right)$ $= 64.8 \text{ ksi} < 90 \text{ ksi}$ **o.k.**

LRFD	ASD
The reduced available tensile force per bolt due to combined forces is: $$\phi r_{nt} = \phi F'_{nt} A_b$$ $$= 0.75(64.9\text{ kips})(0.785\text{ in.}^2)$$ $$= 38.2\text{ kips} > 3.75\text{ kips} \quad \textbf{o.k.}$$	The reduced available tensile force per bolt due to combined forces is: $$\phi r_{nt} = \frac{F'_{nt} A_b}{\Omega}$$ $$= \frac{(65.2\text{ ksi})(0.785\text{ in.}^2)}{2.00}$$ $$= 25.6\text{ kips} > 2.50\text{ kips} \quad \textbf{o.k.}$$

Check prying action on the angles

From AISC *Manual* Part 9:

$$b = \frac{gage - t_w - t}{2}$$
$$= \frac{5\tfrac{1}{2}\text{ in.} - 0.535\text{ in.} - \tfrac{1}{2}\text{ in.}}{2}$$
$$= 2.23\text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$
$$= 2.23\text{ in.} - \frac{1.00\text{ in.}}{2}$$
$$= 1.73\text{ in.}$$

$$a = \frac{2(\text{angle leg}) + t_w - \text{gage}}{2}$$
$$= \frac{2(4.00\text{ in.}) + 0.535\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.52\text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= \left(1.52\text{ in.} + \frac{1.00\text{ in.}}{2}\right) \le \left[1.25(2.23\text{ in.}) + \frac{1.00\text{ in.}}{2}\right]$$
$$= 2.02\text{ in.} < 3.29\text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{1.73\text{ in.}}{2.02\text{ in.}}$$
$$= 0.856$$

$$p = \frac{14\tfrac{1}{2}\text{ in.}}{5}$$
$$= 2.90\text{ in.} \le 2b = 2(2.23\text{ in.}) = 4.46\text{ in.}$$

Using the width dimension along the length of the fitting given in AISC *Specification* Table J3.3 for a 1-in.-diameter bolt:

$$d' = 1\tfrac{1}{16}\text{ in.}$$

$$\begin{aligned}\delta &= 1 - \frac{d'}{p} \\ &= 1 - \frac{1\tfrac{1}{16}\text{ in.}}{2.90\text{ in.}} \\ &= 0.634\end{aligned}$$

From AISC *Manual* Part 9, the available tensile strength including prying action effects is determined as follows:

LRFD	ASD
From AISC *Manual* Equation 9-30a: $$\begin{aligned}t_c &= \sqrt{\frac{4Bb'}{\phi p F_u}} \\ &= \sqrt{\frac{4(38.2\text{ kips})(1.73\text{ in.})}{0.90(2.90\text{ in.})(58\text{ ksi})}} \\ &= 1.32\text{ in.}\end{aligned}$$ $$\begin{aligned}\alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\ &= \frac{1}{0.634(1+0.856)}\left[\left(\frac{1.32\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2 - 1\right] \\ &= 5.07\end{aligned}$$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $$\begin{aligned}Q &= \left(\frac{t}{t_c}\right)^2(1+\delta) \\ &= \left(\frac{\tfrac{1}{2}\text{ in.}}{1.32\text{ in.}}\right)^2(1+0.634) \\ &= 0.234\end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned}T_{avail} &= BQ \\ &= (38.2\text{ kips})(0.234) \\ &= 8.94\text{ kips} > 3.75\text{ kips} \quad \textbf{o.k.}\end{aligned}$$	From AISC *Manual* Equation 9-30b: $$\begin{aligned}t_c &= \sqrt{\frac{4\Omega Bb'}{pF_u}} \\ &= \sqrt{\frac{4(1.67)(25.6\text{ kips})(1.73\text{ in.})}{(2.90\text{ in.})(58\text{ ksi})}} \\ &= 1.33\text{ in.}\end{aligned}$$ $$\begin{aligned}\alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\ &= \frac{1}{0.634(1+0.856)}\left[\left(\frac{1.33\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2 - 1\right] \\ &= 5.16\end{aligned}$$ Because $\alpha' > 1$, determine Q from AISC *Manual* Equation 9-34: $$\begin{aligned}Q &= \left(\frac{t}{t_c}\right)^2(1+\delta) \\ &= \left(\frac{\tfrac{1}{2}\text{ in.}}{1.33\text{ in.}}\right)^2(1+0.634) \\ &= 0.231\end{aligned}$$ From AISC *Manual* Equation 9-31: $$\begin{aligned}T_{avail} &= BQ \\ &= (25.6\text{ kips})(0.231) \\ &= 5.91\text{ kips} > 2.50\text{ kips} \quad \textbf{o.k.}\end{aligned}$$

The transfer force, A_{ub} = 37.5 kips for LRFD and A_{ab} = 25.0 kips for ASD, usually comes from the other side of the column, but it could be induced into the bracing from a column shear. If this is the case, the web may "oil can," and the above prying analysis will be incorrect and unconservative. This occurs only when there is no matching connection on the opposite side of the column web. If the connection on the opposite side is also designed for the transfer force, there is no "oil-canning." Figure 5-13 shows this "oil-canning" effect as a yield line problem where the transverse force required to produce the yield line deformation is (Tamboli, 2010):

$$P_n = 8m_p\left[\sqrt{\frac{2h}{h-g}} + \frac{l}{2(h-g)}\right]$$

where

$$m_p = \frac{F_y\, t_w^2}{4}$$

g = gage of bolts, in.
h = clear distance between flange fillets, in.
l = depth of bolt pattern, in.
t_w = column web thickness, in.

thus:

$$m_p = \frac{(50\text{ ksi})(1.07\text{ in.})^2}{4}$$
$$= 14.3\text{ kip-in./in.}$$

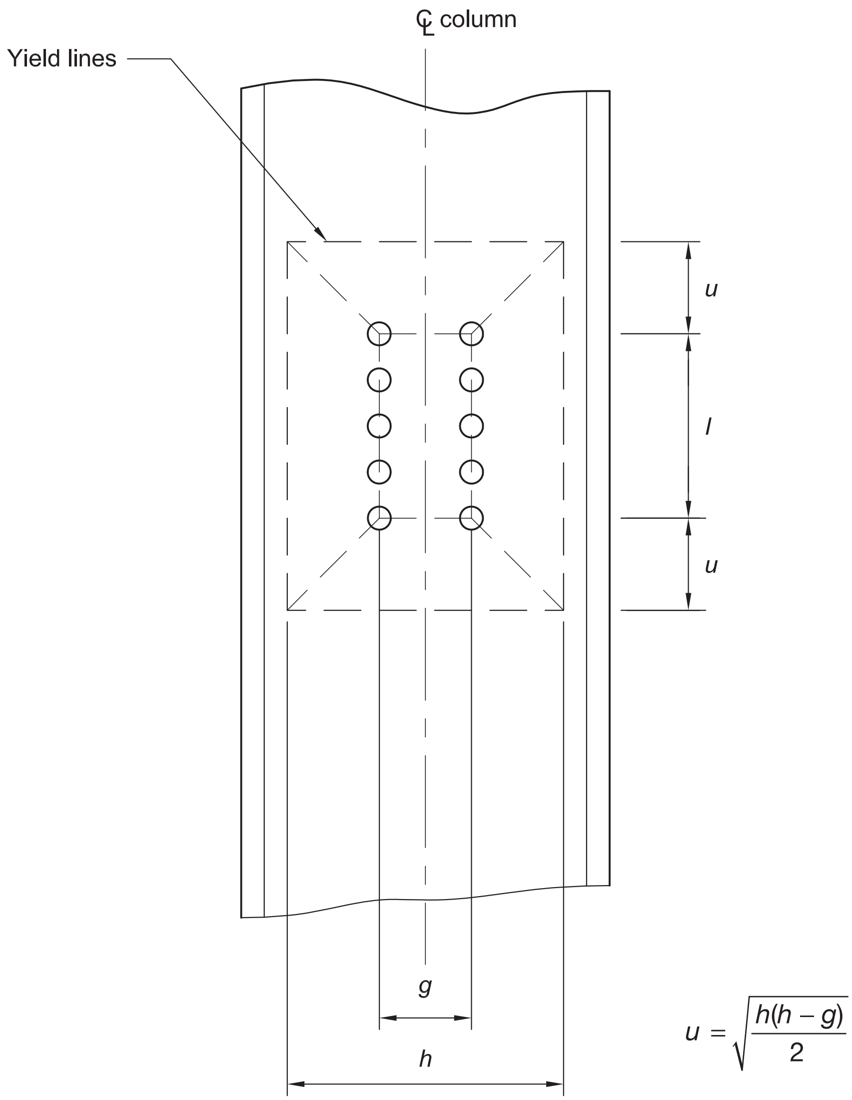


Fig. 5-13. "Oil can" yield line pattern for transversely loaded column web.

Therefore:

$$h = t_w \left(\frac{h}{t_w} \right)$$
$$= (1.07 \text{ in.})(10.7)$$
$$= 11.4 \text{ in.}$$
$$l = 12.0 \text{ in.}$$
$$g = 5\tfrac{1}{2} \text{ in.}$$
$$h - g = 11.4 \text{ in.} - 5\tfrac{1}{2} \text{ in.}$$
$$= 5.90 \text{ in.}$$

Assuming $\phi = 0.90$ and $\Omega = 1.67$, the available tensile strength of the web is:

LRFD	ASD
$\phi P_n = 0.90\left\{8\left(14.3\ \frac{\text{kip-in.}}{\text{in.}}\right)\left[\sqrt{\frac{2(11.4 \text{ in.})}{5.90 \text{ in.}}} + \frac{12.0 \text{ in.}}{2(5.90 \text{ in.})}\right]\right\}$ $= 307 \text{ kips}$	$\frac{P_n}{\Omega} = \frac{1}{1.67}\left\{8\left(14.3\ \frac{\text{kip-in.}}{\text{in.}}\right)\left[\sqrt{\frac{2(11.4 \text{ in.})}{5.90 \text{ in.}}} + \frac{12.0 \text{ in.}}{2(5.90 \text{ in.})}\right]\right\}$ $= 204 \text{ kips}$

The column web itself can carry a transverse load of 307 kips (LRFD) and 204 kips (ASD), but the issue of concern here is stiffness. The authors suggest that unless the oil-can strength of the column web is at least 10 times the specified transfer force the web will not be stiff enough to cause the prying force, q, to develop at the angle tips, i.e., the "a" distance. In the case where the oil-can strength of the column web is less than 10 times the specified transfer force, it is assumed that the web deforms, and q cannot exist because the angle tips separate from the column web. In this case, the prying action analysis must be replaced by a nonprying analysis of the angles, which considers only bending in the angles without the additional prying force. This analysis is provided in AISC *Manual* Part 9, Equation 9-20, specifically the formula for t_{min}, where:

LRFD	ASD
$t_{min} = \sqrt{\frac{4Tb'}{\phi p F_u}}$ Because $\phi P_n = 307 \text{ kips}$ $< 10(37.5 \text{ kips}) = 375 \text{ kips}$ the alternate analysis gives: $t_{min} = \sqrt{\frac{4(3.75 \text{ kips})(1.73 \text{ in.})}{0.90(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 0.414 \text{ in.} < \tfrac{1}{2} \text{ in.}$ **o.k.**	$t_{min} = \sqrt{\frac{4\Omega Tb'}{p F_u}}$ Because $\frac{P_n}{\Omega} = 204 \text{ kips}$ $< 10(25.0 \text{ kips}) = 250 \text{ kips}$ the alternate analysis gives: $t_{min} = \sqrt{\frac{4(1.67)(2.50 \text{ kips})(1.73 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 0.414 \text{ in.} < \tfrac{1}{2} \text{ in.}$ **o.k.**

The 2L4×4×½ have sufficient thickness.

If instead of knowing the required angle thickness, it is desired to know what load the 2L4×4×½ angles can carry, the parameter α' can be set equal to zero, which is equivalent to having the angle legs in single curvature bending due to the prying force, q, being equal to zero.

LRFD	ASD
$t_c = \sqrt{\dfrac{4Bb'}{\phi p F_u}}$ $= 1.32$ in., as before $Q = \left(\dfrac{t}{t_c}\right)^2$ $= \left(\dfrac{\text{½ in.}}{\text{1.32 in.}}\right)^2$ $= 0.143$ $T_{avail} = BQ$ $= (38.2 \text{ kips})(0.143)$ $= 5.46 \text{ kips} > 3.75 \text{ kips}$ **o.k.**	$t_c = \sqrt{\dfrac{4\Omega Bb'}{p F_u}}$ $= 1.33$ in., as before $Q = \left(\dfrac{t}{t_c}\right)^2$ $= \left(\dfrac{\text{½ in.}}{\text{1.33 in.}}\right)^2$ $= 0.141$ $T_{avail} = BQ$ $= (25.6 \text{ kips})(0.141)$ $= 3.61 \text{ kips} > 2.50 \text{ kips}$ **o.k.**

Check bolt bearing on the angles

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

For the inner bolts, the clear distance is:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - (0.5 + 0.5)d_h \\ &= 3.00 \text{ in.} - 1(1\tfrac{1}{16} \text{ in.}) \\ &= 1.94 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.94 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 50.6$ kips $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 52.2$ kips Therefore, $\phi R_n = 50.6$ kips.	$1.2 l_c t F_u/\Omega = 1.2(1.94 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})/2.00$ $= 33.8$ kips $2.4 d t F_u/\Omega = 2.4(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})/2.00$ $= 34.8$ kips Therefore, $R_n/\Omega = 33.8$ kips.

Because the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the inner bolts.

For the end bolts:

$$\begin{aligned} l_c &= 1\tfrac{1}{4} \text{ in.} - 0.5d_h \\ &= 1\tfrac{1}{4} \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.}) \\ &= 0.719 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.719\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 18.8\text{ kips}$ $\phi 2.4 dtF_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 52.2\text{ kips}$ Therefore, $\phi R_n = 18.8$ kips.	$1.2 l_c t F_u/\Omega = 1.2(0.719\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})/2.00$ $= 12.5\text{ kips}$ $2.4 dtF_u/\Omega = (2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})/2.00$ $= 34.8\text{ kips}$ Therefore, $R_n/\Omega = 12.5$ kips.

This controls over the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD).

The total available bolt strength at the angles, including the limit states of bolt shear and bolt bearing, is:

LRFD	ASD
$\phi R_n = 18.8\text{ kips }(2) + 31.8\text{ kips }(8)$ $= 292\text{ kips} > 184\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 12.5\text{ kips}(2) + 21.2\text{ kips}(8)$ $= 195\text{ kips} > 123\text{ kips}$ **o.k.**

Check bolt bearing on the column web

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 dtF_u \quad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

Tearout, the limit state captured by the first part of Equation J3-6a, is not applicable to the column because there is no edge through which the bolts can tear out.

LRFD	ASD
$\phi r_n = \phi 2.4 dtF_u$ $= 0.75(2.4)(1.00\text{ in.})\left(\dfrac{1.07\text{ in.}}{2}\right)(65\text{ ksi})$ $= 62.6\text{ kips}$	$\dfrac{r_n}{\Omega} = \dfrac{2.4 dtF_u}{\Omega}$ $= \dfrac{2.4(1.00\text{ in.})\left(\dfrac{1.07\text{ in.}}{2}\right)(65\text{ ksi})}{2.00}$ $= 41.7\text{ kips}$

Dividing the web by 2 in the above calculation assumes that the same connection and load exist on both sides of the web. It is conservative for other cases.

Because the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the inner bolts. Thus, the total available bolt strength on the column web, including the limit states of bolt shear and bolt bearing, is:

LRFD	ASD
$\phi R_n = (31.8\text{ kips})(10\text{ bolts})$ $= 318\text{ kips} > 184\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (21.2\text{ kips})(10\text{ bolts})$ $= 212\text{ kips} > 123\text{ kips}$ **o.k.**

Design beam web-to-angle weld

The resultant load and load angle are:

LRFD	ASD
$P_u = \sqrt{(184 \text{ kips})^2 + (37.5 \text{ kips})^2}$ $= 188 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{37.5 \text{ kips}}{184 \text{ kips}}\right)$ $= 11.5°$	$P_a = \sqrt{(123 \text{ kips})^2 + (25.0 \text{ kips})^2}$ $= 126 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{25.0 \text{ kips}}{123 \text{ kips}}\right)$ $= 11.5°$

AISC *Manual* Table 8-8 provides coefficients, C, for angle values, θ, equal to 0° and 15°. It could be unconservative to interpolate between the two tables; therefore, the *Manual* discussion recommends that the value from the table with the next lower angle be used, which is 0° in this case. Also, the required force, P_u or P_a, will equal the vertical load at the beam-to-column interface. From AISC *Manual* Table 8-8 and the geometry in Figure 5-11:

$$\text{Angle} = 0°$$
$$kl = 3.50 \text{ in.}$$
$$l = 14\tfrac{1}{2} \text{ in.}$$
$$k = \frac{3.50 \text{ in.}}{14\tfrac{1}{2} \text{ in.}} = 0.241$$

The distance to the center of gravity from the vertical weld line can be determined as follows:

$$\begin{aligned}
x &= \frac{k^2}{1+2k} \\
&= \frac{(0.241)^2}{1+2(0.241)} \\
&= 0.0392 \\
al &= 4.00 \text{ in.} - xl \\
&= 4.00 \text{ in.} - 0.0392(14\tfrac{1}{2} \text{ in.}) \\
&= 3.43 \\
a &= \frac{3.43}{14\tfrac{1}{2} \text{ in.}} \\
&= 0.237
\end{aligned}$$

By interpolation:

$C = 2.73$ from AISC *Manual* Table 8-8

and

$C_1 = 1.0$ for E70XX electrodes

The number of sixteenths of fillet weld required is:

LRFD	ASD
$D_{req} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{184 \text{ kips}}{0.75(2.73)(1.0)(14\tfrac{1}{2} \text{ in.})(2)}$ $= 3.10 \text{ sixteenths}$	$D_{req} = \dfrac{P_a \Omega}{C C_1 l}$ $= \dfrac{(123 \text{ kips})(2.00)}{2.73(1.0)(14\tfrac{1}{2} \text{ in.})(2)}$ $= 3.11 \text{ sixteenths}$

Use a ¼-in. fillet weld, which is also the minimum fillet weld size required according to AISC *Specification* Table J2.4.

Check shear rupture strength of the beam web under the weld

This is a situation where it is not possible by simple analysis to determine the stresses in the beam web under the fillet welds. Therefore, use the following equation to determine the minimum web thickness that will match its available shear rupture strength to that of the weld. Because there is a connection on both sides of the beam web, use AISC *Manual* Equation 9-3:

LRFD	ASD
$t_{min} = \dfrac{6.19D}{F_u}$ $= \dfrac{6.19(3.10 \text{ sixteenths})}{65 \text{ ksi}}$ $= 0.295 \text{ in.} < 0.535 \text{ in.}$ **o.k.**	$t_{min} = \dfrac{6.19D}{F_u}$ $= \dfrac{6.19(3.11 \text{ sixteenths})}{65 \text{ ksi}}$ $= 0.296 \text{ in.} < 0.535 \text{ in.}$ **o.k.**

Note that the calculated required weld size is used rather than the rounded up nominal size of ¼ in. in this calculation.

Check block shear rupture on the beam web

If the tensile transfer force is large enough, it is possible for block shear to control. The beam web can tear out in block shear at the weld lines only due to the axial (not the shear) force in the beam. Therefore, the shear and tension areas are relative to the axial force. This will be a C-shaped failure path. By inspection, the shear yielding component will control over the shear rupture component; therefore, AISC *Specification* Equation J4-5 reduces to:

$$R_n = 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$A_{gv} = t_w l(2)$
$= (0.535 \text{ in.})(3.50 \text{ in.})(2)$
$= 3.75 \text{ in.}^2$

$A_{gt} = t_w l$
$= (0.535 \text{ in.})(14\tfrac{1}{2} \text{ in.})$
$= 7.76 \text{ in.}^2$

$U_{bs} = 1$

The available block shear rupture strength of the beam web at the double angle-to-beam web weld is:

LRFD	ASD
$\phi R_n = \phi(0.60F_yA_{gv} + U_{bs}F_uA_{nt})$ $= 0.75[0.60(50\text{ ksi})(3.75\text{ in.}^2) + 1(65\text{ kips})(7.76\text{ in.}^2)]$ $= 463\text{ kips} > 37.5\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_yA_{gv} + U_{bs}F_uA_{nt}}{\Omega}$ $= \dfrac{(65\text{ ksi})(7.76\text{ in.}^2) + 0.60(50\text{ ksi})(3.75\text{ in.}^2)}{2.00}$ $= 308\text{ kips} > 25.0\text{ kips}$ **o.k.**

Block shear on the outstanding angle legs should also be checked as was done for the gusset-to-column connection. It will not control.

Check shear yielding on the double angle

$$A_{gv} = (14\tfrac{1}{2}\text{ in.})(\tfrac{1}{2}\text{ in.})(2)$$
$$= 14.5\text{ in.}^2$$

From AISC *Specification* Equation J4-3, the available shear yielding strength of the double angle is:

LRFD	ASD
$\phi R_n = \phi 0.60F_yA_{gv}$ $= 1.00(0.60)(36\text{ ksi})(14.5\text{ in.}^2)$ $= 313\text{ kips} > 184\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.6F_yA_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(14.5\text{ in.}^2)}{1.50}$ $= 209\text{ kips} > 123\text{ kips}$ **o.k.**

Check shear rupture on the double angle

$$A_{nv} = A_g - 2(5t)(d_h + \tfrac{1}{16}\text{ in.})$$
$$= 14\tfrac{1}{2}\text{ in.}^2 - 2(5)(\tfrac{1}{2}\text{ in.})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})$$
$$= 8.88\text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi R_n = \phi 0.60F_uA_{nv}$ $= 0.75(0.60)(58\text{ ksi})(8.88\text{ in.}^2)$ $= 232\text{ kips} > 184\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_uA_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(8.88\text{ in.}^2)}{2.00}$ $= 155\text{ kips} > 123\text{ kips}$ **o.k.**

Example 5.6—Corner Connection-to-Column Web: Uniform Force Method Special Case 1

Given:

When the work point is moved from the gravity axis location to a point near the gusset corner, a substantial reduction in gusset size can be achieved. This can be seen by comparing Figures 5-11 and 5-14. In addition to a reduction in gusset size, moving the work point to the corner of the gusset can also result in less demand on the connections and therefore a more economical

structure. However, the beam and the column, if the column is oriented flange-to-view in elevation, must be designed for the resulting eccentricities. If the additional moments cause an increase in member size, then this increase in weight must be weighed against the potential decrease in labor to determine the most economical design.

In Figure 5-14, the work point is located at the center of the column web at the elevation of the beam top flange. This point is very close to the gusset corner and is used in geometry layout because it is known before the connection is designed. The actual position of the gusset corner is not known at this time because the setback may change during fabrication. The brace bevel is calculated from this work point. This example will illustrate the design of the connection shown in Figure 5-14. The theory was discussed in Section 4.2.2.

The details for this connection are the same as those given for Example 5.5.

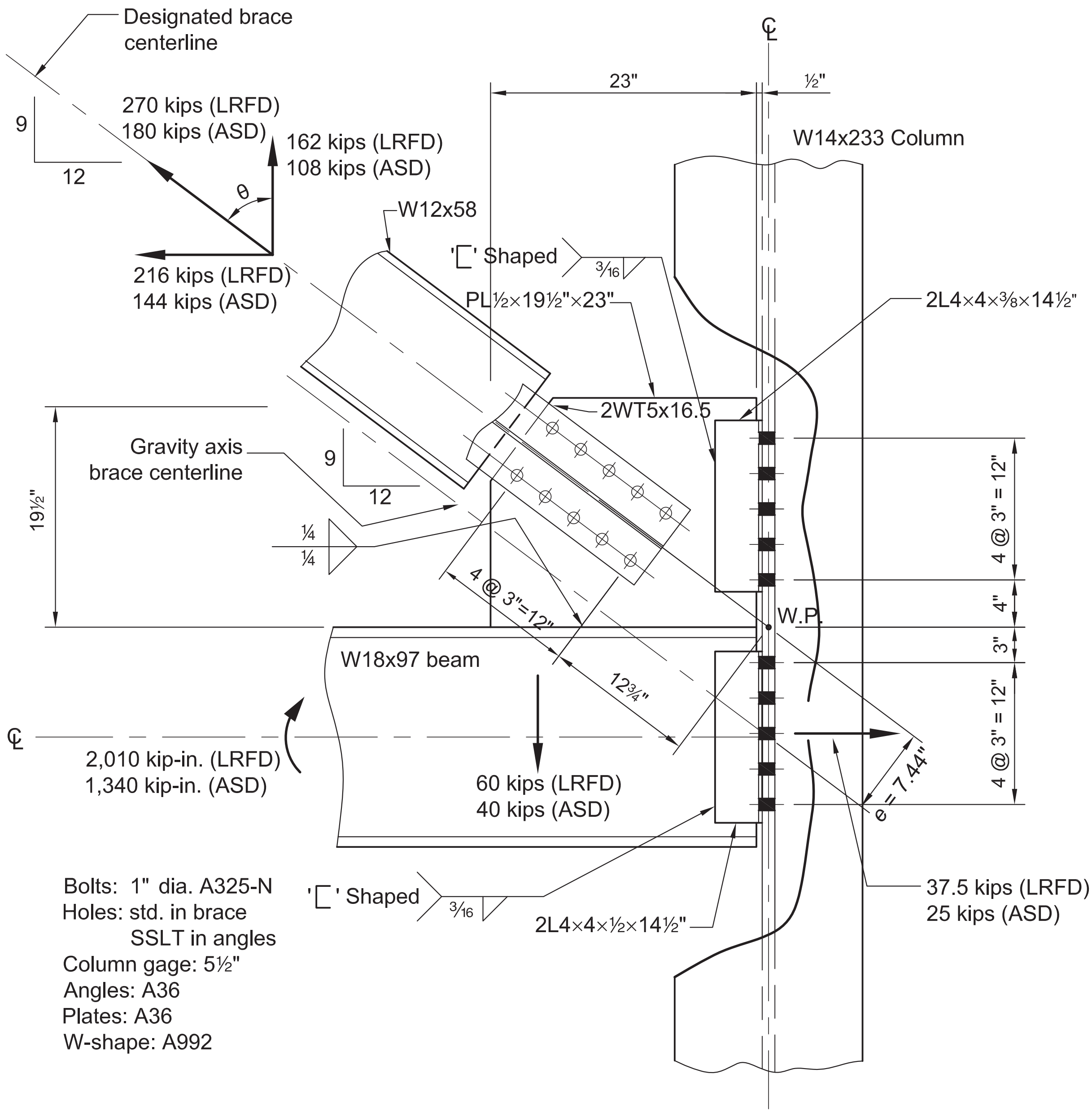


Fig. 5-14. Special Case 1—weak axis.

Solution:

See the Example 5.5 Solution for material and geometric properties, and hole dimensions.

Brace-to-Gusset Connection

Note that checks of the following limit states were included in Example 5.5 and will not be repeated here: bolt shear checks; tensile yielding, tensile rupture, block shear rupture, and bolt bearing on the brace; tensile yielding, block shear rupture, and bolt bearing on the gusset plate; and tensile yielding, tensile rupture, and block shear rupture on the double-tee connecting elements. The limit state for compression buckling on the gusset plate will be affected due to the different gusset plate size and is checked in the following.

Check the gusset plate for compression buckling on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section is:

$$\begin{aligned} l_w &= 5 \text{ in.} + 2(12 \text{ in.})(\tan 30°) \\ &= 18.9 \text{ in.} \end{aligned}$$

Use $K = 0.5$ (Gross, 1990) and the length of buckling, L, is given in Figure 5-14.

$$\begin{aligned} \frac{KL}{r} &= \frac{0.5(12\tfrac{3}{4} \text{ in.})\sqrt{12}}{\tfrac{1}{2} \text{ in.}} \\ &= 44.2 \end{aligned}$$

From AISC *Specification* Section J4.4 and using values from AISC *Manual* Table 4-22, the available strength is:

LRFD	ASD
$\phi_c F_{cr} = 29.3 \text{ ksi}$ $\phi R_n = \phi_c F_{cr} A_g$ $= (29.3 \text{ ksi})(\tfrac{1}{2} \text{ in.})(18.9 \text{ in.})$ $= 277 \text{ kips} > 270 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 19.5 \text{ ksi}$ $\dfrac{R_n}{\Omega} = \dfrac{F_{cr} A_g}{\Omega_c}$ $= (19.5 \text{ ksi})(\tfrac{1}{2} \text{ in.})(18.9 \text{ in.})$ $= 184 \text{ kips} > 180 \text{ kips}$ **o.k.**

Connection Interface Forces

With the brace-to-gusset connection sized, the gusset minimum size can be determined as shown in Figure 5-14. The preliminary gusset size is 19½ in. vertical by 23 in. horizontal. Thus, using terminology from AISC *Manual* Part 13 and dimensions given in Figure 5-14:

$$\begin{aligned} \alpha &= \bar{\alpha} \\ &= \frac{23.0 \text{ in.}}{2} + \tfrac{1}{2} \text{ in.} \\ &= 12.0 \text{ in.} \end{aligned}$$

where α is measured from the face of the column web.

$$\begin{aligned} \tan\theta &= \frac{12}{9} \\ \theta &= \tan^{-1}\left(\frac{12}{9}\right) \\ &= 53.1° \end{aligned}$$

$$e_b = \frac{18.6 \text{ in.}}{2}$$
$$= 9.30 \text{ in.}$$
$$e_c = 0$$

The theoretical value of β such that the connection interfaces are free of moments is:

$$\beta = \frac{\alpha - e_b \tan\theta + e_c}{\tan\theta} \quad \text{(from } \textit{Manual} \text{ Eq. 13-1)}$$
$$= \frac{12.0 \text{ in.} - (9.30 \text{ in.})\left(\frac{12}{9}\right) + 0 \text{ in.}}{\left(\frac{12}{9}\right)}$$
$$= -0.300 \text{ in.}$$

The value of β here is used in subsequent UFM calculations but is otherwise irrelevant because H_{uc} is 0 and therefore there is no moment, $\left[M_c = H_c\left(\beta - \bar{\beta}\right)\right]$.

Note that the work point location will generate shear only on the gusset edges. The calculations shown here are provided for reference to show the general method.

In determining α and β, the gravity brace centerline (that is, the centerline of the brace if it framed into the beam and column centerlines as in Example 5.5) shown in Figure 5-14 is used. The bevel used is the actual bevel. The actual bevel and the gravity bevel will be the same if the connection at the other end of the brace is to a column web and the beam size at the next floor of W18×97 is also the same, which is assumed to be the case here. If they are not the same, the actual bevel should be used in all of the calculations. The fictitious gravity brace centerline of Figure 5-14 is parallel to the actual (true) brace centerline and therefore has the same bevel.

The required axial and shear forces at the gusset-to-beam and gusset-to-column interface are determined as follows:

$$r = \sqrt{\left(\alpha + e_c\right)^2 + \left(\beta + e_b\right)^2} \quad \text{(}\textit{Manual} \text{ Eq. 13-6)}$$
$$= \sqrt{\left(12.0 \text{ in.} + 0 \text{ in.}\right)^2 + \left(-0.300 \text{ in.} + 9.30 \text{ in.}\right)^2}$$
$$= 15.0 \text{ in.}$$

LRFD	ASD
$\frac{P_u}{r} = \frac{270 \text{ kips}}{15.0 \text{ in.}}$ $= 18.0 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ub} = \alpha \frac{P_u}{r}$ $= (12.0 \text{ in.})(18.0 \text{ kip/in.})$ $= 216 \text{ kips}$	$\frac{P_a}{r} = \frac{180 \text{ kips}}{15.0 \text{ in.}}$ $= 12.0 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ab} = \alpha \frac{P_a}{r}$ $= (12.0 \text{ in.})(12.0 \text{ kip/in.})$ $= 144 \text{ kips}$

LRFD	ASD
From AISC *Manual* Equation 13-3: $$H_{uc} = e_c \frac{P_u}{r}$$ $$= (0 \text{ in.})(18 \text{ kip/in.})$$ $$= 0 \text{ kips}$$ $$\Sigma H_u = 216 \text{ kips} + 0 \text{ kips}$$ $$= 216 \text{ kips}$$ $$(270 \text{ kips})\sin 53.1° = 216 \text{ kips} \quad \textbf{o.k.}$$	From AISC *Manual* Equation 13-3: $$H_{ac} = e_c \frac{P_a}{r}$$ $$= (0 \text{ in.})(12 \text{ kip/in.})$$ $$= 0 \text{ kips}$$ $$\Sigma H_a = 144 \text{ kips} + 0 \text{ kips}$$ $$= 144 \text{ kips}$$ $$(180 \text{ kips})\sin 53.1° = 144 \text{ kips} \quad \textbf{o.k}$$
From AISC *Manual* Equation 13-4: $$V_{ub} = e_b \frac{P_u}{r}$$ $$= (9.30 \text{ in.})(18.0 \text{ kip/in.})$$ $$= 167 \text{ kips}$$	From AISC *Manual* Equation 13-4: $$V_{ab} = e_b \frac{P_a}{r}$$ $$= (9.30 \text{ in.})(12.0 \text{ kip/in.})$$ $$= 112 \text{ kips}$$
From AISC *Manual* Equation 13-2: $$V_{uc} = \beta \frac{P_u}{r}$$ $$= (-0.300 \text{ in.})(18.0 \text{ kip/in.})$$ $$= -5.40 \text{ kips}$$ $$\Sigma V_a = 167 \text{ kips} - 5.40 \text{ kips}$$ $$= 162 \text{ kips}$$ $$(270 \text{ kips})\cos 53.1° = 162 \text{ kips} \quad \textbf{o.k.}$$	From AISC *Manual* Equation 13-2: $$V_{ac} = \beta \frac{P_a}{r}$$ $$= (-0.300 \text{ in.})(12.0 \text{ kip/in.})$$ $$= -3.60 \text{ kips}$$ $$\Sigma V_a = 112 \text{ kips} - 3.60 \text{ kips}$$ $$= 108 \text{ kips}$$ $$(180 \text{ kips})\cos 53.1° = 108 \text{ kips} \quad \textbf{o.k.}$$

These forces are shown in Figures 5-15a(a) and 5-15b(a) (UFM admissible forces for gravity axis work point). Superimposed on the above forces are those caused by the nongravity axis work point.

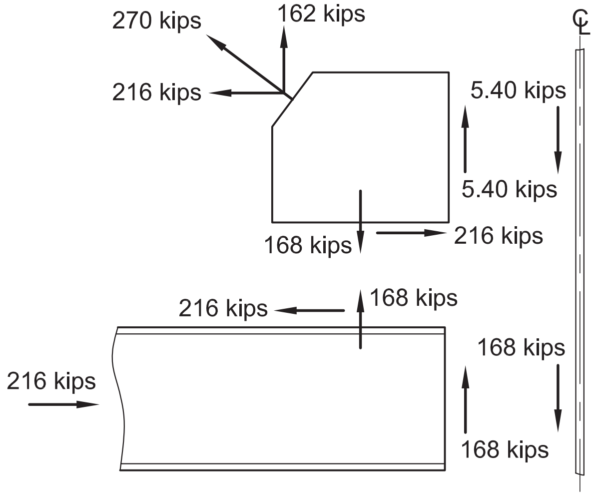
From Figure 4-13,

$$\begin{aligned} e &= (e_b - y)\sin\theta - (e_c - x)\cos\theta \\ &= (9.30 \text{ in.} - 0 \text{ in.})\sin 53.1° - (0 \text{ in.} - 0 \text{ in.})\cos 53.1° \\ &= 7.44 \text{ in.} \end{aligned}$$

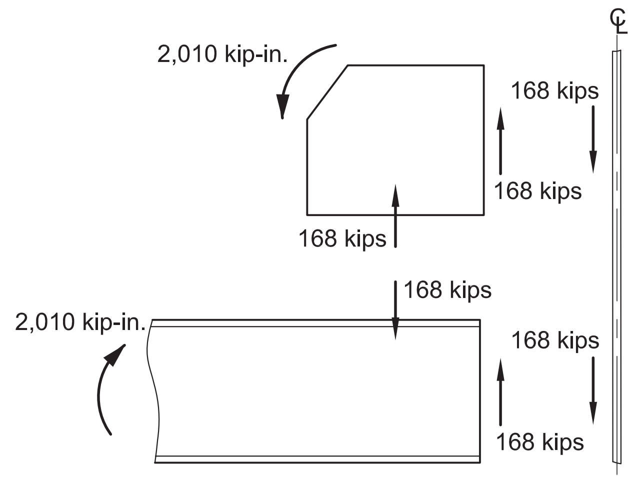
The required flexural strength of the beam is:

LRFD	ASD
$$\begin{aligned} M_u &= P_u e \\ &= (270 \text{ kips})(7.44 \text{ in.}) \\ &= 2{,}010 \text{ kip-in.} \end{aligned}$$	$$\begin{aligned} M_a &= P_a e \\ &= (180 \text{ kips})(7.44 \text{ in.}) \\ &= 1{,}340 \text{ kip-in.} \end{aligned}$$

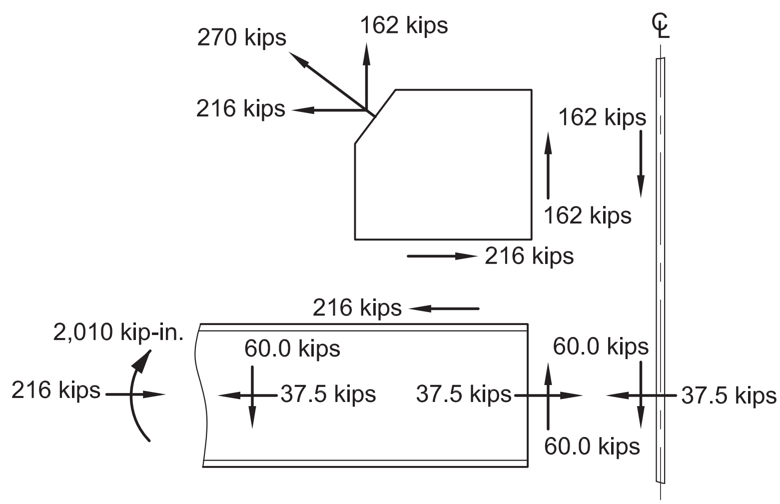
From Section 4.2.2, the quantity η, which is the proportion of the required flexural strength, M_u for LRFD and M_a for ASD, that goes to the beam, is 1.0. This is because the connection to a column web does not mobilize the column stiffness $(I/L)_{col}$ or strength (Z_{col}). The web simply deforms and all of the moment goes to the beam. This distribution was confirmed by Gross (1990) using full scale tests. With no flange engagement, $\eta = 1.0$, and from Section 4.2.2:



(a) UFM admissible forces for gravity axis work point

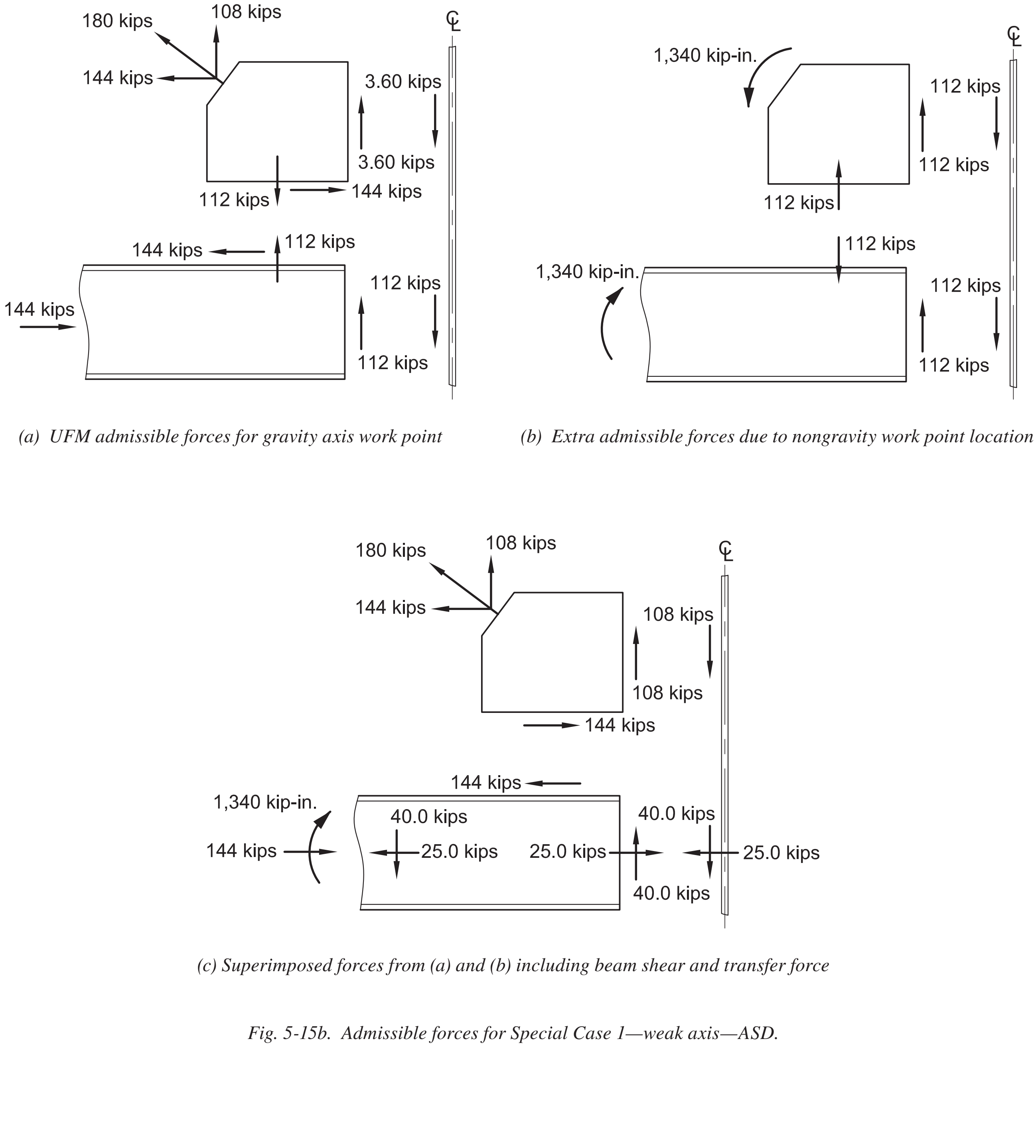


(b) Extra admissible forces due to nongravity work point location



(c) Superimposed forces from (a) and (b) including beam shear and transfer force

Fig. 5-15a. Admissible forces for Special Case 1—weak axis—LRFD.



(a) UFM admissible forces for gravity axis work point

(b) Extra admissible forces due to nongravity work point location

(c) Superimposed forces from (a) and (b) including beam shear and transfer force

Fig. 5-15b. Admissible forces for Special Case 1—weak axis—ASD.

LRFD	ASD
From Equation 4-5: $$H'_u = \frac{(1-\eta)M_u}{\bar{\beta}+e_b} = 0$$ From Equation 4-6: $$V'_u = \frac{M_u - H'_u\beta}{\bar{\alpha}}$$ $$= \frac{2{,}010 \text{ kip-in.} - (0 \text{ kips})(-0.300 \text{ in.})}{12.0 \text{ in.}}$$ $$= 168 \text{ kips}$$	From Equation 4-5: $$H'_a = \frac{(1-\eta)M_a}{\bar{\beta}+e_b} = 0$$ From Equation 4-6: $$V'_a = \frac{M_a - H'_a\beta}{\bar{\alpha}}$$ $$= \frac{1{,}340 \text{ kip-in.} - (0 \text{ kips})(-0.300 \text{ in.})}{12.0 \text{ in.}}$$ $$= 112 \text{ kips}$$

These forces are shown in Figure 5-15a(b) and Figure 5-15b(b) (extra admissible forces due to nongravity work point location). The superimposed forces are also shown in Figure 5-15a(c) and Figure 5-15b(c).

As mentioned earlier in this example, the fact that $\beta - \bar{\beta} \neq 0$ is irrelevant because it is not used in the calculations. The value of $\bar{\beta}$, which is not used, is 10 in. for this example.

Note that the superimposed force distribution of Figure 5-15a(c) and Figure 5-15b(c) is exactly what is obtained from the AISC *Manual* Part 13 presentation of the UFM, Special Case 1. The connection shown in Figure 5-14 is designed for the superimposed interface forces shown in Figure 5-15a(c) and Figure 5-15b(c). The 2,010 kip-in. (LRFD) or 1,340 kip-in. (ASD) moment must be considered in the design of the W18×97 beam.

Gusset-to-Beam Connection

Referring to Figure 5-15a(c) and 5-15b(c) the forces on the interface are:

LRFD	ASD
Required shear strength, V_u = 216 kips Required axial strength, N_u = 0 kips Required flexural strength, M_u = 0 kip-in.	Required shear strength, V_a = 144 kips Required axial strength, N_a = 0 kips Required flexural strength, M_a = 0 kip-in.

Check the gusset plate for shear yielding along the beam flange

Use a gusset plate thickness of ½ in. as shown in Figure 5-14. The available shear yielding strength of the gusset plate is determined from AISC *Specification* Section J4.2, Equation J4-3, as follows:

LRFD	ASD
$$\phi V_n = \phi 0.60 F_y A_{gv}$$ $$= 1.00(0.60)(36 \text{ ksi})(\tfrac{1}{2} \text{ in.})(23.0 \text{ in.})$$ $$= 248 \text{ kips} > 216 \text{ kips} \quad \textbf{o.k.}$$	$$\frac{V_n}{\Omega} = \frac{0.60 F_y A_{gv}}{\Omega}$$ $$= \frac{0.60(36 \text{ ksi})(\tfrac{1}{2} \text{ in.})(23.0 \text{ in.})}{1.50}$$ $$= 166 \text{ kips} > 144 \text{ kips} \quad \textbf{o.k.}$$

Design weld at gusset-to-beam flange connection

The strength determination of fillet welds defined in AISC *Specification* Section J2.4 can be simplified as explained in AISC *Manual* Part 8 to AISC *Manual* Equations 8-2a and 8-2b. The required number of sixteenths of fillet weld on each side of the gusset plate is:

LRFD	ASD
$D_{req} = \dfrac{R_u}{2l(1.392 \text{ kip/in.})}$ $= \dfrac{216 \text{ kips}}{2(23.0 \text{ in.})(1.392 \text{ kip/in.})}$ $= 3.37 \text{ sixteenths}$	$D_{req} = \dfrac{R_a}{2l(0.928 \text{ kip/in.})}$ $= \dfrac{144 \text{ kips}}{2(23.0 \text{ in.})(0.928 \text{ kip/in.})}$ $= 3.37 \text{ sixteenths}$

The weld ductility factor is not used for welds that resist shear forces only.

From AISC *Specification* Table J2.4, the minimum fillet weld size is 3⁄16 in. Use a ¼-in. two-sided fillet weld.

Gusset-to-Column Connection

The forces at the gusset plate-to-column flange interface are:

LRFD	ASD
Required shear strength, $V_u = 162$ kips Required axial strength, $N_u = 0$ kips Required flexural strength, $M_u = 0$ kip-in.	Required shear strength, $V_a = 108$ kips Required axial strength, $N_a = 0$ kips Required flexural strength, $M_a = 0$ kip-in.

Check the gusset plate for shear yielding along the column web

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Section J4.2, Equation J4-3, as follows:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(\tfrac{1}{2} \text{ in.})(19\tfrac{1}{2} \text{ in.})$ $= 211 \text{ kips} > 162 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_g}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(\tfrac{1}{2} \text{ in.})(19\tfrac{1}{2} \text{ in.})}{1.50}$ $= 140 \text{ kips} > 108 \text{ kips}$ **o.k.**

Design gusset plate-to-angles weld

Assume 4-in. angles will be selected. From AISC *Manual* Table 8-8:

$$
\begin{aligned}
&\text{Angle} = 0° \\
&kl = 3.50 \text{ in.} \\
&l = 14\tfrac{1}{2} \text{ in.} \\
&k = \frac{3.50 \text{ in.}}{14\tfrac{1}{2} \text{ in.}} \\
&\quad = 0.241
\end{aligned}
$$

The distance to the center of gravity from the vertical weld line can be determined as follows:

$$
\begin{aligned}
x &= \frac{k^2}{1+2k} \\
&= \frac{(0.241)^2}{1+2(0.241)} \\
&= 0.0392 \\
al &= 4.00 \text{ in.} - xl \\
&= 4.00 \text{ in.} - 0.0392(14\tfrac{1}{2} \text{ in.}) \\
&= 3.43 \\
a &= \frac{al}{l} \\
&= \frac{3.43}{14\tfrac{1}{2} \text{ in.}} \\
&= 0.237
\end{aligned}
$$

From Table 8-8, by interpolation:

$C = 2.73$

and

$C_1 = 1.0$ for E70XX electrodes

The required number of sixteenths of fillet weld size is:

LRFD	ASD
$D_{req} = \dfrac{P_u}{\phi CC_1 l}$ $= \dfrac{162 \text{ kips}}{0.75(2.73)(1.0)(14\tfrac{1}{2} \text{ in.})(2)}$ $= 2.73$ sixteenths	$D_{req} = \dfrac{P_a \Omega}{CC_1 l}$ $= \dfrac{(108 \text{ kips})(2.00)}{2.73(1.0)(14\tfrac{1}{2} \text{ in.})(2)}$ $= 2.73$ sixteenths

Use a 3⁄16-in. fillet weld. The minimum weld size required by AISC *Specification* Table J2.4 will also have to be checked once the double angle size is selected.

Try 2L4×4×5⁄16 to connect the gusset plate to the column web.

Check shear yielding on the angles at the column web connection

$$
\begin{aligned}
A_{gv} &= (14\tfrac{1}{2} \text{ in.})(\tfrac{5}{16} \text{ in.})(2) \\
&= 9.06 \text{ in.}^2
\end{aligned}
$$

The available shear yielding strength is determined from AISC *Specification* Section J4.2(a), Equation J4-3:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(9.06 \text{ in.}^2)$ $= 196 \text{ kips} > 162 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(9.06 \text{ in.}^2)}{1.50}$ $= 130 \text{ kips} > 108 \text{ kips}$ **o.k.**

Check shear rupture on the angles at the column web connection

The net area is determined from AISC *Specification* Section B4.3:

$$A_{nv} = \left[(14\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.}) - 5(\tfrac{5}{16}\text{ in.})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](2)$$
$$= 5.55\text{ in.}^2$$

From AISC *Specification* Section J4.2(b), Equation J4-4:

LRFD	ASD
$\phi V_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(5.55\text{ in.}^2)$ $= 145\text{ kips} < 162\text{ kips}$ **n.g.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(5.55\text{ in.}^2)}{2.00}$ $= 96.6\text{ kips} < 108\text{ kips}$ **n.g.**

Try 2L4×4×⅜ connection angles.

The updated shear rupture strength can be calculated by using a ratio of the new angle thickness to the previous thickness:

LRFD	ASD
$\phi R_n = (149\text{ kips})\left(\dfrac{\tfrac{3}{8}\text{ in.}}{\tfrac{5}{16}\text{ in.}}\right)$ $= 179\text{ kips} > 162\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (96.6\text{ kips})\left(\dfrac{\tfrac{3}{8}\text{ in.}}{\tfrac{5}{16}\text{ in.}}\right)$ $= 116\text{ kips} > 108\text{ kips}$ **o.k.**

Use 2L4×4×⅜.

Check block shear rupture on the angles

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(}Spec\text{. Eq. J4-5)}$$

The edge distance for the net tension area calculation, given the 5½-in. column bolt gage, is:

$$l_{eh} = \frac{4.00\text{ in.} + 4.00\text{ in.} + \tfrac{1}{2}\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.50\text{ in.}$$

Shear yielding component:

$$A_{gv} = (13.25\text{ in.})(\tfrac{3}{8}\text{ in.})(2)$$
$$= 9.94\text{ in.}^2$$

$$0.60 F_y A_{gv} = 0.60(36\text{ ksi})(9.94\text{ in.}^2)$$
$$= 215\text{ kips}$$

Shear rupture component:

$$A_{nv} = 9.94\text{ in.}^2 - 4.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{8}\text{ in.})(2)$$
$$= 6.14\text{ in.}^2$$

$$0.60 F_u A_{nv} = 0.60(58\text{ ksi})(6.14\text{ in.}^2)$$
$$= 214\text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = \left[(1.50\text{ in.})(\tfrac{3}{8}\text{ in.}) - 0.5(1\tfrac{5}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{8}\text{ in.})\right](2)$$
$$= 0.609\text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 1(58\text{ ksi})\left(0.609\text{ in.}^2\right)$$
$$= 35.3\text{ kips}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 214\text{ kips} + 35.3\text{ kips}$$
$$= 249\text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 215\text{ kips} + 35.3\text{ kips}$$
$$= 250\text{ kips}$$

Therefore, $R_n = 249$ kips.

LRFD	ASD
$\phi R_n = 0.75(249\text{ kips})$ $= 187\text{ kips} > 162\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{249\text{ kips}}{2.00}$ $= 125\text{ kips} > 108\text{ kips}$ **o.k.**

Check bolt shear strength

From AISC *Manual* Table 7-1, for 1-in.-diameter ASTM A325-N bolts in single shear, the available shear strength is:

LRFD	ASD
$\phi R_n = (31.8\text{ kips/bolt})(10\text{ bolts})$ $= 318\text{ kips} > 162\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \left(21.2\text{ kips/bolt}\right)\left(10\text{ bolts}\right)$ $= 212\text{ kips} > 108\text{ kips}$ **o.k.**

Check bolt bearing on the angles

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2l_ctF_u \le 2.4dtF_u \qquad (Spec.\ \text{Eq. J3-6a})$$

For the inner bolts, the clear distance is:

$$l_c = 3.00\text{ in.} - (0.5 + 0.5)d_h$$
$$= 3.00\text{ in.} - 1(1\tfrac{1}{16}\text{ in.})$$
$$= 1.94\text{ in.}$$

The available bearing strength per bolt is determined as follows for the inner bolts:

LRFD	ASD
$\phi 1.2l_ctF_u = 0.75(1.2)(1.94\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 38.0\text{ kips/bolt}$	$1.2l_ctF_u/\Omega = 1.2(1.94\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 25.3\text{ kips/bolt}$
$\phi 2.4dtF_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 39.2\text{ kips/bolt}$	$2.4dtF_u/\Omega = 2.4(1.00\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 26.1\text{ kips/bolt}$
Therefore, $\phi R_n = 38.0$ kips/bolt.	Therefore, $R_n/\Omega = 25.3$ kips/bolt.

Because these strengths are greater than the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt shear controls the strength of the inner bolts.

For the edge bolts, the clear distance is:

$$\begin{aligned} l_c &= 1\tfrac{1}{4}\text{ in.} - 0.5d_h \\ &= 1\tfrac{1}{4}\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.}) \\ &= 0.719\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.719\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 14.1\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 39.2\text{ kips/bolt}$ Therefore, $\phi R_n = 14.1$ kips.	$1.2 l_c t F_u/\Omega = 1.2(0.719\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 9.38\text{ kips/bolt}$ $2.4 d t F_u/\Omega = 2.4(1.00\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 26.1\text{ kips/bolt}$ Therefore, $R_n/\Omega = 9.38$ kips.

Because these strengths are less than the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt bearing controls the strength of the edge bolts.

To determine the available strength of the bolt group, sum the individual effective strengths for each bolt. The total available strength of the bolt group is:

LRFD	ASD
$\phi R_n = (14.1\text{ kips/bolt})(2\text{ bolts}) + (31.8\text{ kips/bolt})(8\text{ bolts})$ $= 283\text{ kips} > 162\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (9.38\text{ kips/bolt})(2\text{ bolts}) + (21.2\text{ kips/bolt})(8\text{ bolts})$ $= 188\text{ kips} > 108\text{ kips}$ **o.k.**

The limit state of bolt bearing on the column web, with $t_w = 1.07$ in., is ok by inspection.

The 2L4×4×⅜ angles are satisfactory.

Beam-to-Column Connection

The forces on this interface, as shown in Figure 5-15, are:

LRFD	ASD
Required shear strength, $V_u = 60.0$ kips Required axial strength, $N_u = 37.5$ kips Required flexural strength, $M_u = 0$ kip-in.	Required shear strength, $V_a = 40.0$ kips Required axial strength, $N_a = 25.0$ kips Required flexural strength, $M_a = 0$ kip-in.

Check shear yielding on gross area of the beam

From AISC *Manual* Table 3-6:

LRFD	ASD
$\phi_v V_n = 299\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_v} = 199\text{ kips} > 40.0\text{ kips}$ **o.k.**

Design angle-to-beam web weld

The resultant load and angle of the resultant load are:

LRFD	ASD
$R_u = \sqrt{(37.5 \text{ kips})^2 + (60.0 \text{ kips})^2}$ $= 70.8 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{37.5 \text{ kips}}{60.0 \text{ kips}}\right)$ $= 32.0° \text{ from vertical}$	$R_a = \sqrt{(25.0 \text{ kips})^2 + (40.0 \text{ kips})^2}$ $= 47.2 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{25.0 \text{ kips}}{40.0 \text{ kips}}\right)$ $= 32.0° \text{ from vertical}$

From AISC *Manual* Table 8-8 and Example 5.5:

$l = 14\tfrac{1}{2}$ in.
$kl = 3.50$ in.
$k = 0.241$ in.
$x = 0.0392$ in.
$al = 3.43$ in.
$a = 0.237$ in.

The AISC *Manual* discussion says to use the table for the next lower angle; therefore, from Table 8-8 for $\theta = 30°$, by interpolation:

$C = 2.86$
and
$C_1 = 1.0$ for E70XX electrodes

The required number of sixteenths of fillet weld is:

LRFD	ASD
$D_{req} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{70.8 \text{ kips}}{0.75(2.86)(1.00)(14\tfrac{1}{2} \text{ in.})(2)}$ $= 1.14 \text{ sixteenths}$	$D_{req} = \dfrac{P_a \Omega}{C C_1 l}$ $= \dfrac{(47.2 \text{ kips})(2.00)}{2.86(14\tfrac{1}{2} \text{ in.})(2 \text{ welds})}$ $= 1.14 \text{ sixteenths}$

Use a $\tfrac{3}{16}$-in. fillet weld, which is the minimum permitted in AISC *Specification* Table J2.4 based on the thickness of the thinner part joined (assume $\tfrac{1}{4}$ in. $< t_{min} < \tfrac{1}{2}$ in.).

Try 2L4×4×$\tfrac{5}{16}$ to join the beam to the column.

Check shear yielding on the gross area of the angles

$A_{gv} = (14\tfrac{1}{2} \text{ in.})(\tfrac{5}{16} \text{ in.})(2)$
$= 9.06 \text{ in.}^2$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi V_n = 1.00(0.60)(36 \text{ ksi})(9.06 \text{ in.}^2)$ $= 196 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60(36 \text{ ksi})(9.06 \text{ in.}^2)}{1.50}$ $= 130 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Check shear rupture on the net section of the angles at the column web connection

$$A_{nv} = 2\left[(14\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.}) - 5(\tfrac{5}{16}\text{ in.})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right]$$
$$= 5.55\text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi V_n = 0.75(0.60)(58\text{ ksi})\left(5.55\text{ in.}^2\right)$ $= 145\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60(58\text{ ksi})\left(5.55\text{ in.}^2\right)}{2.00}$ $= 96.6\text{ kips} > 40.0\text{ kips}$ **o.k.**

Check block shear rupture on the angles at the column web connection

Check block shear relative to the required shear strength of the connection. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

The edge distance for the net tension area calculation, given the 5½-in. column bolt gage, is:

$$l_{eh} = \frac{4.00\text{ in.} + 4.00\text{ in.} + \tfrac{1}{2}\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.50\text{ in.}$$

Shear yielding component:

$$A_{gv} = (13.25\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 8.28\text{ in.}^2$$

$$0.60F_y A_{gv} = 0.60(36\text{ ksi})\left(8.28\text{ in.}^2\right)$$
$$= 179\text{ kips}$$

Shear rupture component:

$$A_{nv} = 8.28\text{ in.}^2 - 4.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 5.12\text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(58\text{ ksi})\left(5.12\text{ in.}^2\right)$$
$$= 178\text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = \left[1.50\text{ in.} - 0.5(1\tfrac{5}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{16}\text{ in.})(2)$$
$$= 0.508\text{ in.}^2$$

$$U_{bs}F_u A_{nt} = 1(58\text{ ksi})\left(0.508\text{ in.}^2\right)$$
$$= 29.5\text{ kips}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 178\text{ kips} + 29.5\text{ kips}$$
$$= 208\text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 179\text{ kips} + 29.5\text{ kips}$$
$$= 209\text{ kips}$$

Therefore, $R_n = 208$ kips.

LRFD	ASD
$\phi R_n = 0.75(208 \text{ kips})$ $= 156 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{208 \text{ kips}}{2.00}$ $= 104 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Design bolts at beam-to-column connection

Using the available strengths from AISC *Manual* Tables 7-1 and 7-2, check the shear strength and tensile strength of the connection:

LRFD	ASD
$\phi R_{nv} = (31.8 \text{ kips/bolt})(10 \text{ bolts})$ $= 318 \text{ kips} > 60.0 \text{ kips}$ **o.k.** $\phi R_{nt} = (53.0 \text{ kips/bolt})(10 \text{ bolts})$ $= 530 \text{ kips} > 37.5 \text{ kips}$ **o.k.**	$\dfrac{R_{nv}}{\Omega} = (21.2 \text{ kips/bolt})(10 \text{ bolts})$ $= 212 \text{ kips} > 40.0 \text{ kips}$ **o.k.** $\dfrac{R_{nt}}{\Omega} = (35.3 \text{ kips/bolt})(10 \text{ bolts})$ $= 353 \text{ kips} > 25 \text{ kips}$ **o.k.**

The tensile strength requires an additional check due to the combination of tension and shear. From AISC *Specification* Section J3.7 and AISC *Specification* Table J3.2, the available tensile strength of the bolts subject to tension and shear is:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}\left(\dfrac{60.0 \text{ kips}}{10(0.785 \text{ in.}^2)}\right)$ $= 100 \text{ ksi} > 90 \text{ ksi}$ Therefore, $F'_{nt} = 90$ ksi and there is no reduction in the available tensile strength.	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ $F_{nt} = 90 \text{ ksi}$ $F_{nv} = 54 \text{ ksi}$ $F'_{nt} = 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}\left(\dfrac{40.0 \text{ kips}}{10(0.785 \text{ in.}^2)}\right)$ $= 100 \text{ ksi} > 90 \text{ ksi}$ Therefore, $F'_{nt} = 90$ ksi and there is no reduction in the available tensile strength.

Check bolt bearing on angles

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad (\textit{Spec}.\ \text{Eq. J3-6a})$$

For the inner bolts, the clear distance is:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - (0.5 + 0.5)d_h \\ &= 3.00 \text{ in.} - 1(1\tfrac{1}{16} \text{ in.}) \\ &= 1.94 \text{ in.} \end{aligned}$$

The available bearing strength per bolt is determined as follows for the inner bolts:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.94 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})$ $= 31.6 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})$ $= 32.6 \text{ kips/bolt}$ Therefore, $\phi R_n = 31.6$ kips/bolt.	$1.2 l_c t F_u/\Omega = 1.2(1.94 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})/2.00$ $= 21.1 \text{ kips/bolt}$ $2.4 d t F_u/\Omega = 2.4(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})/2.00$ $= 21.8 \text{ kips/bolt}$ Therefore, $R_n/\Omega = 21.1$ kips/bolt.

Because these strengths are less than the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt bearing controls the strength of the inner bolts.

For the edge bolts:

$$
\begin{aligned}
l_c &= 1\tfrac{1}{4} \text{ in.} - 0.5 d_h \\
&= 1\tfrac{1}{4} \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.}) \\
&= 0.719 \text{ in.}
\end{aligned}
$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.719 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})$ $= 11.7 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})$ $= 32.6 \text{ kips/bolt}$ Therefore, $\phi R_n = 11.7$ kips.	$1.2 l_c t F_u/\Omega = 1.2(0.719 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})/2.00$ $= 7.8\ 2 \text{ kips/bolt}$ $2.4 d t F_u/\Omega = (2.4)(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})/2.00$ $= 21.8 \text{ kips/bolt}$ Therefore, $R_n/\Omega = 7.82$ kips.

Because these strengths are less than the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt bearing controls the strength of the edge bolts.

Thus, the strength of the bolt group is:

LRFD	ASD
$\phi R_n = (11.7 \text{ kips/bolt})(2 \text{ bolts}) + (31.6 \text{ kips/bolt})(8 \text{ bolts})$ $= 276 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (7.82 \text{ kips/bolt})(2 \text{ bolts}) + (21.1 \text{ kips/bolt})(8 \text{ bolts})$ $= 184 \text{ kips} > 40 \text{ kips}$ **o.k.**

Bearing on the column web is o.k. by inspection because $t_w = 1.07$ in. is much greater than the angle thickness = $\tfrac{5}{16}$ in.

Check prying action on angles

The example given in Example 5.5 had the same axial transfer force of 37.5 kips (LRFD) or 25.0 kips (ASD) and, because of column "oil-canning," a ½-in.-thick angle was required. The ½-in.-thick angles will be required in this example also; therefore, use 2L4×4×½.

Example 5.7—Corner Connection-to-Column Web: Uniform Force Method Special Case 2

Given:

Special Case 2 was discussed for strong-axis connections in Example 5.3. This case is used when the beam web is too light to carry the combined beam-to-column shear load of $V_b + R$. Some of the vertical force that is typically assigned to the gusset-to-beam connection, V_b, can instead be carried by the gusset-to-column connection, thereby reducing the beam-to-column shear force. The theory is discussed in Section 4.2.3 where the quantity ΔV_b is introduced.

Using the uniform force method Special Case 2, consider again the design of the connection given in Example 5.5 with a W18×35 beam in place of the W18×97 beam shown in Figure 5-11, and a required shear strength, R_u = 90.0 kips (LRFD) or R_a = 60.0 kips (ASD). The brace-to-gusset connection will not change from the Example 5.5 design; therefore, the gusset plate size and thickness determined in Example 5.5 is a good starting point.

Solution:

From AISC *Manual* Table 1-1, the beam geometric properties are:

Beam
W18×35
$d = 17.7$ in. $\quad t_w = 0.300$ in. $\quad t_f = 0.425$ in. $\quad k_{des} = 0.827$ in.

From Figure 5-11, the connection geometry is:

$$e_c = 0$$

$$\begin{aligned} e_b &= \frac{17.7\text{ in.}}{2} \\ &= 8.85\text{ in.} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{12}{9} \\ &= 1.33 \end{aligned}$$

$$\theta = 53.1°$$

$$\begin{aligned} \alpha &= \bar{\alpha} \\ &= \frac{31\tfrac{1}{2}\text{ in.}}{2} + \tfrac{1}{2}\text{ in.} \\ &= 16.3\text{ in.} \end{aligned}$$

$$\begin{aligned} \bar{\beta} &= \frac{17\tfrac{3}{4}\text{ in.}}{2} \\ &= 8.88\text{ in.} \end{aligned}$$

$$\begin{aligned} \beta &= \frac{\alpha - e_b\tan\theta - e_c}{\tan\theta} \qquad \text{(from } \textit{Manual} \text{ Eq. 13-1)} \\ &= \frac{16.3\text{ in.} - (8.85\text{ in.})(1.33) - 0}{1.33} \\ &= 3.41\text{ in.} \end{aligned}$$

This value of β is used in all the subsequent equations. The fact that $\beta \neq \bar{\beta}$ is unimportant, because $\bar{\beta} - \beta$ is not used.

$$\begin{aligned} r &= \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad (\textit{Manual} \text{ Eq. 13-6}) \\ &= \sqrt{(16.3\text{ in.} + 0)^2 + (3.41\text{ in.} + 8.85\text{ in.})^2} \\ &= 20.4\text{ in.} \end{aligned}$$

LRFD	ASD
$\frac{P_u}{r} = \frac{270 \text{ kips}}{20.4 \text{ in.}}$ $= 13.2 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ub} = \alpha \frac{P_u}{r}$ $= (16.3 \text{ in.})(13.2 \text{ kip/in.})$ $= 215 \text{ kips}$ From AISC *Manual* Equation 13-3: $H_{uc} = e_c \frac{P_u}{r}$ $= (0 \text{ in.})(13.2 \text{ kip/in.})$ $= 0 \text{ kips}$ $\Sigma H_u = 215 \text{ kips}$ $(270 \text{ kips})\sin 53.1° = 216 \text{ kips}$ **o.k.** From AISC *Manual* Equation 13-4: $V_{ub} = e_b \frac{P_u}{r}$ $= (8.85 \text{ in.})(13.2 \text{ kip/in.})$ $= 117 \text{ kips}$ From AISC *Manual* Equation 13-2: $V_{uc} = \beta \frac{P_u}{r}$ $= (3.41 \text{ in.})(13.2 \text{ kip/in.})$ $= 45.0 \text{ kips}$ $\Sigma V_u = 45.0 \text{ kips} + 117 \text{ kips}$ $= 162 \text{ kips}$ $(270 \text{ kips})\cos 53.1° = 162 \text{ kips}$ **o.k.**	$\frac{P_a}{r} = \frac{180 \text{ kips}}{20.4 \text{ in.}}$ $= 8.82 \text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ab} = \alpha \frac{P_a}{r}$ $= (16.3 \text{ in.})(8.82 \text{ kip/in.})$ $= 144 \text{ kips}$ From AISC *Manual* Equation 13-3: $H_{ac} = e_c \frac{P_a}{r}$ $= (0 \text{ in.})(8.82 \text{ kip/in.})$ $= 0 \text{ kips}$ $\Sigma H_a = 144 \text{ kips}$ $(180 \text{ kips})\sin 53.1° = 144 \text{ kips}$ **o.k.** From AISC *Manual* Equation 13-4: $V_{ab} = e_b \frac{P_a}{r}$ $= (8.85 \text{ in.})(8.82 \text{ kip/in.})$ $= 78.1 \text{ kips}$ From AISC *Manual* Equation 13-2: $V_{ac} = \beta \frac{P_a}{r}$ $= (3.41 \text{ in.})(8.82 \text{ kip/in.})$ $= 30.1 \text{ kips}$ $\Sigma V_a = 30.1 \text{ kips} + 78.1 \text{ kips}$ $= 108 \text{ kips}$ $(180 \text{ kips})\cos 53.1° = 108 \text{ kips}$ **o.k.**

Figures 5-16a and 5-16b show the admissible force fields for this case. The total required shear strength of the beam-to-column connection is:

LRFD	ASD
$V_u = V_{ub} + R_u$ $= 117 \text{ kips} + 90.0 \text{ kips}$ $= 207 \text{ kips}$	$V_a = V_{ab} + R_a$ $= 78.1 \text{ kips} + 60.0 \text{ kips}$ $= 138 \text{ kips}$

From AISC *Manual* Table 3-6, the available shear strength of the W18×35 is:

LRFD	ASD
$\phi_v V_n = 159 \text{ kips} < 207 \text{ kips}$ **n.g.**	$V_n/\Omega_v = 106 \text{ kips} < 138 \text{ kips}$ **n.g.**

This means that a web doubler plate is required. However, the required shear strength of the beam can be reduced by moving a portion of the V_{ub} (LRFD) or V_{ab} (ASD) force to the gusset-to column interface, as follows:

LRFD	ASD
$V_{ub} - \Delta V_{ub} + R_u = 159 \text{ kips}$ Solving for ΔV_{ub}, $\Delta V_{ub} = 117 \text{ kips} + 90.0 \text{ kips} - 159 \text{ kips}$ $= 48.0 \text{ kips}$	$V_{ab} - \Delta V_{ab} + R_a = 106 \text{ kips}$ Solving for ΔV_{ab}, $\Delta V_{ab} = 78.1 \text{ kips} + 60.0 \text{ kips} - 106 \text{ kips}$ $= 32.1 \text{ kips}$

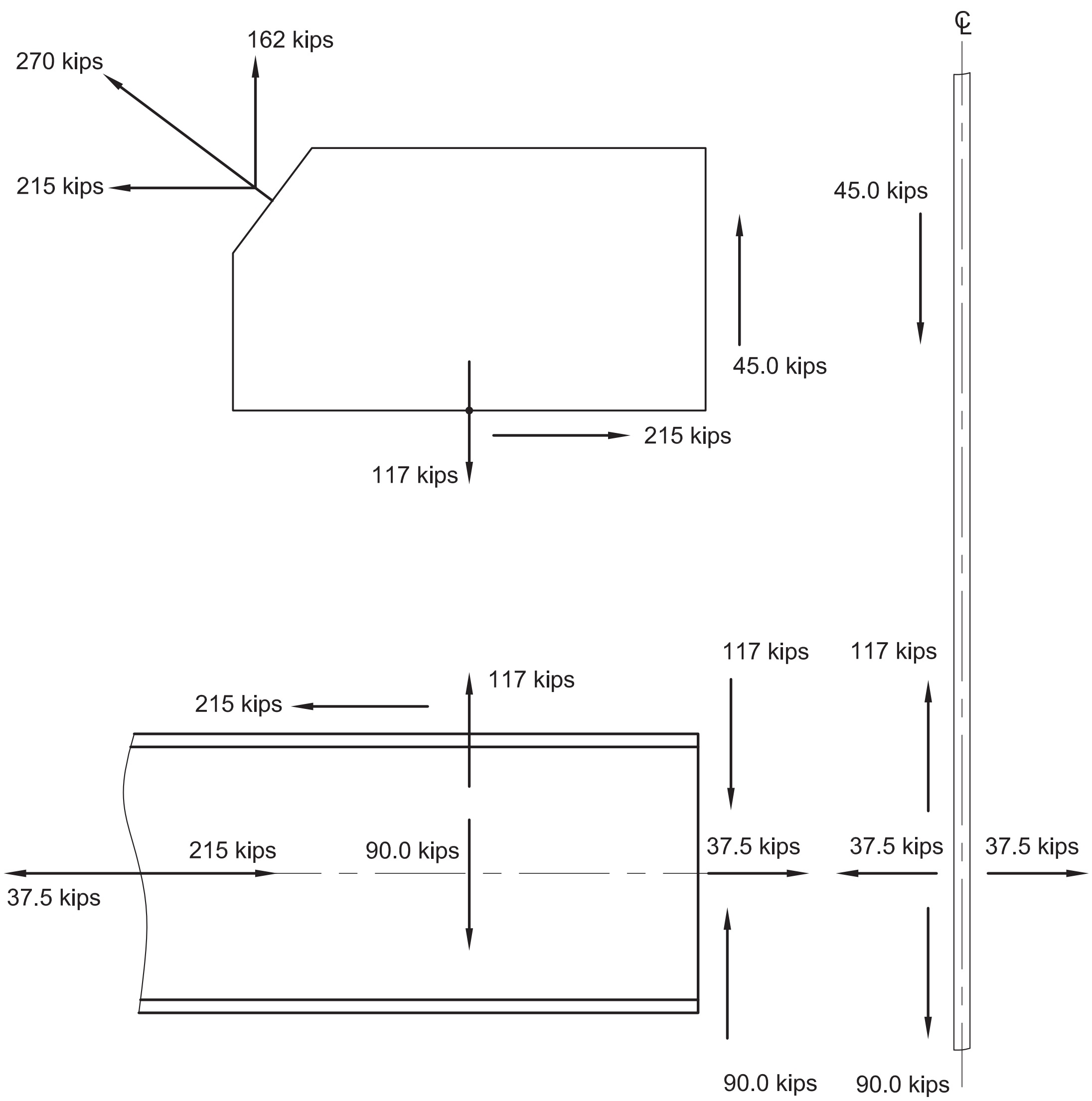


Fig. 5-16a. UFM General Case: Admissible force fields for Example 5.7 connection—LRFD.

From Section 4.2.3, Equation 4-10:

LRFD	ASD
$M_{ub} = V_{ub}\left(\alpha - \overline{\alpha}\right) + \Delta V_{ub}\overline{\alpha}$ $= (117 \text{ kips})(0) + (48.0 \text{ kips})(16.3 \text{ in.})$ $= 782$ kip-in.	$M_{ab} = V_{ab}\left(\alpha - \overline{\alpha}\right) + \Delta V_{ab}\overline{\alpha}$ $= (78.1 \text{ kips})(0) + (32.1 \text{ kips})(16.3 \text{ in.})$ $= 523$ kip-in.

Figure 5-17 shows the Special Case 2 interface forces.

Brace-to-Gusset Connection

The calculations are the same as for the general case in Example 5.5.

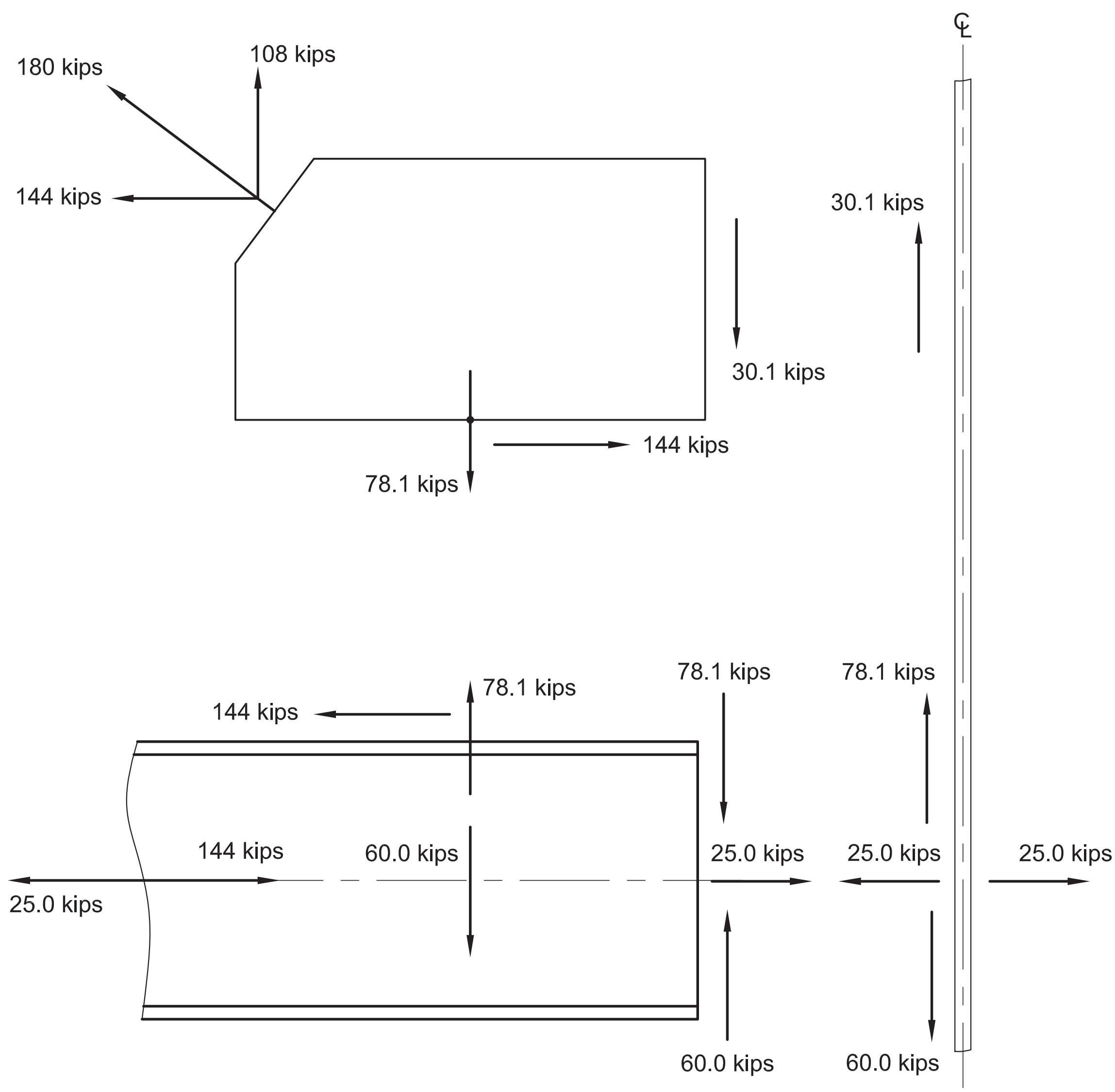


Fig. 5-16b. UFM General Case: Admissible force fields for Example 5.7 connection—ASD.

Gusset-to-Beam Connection

From Figures 5-16 and Figures 5-17 and AISC *Manual* Part 13 for Special Case 2, the required strengths at the gusset-to-beam connection are:

LRFD	ASD
Required shear strength, $H_u = 215$ kips Required normal strength, $$\begin{aligned} V_u &= V_{ub} - \Delta V_{ub} \\ &= 117 \text{ kips} - 48.0 \text{ kips} \\ &= 69.0 \text{ kips} \end{aligned}$$ Required flexural strength, $M_u = 782$ kip-in.	Required shear strength, $H_a = 144$ kips Required normal strength, $$\begin{aligned} V_a &= V_{ab} - \Delta V_{ab} \\ &= 78.1 \text{ kips} - 32.1 \text{ kips} \\ &= 46.0 \text{ kips} \end{aligned}$$ Required flexural strength, $M_a = 523$ kip-in.

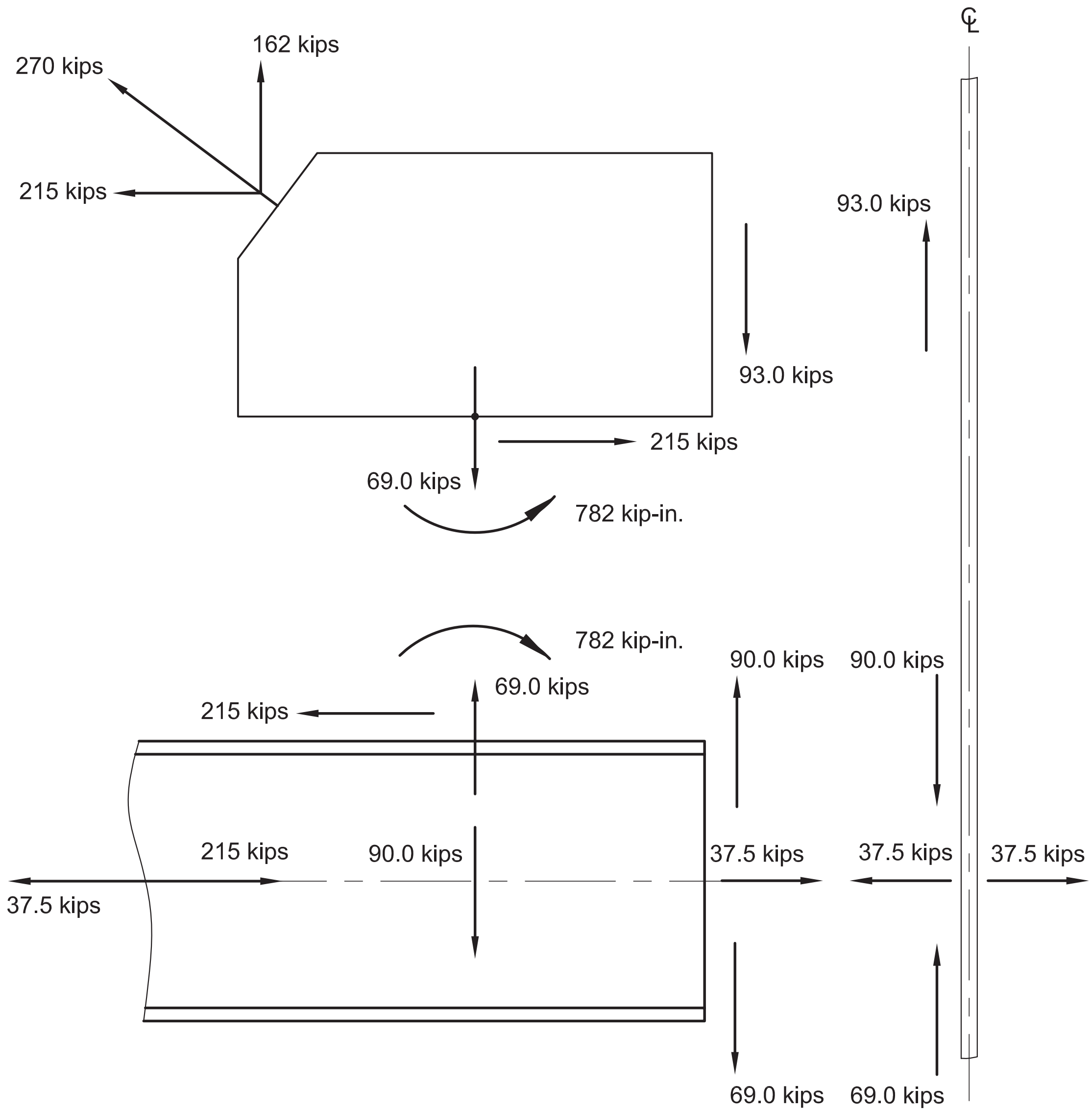


Fig. 5-17a. Admissible forces Special Case 2—LRFD.

Check the gusset plate for tensile yielding and shear yielding along the beam flange

LRFD	ASD
Check shear yielding $$f_{uv} = \frac{H_u}{A_g}$$ $$= \frac{215 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$$ $$= 13.7 \text{ ksi} < 1.00(0.60)(36 \text{ ksi}) = 21.6 \text{ ksi} \quad \textbf{o.k.}$$	Check shear yielding $$f_{av} = \frac{H_a}{A_g}$$ $$= \frac{144 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$$ $$= 9.14 \text{ ksi} < \frac{0.60(36 \text{ ksi})}{1.50} = 14.4 \text{ ksi} \quad \textbf{o.k.}$$

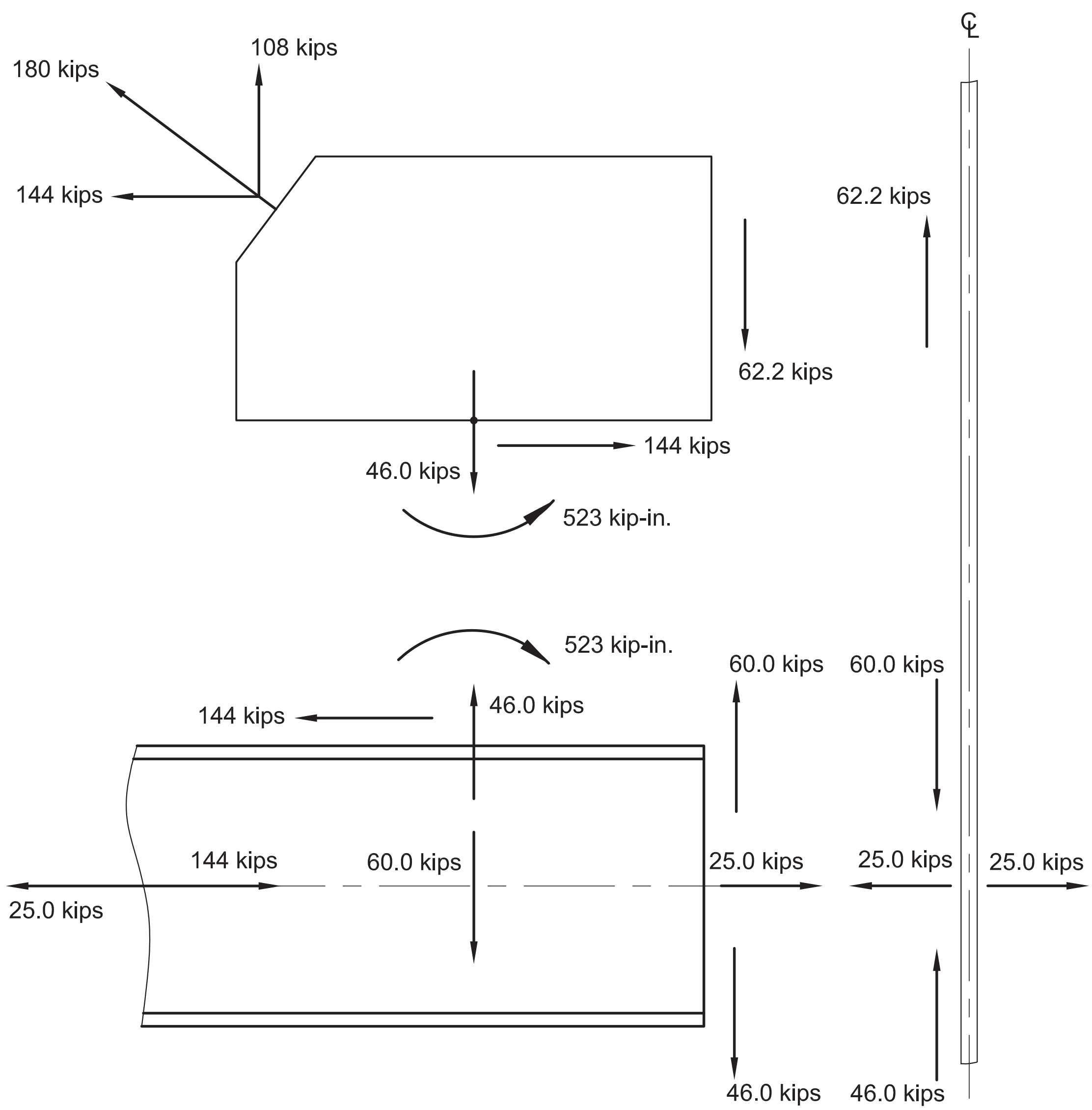


Fig. 5-17b. Admissible forces Special Case 2—ASD.

LRFD	ASD
$f_{ua} = \dfrac{V_u}{A_g}$ $= \dfrac{69.0 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$ $= 4.38 \text{ ksi}$ $f_{ub} = \dfrac{M_u}{Z}$ $= \dfrac{(782 \text{ kip-in.})(4)}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})^2}$ $= 6.30 \text{ ksi}$ Total normal stress $= f_{un}$ Check tensile yielding $f_{un} = f_{ua} + f_{ub}$ $= 4.38 \text{ ksi} + 6.30 \text{ ksi}$ $= 10.7 \text{ ksi} < 0.90(36 \text{ ksi}) = 32.4 \text{ ksi}$ **o.k.**	$f_{aa} = \dfrac{V_a}{A_g}$ $= \dfrac{46.0 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$ $= 2.92 \text{ ksi}$ $f_{ab} = \dfrac{M_a}{Z}$ $= \dfrac{(523 \text{ kip-in.})(4)}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})^2}$ $= 4.22 \text{ ksi}$ Total normal stress $= f_{an}$ Check tensile yielding $f_{an} = f_{aa} + f_{ab}$ $= 2.92 \text{ ksi} + 4.22 \text{ ksi}$ $= 7.14 \text{ ksi} < \dfrac{36 \text{ ksi}}{1.67} = 21.6 \text{ ksi}$ **o.k.**

Design weld at gusset-to-beam flange connection

LRFD	ASD
$R_u = \sqrt{(215 \text{ kips})^2 + (69.0 \text{ kips})^2}$ $= 226 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{69.0 \text{ kips}}{215 \text{ kips}}\right)$ $= 17.8°$ Measured relative to the horizontal	$R_a = \sqrt{(144 \text{ kips})^2 + (46.0 \text{ kips})^2}$ $= 151 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{46.0 \text{ kips}}{144 \text{ kips}}\right)$ $= 17.7°$ Measured relative to the horizontal

The effective eccentricity is:

LRFD	ASD
$al = \dfrac{M_u}{H_u}$ $= \dfrac{782 \text{ kip-in.}}{215 \text{ kips}}$ $= 3.64 \text{ in.}$	$al = \dfrac{M_a}{H_a}$ $= \dfrac{523 \text{ kip-in.}}{144 \text{ kips}}$ $= 3.63 \text{ in.}$

From AISC *Manual* Table 8-4:

Angle = 15°
k = 0
al = 3.63 in.
a = 3.63 in./31½ in.
= 0.115
C = 3.76
C_1 = 1.0 for E70XX electrodes

LRFD	ASD
$D_{req} = \dfrac{R_u(1.25)}{\phi C C_1 l}$ $= \dfrac{(226 \text{ kips})(1.25)}{0.75(3.76)(1.0)(31\frac{1}{2} \text{ in.})}$ $= 3.18$	$D_{req} = \dfrac{R_a \Omega(1.25)}{C C_1 l}$ $= \dfrac{(151 \text{ kips})(2.00)(1.25)}{3.76(1.0)(31\frac{1}{2} \text{ in.})}$ $= 3.19$

Use a ¼-in. double-sided fillet weld. The 1.25 factor in the preceding calculation is the weld ductility factor which is explained in AISC *Manual* Part 13.

An alternative method to size the weld is to introduce an equivalent normal force:

The maximum stress is $f_n = f_a + f_b$ = 10.4 ksi (LRFD) or 7.14 ksi (ASD). This maximum stress occurs over half of the 31½-in. gusset edge; the other half of the gusset edge has a stress of $f_a - f_b$. It is convenient when dealing with web yielding and crippling to work with an equivalent normal force that produces the same maximum stress. The maximum equivalent normal force is (see Appendix B, Figure B-1):

LRFD	ASD
$N_e = V_u + \dfrac{M_u(2)}{(l_g/2)}$ $= V_u + \dfrac{4M_u}{l_g}$ $= 69.0 \text{ kips} + \dfrac{4(782 \text{ kip-in.})}{31\frac{1}{2} \text{ in.}}$ $= 168 \text{ kips}$	$N_e = V_a + \dfrac{M_a(2)}{(l_g/2)}$ $= V_a + \dfrac{4M_a}{l_g}$ $= 46.0 \text{ kips} + \dfrac{4(523 \text{ kip-in.})}{31\frac{1}{2} \text{ in.}}$ $= 112 \text{ kips}$

Check that the normal stress is the same as previously calculated:

LRFD	ASD
$f_{un} = \dfrac{N_e}{t_g l_g}$ $= \dfrac{168 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$ $= 10.7 \text{ ksi}$ **o.k.** $N_{u\,max} = N_e$ $= 168 \text{ kips}$	$f_{an} = \dfrac{N_e}{t_g l_g}$ $= \dfrac{112 \text{ kips}}{(\frac{1}{2} \text{ in.})(31\frac{1}{2} \text{ in.})}$ $= 7.11 \text{ ksi}$ **o.k.** $N_{a\,max} = N_e$ $= 112 \text{ kips}$

Except for small differences due to rounding, f_n is the same as previously computed.

The required weld size is determined using the directional strength increase and the simplified equation given in AISC *Manual* Part 8 as Equation 8-2:

LRFD	ASD
$R_u = \sqrt{N_{u\ max}^2 + H_u^2}$ $= \sqrt{(168\text{ kips})^2 + (215\text{ kips})^2}$ $= 273\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{168\text{ kips}}{215\text{ kips}}\right)$ $= 38.0°$ $D_{req} = \dfrac{1.25R_u}{2l(1.392\text{ kip/in.})(1+0.50\sin^{1.5}\theta)}$ $= \dfrac{1.25(273\text{ kips})}{2(31\tfrac{1}{2}\text{ in.})(1.392\text{ kip/in.})(1+0.50\sin^{1.5}38.0°)}$ $= 3.13\text{ sixteenths}$	$R_a = \sqrt{N_{a\ max}^2 + H_a^2}$ $= \sqrt{(112\text{ kips})^2 + (144\text{ kips})^2}$ $= 182\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{112\text{ kips}}{144\text{ kips}}\right)$ $= 37.9°$ $D_{req} = \dfrac{1.25R_u}{2l(0.928\text{ kip/in.})(1+0.50\sin^{1.5}\theta)}$ $= \dfrac{1.25(182\text{ kips})}{2(31\tfrac{1}{2}\text{ in.})(0.928\text{ kip/in.})(1+0.50\sin^{1.5}37.9°)}$ $= 3.14\text{ sixteenths}$

This is essentially the same value as obtained from the AISC *Manual* Table 8-4. Note that the alternative method as used here is a conservative variation on the approach presented by Hewitt and Thornton (2004) and discussed in AISC *Manual* Part 13, in that the ductility factor is applied to the peak rather than to the average stress. If the more exact procedure of Hewitt and Thornton were used (see Appendix B):

LRFD	ASD
$R_{u\ max} = \sqrt{N_{u\ max}{}^2 + H_u{}^2}$ $= \sqrt{(168\text{ kips})^2 + (215\text{ kips})^2}$ $= 273\text{ kips}$ $N_{u\ min} = \left\lvert V_u - \dfrac{4M_u}{l}\right\rvert$ $= \left\lvert 69.0\text{ kips} - \dfrac{4(782\text{ kip-in.})}{31\tfrac{1}{2}\text{ in.}}\right\rvert$ $= 30.3\text{ kips}$ $R_{u\ min} = \sqrt{N_{u\ min}{}^2 + H_u{}^2}$ $= \sqrt{(30.3\text{ kips})^2 + (215\text{ kips})^2}$ $= 217\text{ kips}$ $R_{avg} = \dfrac{R_{u\ max} + R_{u\ min}}{2}$ $= \dfrac{273\text{ kips} + 217\text{ kips}}{2}$ $= 245\text{ kips}$	$R_{a\ max} = \sqrt{N_{a\ max}{}^2 + H_a{}^2}$ $= \sqrt{(112\text{ kips})^2 + (144\text{ kips})^2}$ $= 182\text{ kips}$ $N_{a\ min} = \left\lvert V_a - \dfrac{4M_a}{l}\right\rvert$ $= \left\lvert 46.0\text{ kips} - \dfrac{4(523\text{ kip-in.})}{31\tfrac{1}{2}\text{ in.}}\right\rvert$ $= 20.4\text{ kips}$ $R_{a\ min} = \sqrt{N_{a\ min}{}^2 + H_a{}^2}$ $= \sqrt{(20.4\text{ kips})^2 + (144\text{ kips})^2}$ $= 145\text{ kips}$ $R_{avg} = \dfrac{R_{a\ max} + R_{a\ min}}{2}$ $= \dfrac{182\text{ kips} + 145\text{ kips}}{2}$ $= 164\text{ kips}$

The weld size is based on max ($R_{a\ max}$, $1.25R_{avg}$):

LRFD	ASD
$R_u = \max\left[273 \text{ kips}, 1.25(245 \text{ kips})\right]$ $= 306 \text{ kips}$ $D_{req} = \dfrac{R_u}{2l(1.392 \text{ kip/in.})\left(1+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{306 \text{ kips}}{2(31\tfrac{1}{2} \text{ in.})(1.392 \text{ kip/in.})\left(1+0.50\sin^{1.5}38.0°\right)}$ $= 2.81 \text{ sixteenths}$	$R_a = \max\left[182 \text{ kips}, 1.25(164 \text{ kips})\right]$ $= 205 \text{ kips}$ $D_{req} = \dfrac{R_a}{2l(0.928 \text{ kip/in.})\left(1+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{205 \text{ kips}}{2(31\tfrac{1}{2} \text{ in.})(0.928 \text{ kip/in.})\left(1+0.50\sin^{1.5}37.9°\right)}$ $= 2.83 \text{ sixteenths}$

If D = 2.81 (LRFD) or 2.83 (ASD) sixteenths is taken as the most accurate value, the AISC *Manual* Table 8-4 method is conservative by approximately $\dfrac{3.19-2.83}{2.83}(100\%) = 12.7\%$ (ASD) and the simplified Hewitt-Thornton method is conservative by $\dfrac{3.14-2.83}{2.83}(100\%) = 11.0\%$ (ASD). In any case, a ¼-in. fillet weld is used here because it is the minimum size used by many fabricators. (Note that AISC *Specification* Table J2.4 requires a minimum $\frac{3}{16}$-in. fillet weld based on the thinner part joined.) These comparisons are intended to demonstrate that there are several ways to perform these calculations that yield similar results.

Beam Checks

Check beam web local yielding

The normal force is assumed to act at the center of the gusset-to-beam interface, or $31\tfrac{1}{2} \text{ in.}/2 = 15.8$ in. from the end of the beam. Because this distance is less than or equal to the depth of the beam (17.7 in.), use AISC *Specification* Equation J10-3:

LRFD	ASD
$\phi R_n = \phi F_y t_w \left(2.5k + l_b\right)$ $= 1.00(50 \text{ ksi})(0.300 \text{ in.})\left[2.5(0.827 \text{ in.}) + 31\tfrac{1}{2} \text{ in.}\right]$ $= 504 \text{ kips} > N_e = 168 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w \left(2.5k + l_b\right)}{\Omega}$ $= \dfrac{(50 \text{ ksi})(0.300 \text{ in.})\left[2.5(0.827 \text{ in.}) + 31\tfrac{1}{2} \text{ in.}\right]}{1.50}$ $= 336 \text{ kips} > N_e = 112 \text{ kips}$ **o.k.**

Check beam web local crippling

The normal force is applied at 15.8 in. from the beam end, which is greater than $d/2$, where d is the depth of the beam. Therefore, determine the beam web local crippling strength from AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}} \geq N_e$ $= 0.75(0.80)(0.300 \text{ in.})^2\left[1 + 3\left(\dfrac{31\tfrac{1}{2} \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 322 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1}{\Omega} 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}} \geq N_e$ $= \dfrac{1}{2.00}(0.80)(0.300 \text{ in.})^2\left[1 + 3\left(\dfrac{31\tfrac{1}{2} \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 215 \text{ kips} > 112 \text{ kips}$ **o.k.**

Gusset-to-Column Connection

From Figure 5-17, the required shear strengths are 93.0 kips (LRFD) and 62.2 kips (ASD).

The connection angle sizes are as given in Figure 5-11.

Check bolt shear strength

From AISC *Manual* Table 7-1 for a 1-in.-diameter ASTM A325-N bolt in single shear, the required shear strength is:

LRFD	ASD
$\phi r_n = 31.8 \text{ kips}$	$\dfrac{r_n}{\Omega} = 21.2 \text{ kips}$

The required shear strength per bolt for the six-bolt double-angle connection is:

LRFD	ASD
$r_u = \dfrac{93.0 \text{ kips}}{6}$ $= 15.5 \text{ kips} < 31.8 \text{ kips}$ **o.k.**	$r_a = \dfrac{62.2 \text{ kips}}{6}$ $= 10.4 \text{ kips} < 21.2 \text{ kips}$ **o.k.**

Check bolt bearing on the angles

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

For the inner bolts, the spacing between bolts is large (5.0 in. as shown in Figure 5-11); therefore, the first part of the equation (tearout) will not control by inspection.

LRFD	ASD
$\phi r_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})$ $= 32.6 \text{ kips}$	$\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00 \text{ in.})(\tfrac{5}{16} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 21.8 \text{ kips}$

This does not control over the available bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD).

For the top bolts, the clear distance, based on the 1¼-in. edge distance is:

$$\begin{aligned} l_c &= 1\tfrac{1}{4} \text{ in.} - 0.5 d_h \\ &= 1\tfrac{1}{4} \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.}) \\ &= 0.719 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2l_c tF_u = 0.75(1.2)(0.719\text{ in.})(\tfrac{5}{16}\text{ in.})(58\text{ ksi})$ $= 11.7\text{ kips/bolt}$ $\phi 2.4dtF_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{5}{16}\text{ in.})(58\text{ ksi})$ $= 32.6\text{ kips/bolt}$ Therefore, $\phi r_n = 11.7$ kips/bolt.	$\dfrac{1.2l_c tF_u}{\Omega} = \dfrac{1.2(0.719\text{ in.})(\tfrac{5}{16}\text{ in.})(58\text{ ksi})}{2.00}$ $= 7.82\text{ kips/bolt}$ $\dfrac{2.4dtF_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(\tfrac{5}{16}\text{ in.})(58\text{ ksi})}{2.00}$ $= 21.8\text{ kips/bolt}$ Therefore, $r_n/\Omega = 7.82$ kips/bolt.

For the top bolts, because the available strength due to bolt bearing on the angles is less than the bolt shear strength determined previously (31.8 kips for LRFD and 21.2 kips for ASD), the limit state of bolt bearing controls the strength of the top bolts.

Check bolt bearing on the column web

There are no end bolts on the column web because the column is continuous. Similar to the inner bolts bearing on the angles, the first part of AISC *Specification* Equation J3-6a (tearout) does not control. Therefore:

LRFD	ASD
$\phi r_n = \phi 2.4dtF_u$ $= 0.75(2.4)(1.00\text{ in.})\left(\dfrac{1.07\text{ in.}}{2}\right)(65\text{ ksi})$ $= 62.6\text{ kips} > 31.8\text{ kips}$ bolt shear controls	$\dfrac{r_n}{\Omega} = \dfrac{2.4dtF_u}{\Omega}$ $= \dfrac{2.4(1.00\text{ in.})\left(\dfrac{1.07\text{ in.}}{2}\right)(65\text{ ksi})}{2.00}$ $= 41.7\text{ kips} > 21.2\text{ kips}$ bolt shear controls

Note that in this check, one half of the column web thickness is used. The other one half is reserved for connections, if any, on the other side of the column web. If there are no connections on the other side, the entire web thickness of 1.07 in. may be used here. This is the simplest way to deal with this problem, and is correct for connections of the same size and load on both sides. It is conservative for any other case.

The available bearing strength of the bolts on the angles and the column web is:

LRFD	ASD
$\phi R_n = 2(11.7\text{ kips}) + 4(31.8\text{ kips})$ $= 151\text{ kips} > 93.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2(7.82\text{ kips}) + 4(21.2\text{ kips})$ $= 100\text{ kips} > 62.2\text{ kips}$ **o.k.**

Check shear yielding on the angles

The available shear yielding strength from AISC *Specification* Section J4.2(a) is:

LRFD	ASD
$\phi R_n = \phi 0.60F_y A_{gv}$ $= \phi 0.60F_y l_a t_a(2)$ $= 1.00(0.60)(36\text{ ksi})(12\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$ $= 169\text{ kips} > 93.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_y A_{gv}}{\Omega}$ $= \dfrac{0.60F_y l_a t_a(2)}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)}{1.50}$ $= 113\text{ kips} > 62.2\text{ kips}$ **o.k.**

Check shear rupture on the angles

The net area is determined from AISC *Specification* Section J4.3:

$$A_{nv} = \left[12\tfrac{1}{2}\text{ in.} - 3(1\tfrac{1}{16} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{16}\text{ in.})(2)$$
$$= 5.70\text{ in.}^2$$

The available shear rupture strength from AISC *Specification* Section J4.2(b) is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(5.70\text{ in.}^2)$ $= 149\text{ kips} > 93.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(5.70\text{ in.}^2)}{2.00}$ $= 99.2\text{ kips} > 62.2\text{ kips}$ **o.k.**

Check block shear rupture on the angles

The controlling block shear failure path is assumed to start at the top of the angles and follows the vertical bolt lines and then turns perpendicular to the edge of each angle. For the tension area calculated in the following, the net area is based on the length of the short-slotted hole dimension given in AISC *Specification* Table J3.3 and the requirements for net area determination in AISC *Specification* Section B4.3b. For the shear area, the net area is based on the width of the short-slotted hole, which is the same as for a standard hole.

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = (12\tfrac{1}{2}\text{ in.} - 1\tfrac{1}{4}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 7.03\text{ in.}^2$$

$$0.60 F_y A_{gv} = 0.60(36\text{ ksi})(7.03\text{ in.}^2)$$
$$= 152\text{ kips}$$

Shear rupture component:

$$A_{nv} = 7.03\text{ in.}^2 - 2.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{16}\text{ in.})(2)$$
$$= 5.27\text{ in.}^2$$

$$0.60 F_u A_{nv} = 0.60(58\text{ ksi})(5.27\text{ in.}^2)$$
$$= 183\text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$L_e = \frac{4.00\text{ in.} + 4.00\text{ in.} + \tfrac{1}{2}\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2} - 0.50(1\tfrac{5}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})$$
$$= 0.813\text{ in.}$$

$$A_{nt} = tL_e(2)$$
$$= (\tfrac{5}{16}\text{ in.})(0.813\text{ in.})(2)$$
$$= 0.508\text{ in.}^2$$

$$U_{bs} F_u A_{nt} = 1(58\text{ ksi})(0.508\text{ in.}^2)$$
$$= 29.5\text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 183 \text{ kips} + 29.5 \text{ kips}$$
$$= 213 \text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 152 \text{ kips} + 29.5 \text{ kips}$$
$$= 182 \text{ kips}$$

Therefore, $R_n = 182$ kips.

LRFD	ASD
$\phi R_n = 0.75(182 \text{ kips})$ $= 137 \text{ kips} > 93.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{182 \text{ kips}}{2.00}$ $= 91.0 \text{ kips} > 62.2 \text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

It is possible for the gusset to fail in block shear rupture under the angles at the weld lines. This will be an L-shaped tearout. By inspection, the shear yielding component will control over the shear rupture component; therefore, AISC *Specification* Equation J4-5 reduces to:

$$R_n = 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$A_{gv} = t_g(L_{ev} + L_{angle})$
$= (\tfrac{1}{2} \text{ in.})(2\tfrac{1}{2} \text{ in.} + 12\tfrac{1}{2} \text{ in.})$
$= 7.50 \text{ in.}^2$

$A_{nt} = t_g L_{eh}$
$= (\tfrac{1}{2} \text{ in.})(3.50 \text{ in.})$
$= 1.75 \text{ in.}^2$

$U_{bs} = 1$

The available block shear rupture strength of the gusset plate at the angle-to-gusset plate weld is:

LRFD	ASD
$\phi R_n = \phi\left(0.60F_yA_{gv} + U_{bs}F_uA_{nt}\right)$ $= 0.75\left[0.60(36 \text{ ksi})\left(7.50 \text{ in.}^2\right) + 1(58 \text{ ksi})(1.75 \text{ in.})\right]$ $= 197 \text{ kips} > 93.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_yA_{gv} + U_{bs}F_uA_{nt}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})\left(7.50 \text{ in.}^2\right) + 1(58 \text{ ksi})\left(1.75 \text{ in.}^2\right)}{2.00}$ $= 132 \text{ kips} > 62.2 \text{ kips}$ **o.k.**

Check shear yielding on the gusset plate

On a vertical section of the gusset plate, the available shear yielding strength is, from AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60F_yA_{gv}$ $= \phi 0.60F_yl_vt_g$ $= 1.00(0.60)(36 \text{ ksi})(17\tfrac{3}{4} \text{ in.})(\tfrac{1}{2} \text{ in.})$ $= 192 \text{ kips} > 93.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_yA_{gv}}{\Omega}$ $= \dfrac{0.60F_yl_vt_g}{1.50}$ $= \dfrac{0.60(36 \text{ ksi})(17\tfrac{3}{4} \text{ in.})(\tfrac{1}{2} \text{ in.})}{1.50}$ $= 128 \text{ kips} > 62.2 \text{ kips}$ **o.k.**

Design gusset plate-to-angles weld

Use AISC *Manual* Table 10-2, Case I, conservatively using $L = 11\frac{1}{2}$ in. for the $12\frac{1}{2}$-in. connection in this example. The angle leg used in this example is slightly larger than that assumed in Table 10-2, which is also conservative. A $\frac{3}{16}$-in. fillet weld (Weld A) will have an available strength of:

LRFD	ASD
$\phi R_n = 142 \text{ kips} > 93.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 95.0 \text{ kips} > 62.2 \text{ kips}$ **o.k.**

Use a $\frac{3}{16}$-in. fillet weld, which is also the minimum required fillet weld size according to AISC *Specification* Table J2.4.

Beam-to-Column Connection

From Figure 5-17, the beam-to-column interface forces are:

LRFD	ASD
Required shear strength: $V_u = 90.0 \text{ kips} + 69.0 \text{ kips}$ $= 159 \text{ kips}$ Required axial strength: $A_{ub} = 37.5 \text{ kips transfer force}$	Required shear strength: $V_a = 60.0 \text{ kips} + 46.0 \text{ kips}$ $= 106 \text{ kips}$ Required axial strength: $A_{ab} = 25.0 \text{ kips transfer force}$

The required shear strength is less than that used in Example 5.5 (Figure 5-11) and the calculations will not be repeated here. The final design is the same as that of Figure 5-11, with the beam size changed to a W18×35.

Example 5.8—Corner Connection-to-Column Web with Gusset Connected to Beam Only: Uniform Force Method Special Case 3

This case is discussed in Section 4.2.4, shown in Figure 4-16, and addressed in an example (for the case where the beam is connected to the column flange or strong axis) in Example 5.4. A similar example will be considered here in the weak-axis configuration, that is, where the beam connects to the column web. The connection is shown in Figure 5-18. The optional flat bar (FB) configuration will be used, with a thickness of ⅝ in. The column is a W12×58 flange to view, the beam is a W18×35, and the brace is 2L4×4×⅜. All properties and loads are the same as those given for Example 5.4, and as shown in Figure 5-10.

The required strengths are:

LRFD	ASD
Brace required strength, $P_u = \pm 100$ kips Beam required shear strength, $V_u = 30.0$ kips	Brace required strength, $P_a = \pm 66.7$ kips Beam required shear strength, $V_a = 20.0$ kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W18×35
$t_w = 0.300$ in. $d = 17.7$ in. $b_f = 6.00$ in. $t_f = 0.425$ in. $k_{des} = 0.827$ in.

Column
W12×58
$t_w = 0.360$ in. $d = 12.2$ in. $b_f = 10.0$ in. $t_f = 0.640$ in.

Brace
2L4×4×3⁄8 in.
$A_g = 5.72$ in.^{2} $\bar{x} = 1.13$ in.

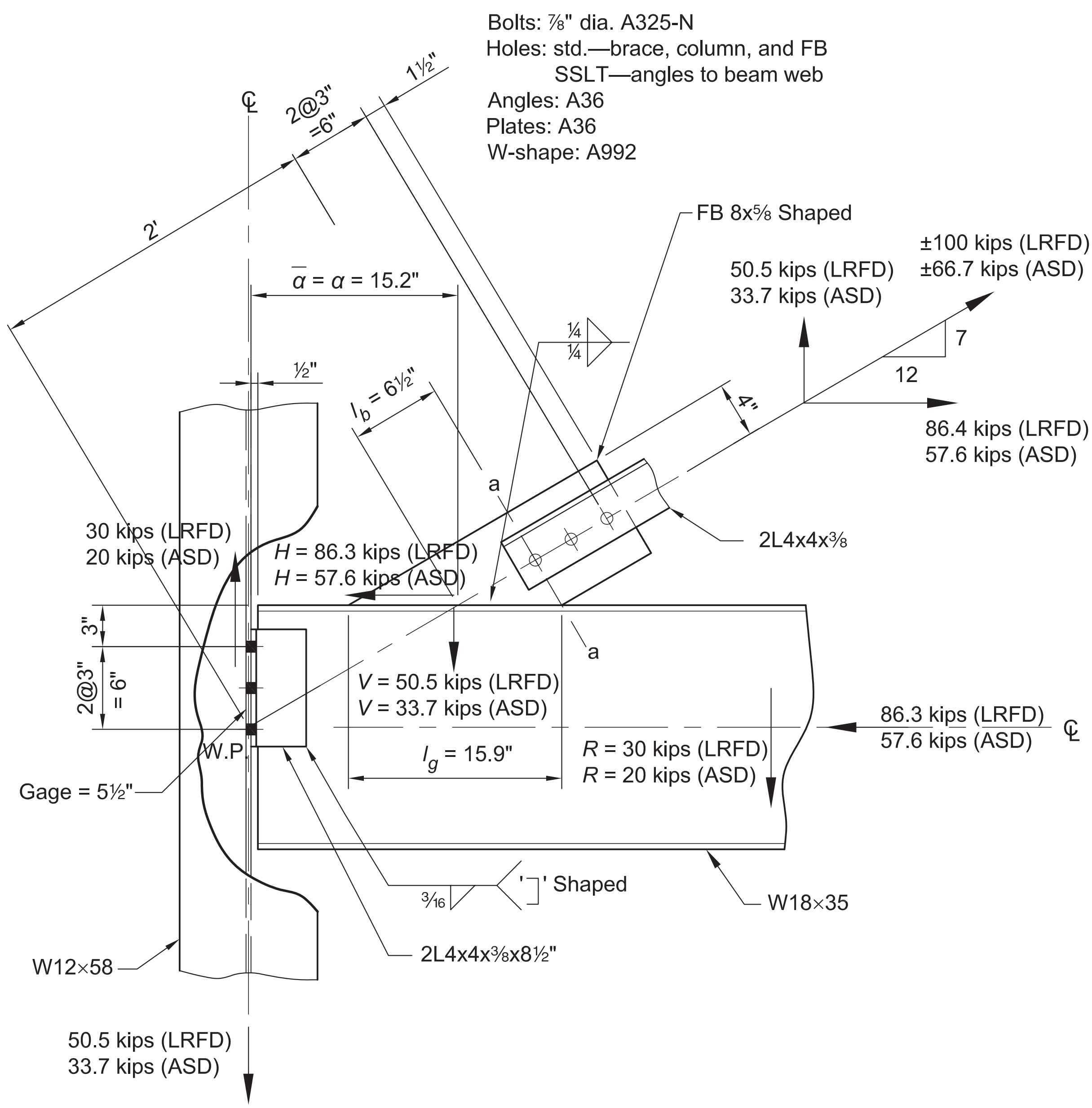


Fig. 5-18. Special Case 3—weak axis.

From AISC *Specification* Table J3.3, the hole dimensions for ⅞-in.-diameter bolts are as follows:

Brace, column and flat bar
Standard: d_h = 15⁄16 in. diameter

Angles
Short slots: 15⁄16 in. × ⅛ in.

Connection Interface Forces

The forces at the gusset-to-beam and gusset-to-column interfaces are determined using Special Case 3 of the UFM as discussed in Section 4.2.4 and AISC *Manual* Part 13.

From Figure 5-18 and the given beam and column geometry:

$$
\begin{aligned}
e_b &= \frac{d_b}{2} \\
&= \frac{17.7 \text{ in.}}{2} \\
&= 8.85 \text{ in.} \\
e_c &= 0 \text{ in.} \\
\tan\theta &= \frac{12}{7} \\
\theta &= \tan^{-1}\left(\frac{12}{7}\right) \\
&= 59.7°
\end{aligned}
$$

Because there is no connection of the gusset plate to the column, $\beta = 0$, and

$$
\begin{aligned}
\alpha &= e_b \tan\theta - e_c \qquad \text{(\textit{Manual} Eq. 13-1)} \\
&= (8.85 \text{ in.})\left(\frac{12}{7}\right) - 0 \text{ in.} \\
&= 15.2 \text{ in.}
\end{aligned}
$$

The forces can be calculated from Figure 4-16 or by inspection of Figure 5-18. Because $e_c = 0$, $M_{bc} = 0$. With the flat bar (FB) straddling the brace line of action, $\overline{\alpha} = \alpha$; therefore, $M_b = 0$.

The forces on the gusset-to-beam connection are:

LRFD	ASD
$H_{ub} = H$ $= P_u \sin\theta$ $= (100 \text{ kips})\sin 59.7°$ $= 86.3$ kips $V_{ub} = V$ $= P_u \cos\theta$ $= (100 \text{ kips})\cos 59.7°$ $= 50.5$ kips	$H_{ab} = H$ $= P_a \sin\theta$ $= (66.7 \text{ kips})\sin 59.7°$ $= 57.6$ kips $V_{ab} = V$ $= P_a \cos\theta$ $= (66.7 \text{ kips})\cos 59.7°$ $= 33.7$ kips

The forces on the beam-to-column interface are:

LRFD	ASD
$V_{bc} = V_{ub} + R$ $= 50.5\text{ kips} + 30\text{ kips}$ $= 80.5\text{ kips}$	$V_{bc} = V_{ab} + R$ $= 33.7\text{ kips} + 20\text{ kips}$ $= 53.7\text{ kips}$

Brace-to-Gusset Connection

The contact length between the flat bar (FB) and the beam flange is:

$$\begin{aligned} l_g &= \frac{8.00\text{ in.}}{\cos\theta} \\ &= \frac{8.00\text{ in.}}{\cos 59.7^\circ} \\ &= 15.9\text{ in.} \end{aligned}$$

The calculations for the design of this connection are the same as those shown in Example 5.4, except as follows.

Check the flat bar for tensile yielding on the Whitmore section

As explained in AISC *Manual* Part 9, the gross area may be limited by the Whitmore section, where the width of the Whitmore section is:

$$\begin{aligned} l_w &= 2(6.00\text{ in.})(\tan 30^\circ) \\ &= 6.93\text{ in.} \end{aligned}$$

If l_w were greater than 8 in., a Whitmore width of 8 in. would be used because 8 in. is the full width of the flat bar.

$$\begin{aligned} A_w &= (6.93\text{ in.})(\tfrac{5}{8}\text{ in.}) \\ &= 4.33\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(36\text{ ksi})(4.33\text{ in.}^2)$ $= 140\text{ kips} > 100\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{(36\text{ ksi})(4.33\text{ in.}^2)}{1.67}$ $= 93.3\text{ kips} > 66.7\text{ kips}$ **o.k.**

Check the flat bar for compression buckling on the Whitmore section

The available compressive strength of the flat bar based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4. Use an effective length factor of $K = 1.2$ due to the possibility of sidesway buckling (see AISC *Specification* Commentary Table C-A-7.1). From Figure 5-18, the unbraced length of the flat bar is shown as 6½ in.

$$\begin{aligned} \frac{KL}{r} &= \frac{1.2(6\tfrac{1}{2}\text{ in.})}{(\tfrac{5}{8}\text{ in.})/\sqrt{12}} \\ &= 43.2 \end{aligned}$$

Because $KL/r > 25$, use AISC *Manual* Table 4-22 to determine the available critical stress for the 36-ksi flat bar. Then, the available compressive strength can be determined from AISC *Specification* Sections E1 and E3, with $A_g = A_w$:

LRFD	ASD
$\phi_c F_{cr} = 29.4 \text{ ksi}$ $\phi_c P_n = \phi_c F_{cr} A_w$ $= (29.4 \text{ ksi})(4.33 \text{ in.}^2)$ $= 127 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 19.6 \text{ ksi}$ $\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= (19.6 \text{ ksi})(4.33 \text{ in.}^2)$ $= 84.9 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check tension rupture on the net section of the flat bar

Because Section a-a of Figure 5-18 passes through a flat bar cross section, net tension is a more likely failure mode (limit state).

The net area on the section marked a-a through the flat bar on Figure 5-18 is determined in accordance with AISC *Specification* Section B4.3, with the bolt hole diameter, $d_h = \frac{15}{16}$ in., from AISC *Specification* Table J3.3:

$$\begin{aligned} A_n &= \left[8.00 \text{ in.} - 1(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.}) \\ &= 4.38 \text{ in.}^2 \end{aligned}$$

The available tensile rupture strength of the flat bar according to AISC *Specification* Equation J4-2, with $A_e = A_n$ ($U = 1$), is:

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75(58 \text{ ksi})(4.38 \text{ in.}^2)$ $= 191 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(58 \text{ ksi})(4.38 \text{ in.}^2)}{2.00}$ $= 127 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

Check block shear rupture on the flat bar

The available strength for the limit state of block shear rupture on the flat bar is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}. \text{ Eq. J4-5})$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= (7.50 \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 4.69 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_y A_{gv} &= 0.60(36 \text{ ksi})(4.69 \text{ in.}^2) \\ &= 101 \text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 4.69 \text{ in.}^2 - 2.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 3.13 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_u A_{nv} &= 0.60(58 \text{ ksi})(3.13 \text{ in.}^2) \\ &= 109 \text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{nt} &= \left[4.00 \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.}) \\ &= 2.19 \text{ in.}^2 \end{aligned}$$

$$U_{bs}F_uA_{nt} = 1(58\text{ ksi})\left(2.19\text{ in.}^2\right)$$
$$= 127\text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 109\text{ kips} + 127\text{ kips}$$
$$= 236\text{ kips}$$

$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 101\text{ kips} + 127\text{ kips}$$
$$= 228\text{ kips}$$

Therefore, $R_n = 228$ kips.

LRFD	ASD
$\phi R_n = 0.75(228\text{ kips})$ $= 171\text{ kips} > 100\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_uA_{nt} + 0.6F_yA_{gv}}{\Omega}$ $= \dfrac{228\text{ kips}}{2.00}$ $= 114\text{ kips} > 66.7\text{ kips}$ **o.k.**

Gusset-to-Beam Connection

Check gusset plate for shear yielding and tensile yielding along the beam flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

LRFD	ASD
$\phi V_n = \phi 0.60F_yA_{gv}$ $= 1.00(0.60)(36\text{ ksi})(5/8\text{ in.})(15.9\text{ in.})$ $= 215\text{ kips} > 86.3\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60F_yA_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi.})(5/8\text{ in.})(15.9\text{ in.})}{1.50}$ $= 143\text{ kips} > 57.6\text{ kips}$ **o.k.**
$\phi N_n = \phi F_yA_g$ $= 0.90(36\text{ ksi})(5/8\text{ in.})(15.9\text{ in.})$ $= 322\text{ kips} > 50.5\text{ kips}$ **o.k.**	$\dfrac{N_n}{\Omega} = \dfrac{F_yA_g}{\Omega}$ $= \dfrac{(36\text{ ksi})(5/8\text{ in.})(15.9\text{ in.})}{1.67}$ $= 214\text{ kips} > 33.7\text{ kips}$ **o.k.**

Note that as shown previously, the interaction of the forces at the gusset-to-beam interface is assumed to have no impact on the design.

Design weld at gusset-to-beam flange connection

The resultant load and load angle are:

LRFD	ASD
$R_u = \sqrt{(86.3\text{ kips})^2 + (50.5\text{ kips})^2}$ $= 100\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{50.5\text{ kips}}{86.3\text{ kips}}\right)$ $= 30.3°$	$R_a = \sqrt{(57.6\text{ kips})^2 + (33.7\text{ kips})^2}$ $= 66.7\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{33.7\text{ kips}}{57.6\text{ kips}}\right)$ $= 30.3°$

From AISC *Manual* Table 8-4:

Angle = 30°
a = 0
k = 0
C = 4.37
C_1 = 1.0 for E70XX electrodes

Use AISC *Manual* Table 8-4 equation for D_{min} to determine the number of sixteenths of weld required:

LRFD	ASD
$D_{req} = \dfrac{R_u}{\phi C C_1 l}$ $= \dfrac{100 \text{ kips}}{0.75(4.37)(1.0)(15.9 \text{ in.})}$ $= 1.92$ sixteenths	$D_{req} = \dfrac{R_a \Omega}{C C_1 l}$ $= \dfrac{(66.7 \text{ kips})(2.00)}{(4.37)(1.0)(15.9 \text{ in.})}$ $= 1.92$ sixteenths

Note that the ductility factor need not be used here because the stress is uniform and statically determinate. An Appendix B approach could also be used. Alternatively, the AISC *Manual* Equation 8-2 with the directional strength increase may be used.

Use a double-sided 3⁄16-in. fillet weld based on the minimum weld size required by AISC *Specification* Table J2.4.

Check beam web local yielding

From AISC *Specification* Equation J10-3, because the resultant force is applied at a distance, α = 15.2 in., from the member end that is less than or equal to the depth of the W18×35 beam, the available strength for web local yielding is:

LRFD	ASD
$\phi R_n = \phi F_y t_w (l_b + 2.5k_{des})$ $= 1.00(50 \text{ ksi})(0.300 \text{ in.})[15.9 \text{ in.} + 2.5(0.827 \text{ in.})]$ $= 270 \text{ kips} > 50.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w}{\Omega}[l_b + 2.5k_{des}]$ $= \dfrac{(50 \text{ ksi})(0.300 \text{ in.})}{1.50}[15.9 \text{ in.} + 2.5(0.827 \text{ in.})]$ $= 180 \text{ kips} > 33.7 \text{ kips}$ **o.k.**

Check beam web local crippling

Using AISC *Specification* Equation J10-4 because the force to be resisted is applied a distance from the member end, α = 15.2 in., that is greater than or equal to $d/2$, the available strength for web local crippling is:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_{yw} t_f}{t_w}}$ $= 0.75(0.80)(0.300 \text{ in.})^2\left[1 + 3\left(\dfrac{15.9 \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 201 \text{ kips} > 50.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2}{\Omega}\left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_{yw} t_f}{t_w}}$ $= \dfrac{(0.80)(0.300 \text{ in.})^2}{2.00}\left[1 + 3\left(\dfrac{15.9 \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 134 \text{ kips} > 33.7 \text{ kips}$ **o.k.**

Beam-to-Column Connection

The required shear strength is:

LRFD	ASD
$V_u = 30.0 \text{ kips} + 50.5 \text{ kips}$ $= 80.5 \text{ kips}$	$V_a = 20.0 \text{ kips} + 33.7 \text{ kips}$ $= 53.7 \text{ kips}$

Use a double-angle connection as shown in Figure 5-18.

Design bolts at beam-to-column connection

Use ⅞-in.-diameter ASTM A325-N bolts with horizontal short slots in the angles. From AISC *Manual* Table 7-1, the available shear strength is:

LRFD	ASD
$\phi r_{nv} = 24.3 \text{ kips/bolt}$	$\dfrac{r_{nv}}{\Omega} = 16.2 \text{ kips/bolt}$

The number of bolts required is:

LRFD	ASD
$n = \dfrac{80.5 \text{ kips}}{24.3 \text{ kips/bolt}}$ $= 3.31 \text{ bolts}$	$n = \dfrac{53.7 \text{ kips}}{16.2 \text{ kips/bolt}}$ $= 3.31 \text{ bolts}$

The AISC *Manual* recommends a minimum depth connection for a W18 to be 3 rows of bolts, or 6 bolts total. Thus, try a minimum depth connection as shown in Figure 5-18 with six bolts arranged in three rows of 2 on the column, as indicated. Try angles 2L4×4×⅜×8½.

Check bolt bearing on the connection angles

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength.

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(}Spec.\text{ Eq. J3-6a)}$$

For the inner bolts, the clear distance is:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - (0.5 + 0.5) d_h \\ &= 3.00 \text{ in.} - 1(\tfrac{15}{16} \text{ in.}) \\ &= 2.06 \text{ in.} \end{aligned}$$

For the inner bolts, the bearing strength per bolt is:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06 \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})$ $= 40.3 \text{ kips/bolt}$	$1.2 l_c t F_u / \Omega = 1.2(2.06 \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})/200$ $= 26.9 \text{ kips/bolt}$
$\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})$ $= 34.3 \text{ kips/bolt}$	$2.4 d t F_u / \Omega = 2.4(\tfrac{7}{8} \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})/2.00$ $= 22.8 \text{ kips/bolt}$
Therefore, $\phi r_n = 34.3$ kips/bolt.	Therefore, $r_n/\Omega = 22.8$ kips/bolt.

Because the available bolt shear strength determined previously (24.3 kips for LRFD and 16.2 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the inner bolts.

For the end bolts, based on a 1¼-in. edge distance, the clear distance is:

$$\begin{aligned} l_c &= 1\tfrac{1}{4}\text{ in.} - 0.5d_h \\ &= 1\tfrac{1}{4}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.}) \\ &= 0.781\text{ in.} \end{aligned}$$

For the end bolts, the available bearing strength per bolt is:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.781\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 15.3\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $= 34.3\text{ kips/bolt}$ Therefore, $\phi r_n = 15.3$ kips/bolt.	$1.2 l_c t F_u / \Omega = 1.2(0.781\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 10.2\text{ kips/bolt}$ $2.4 d t F_u / \Omega = 2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})/2.00$ $= 22.8\text{ kips/bolt}$ Therefore, $r_n/\Omega = 10.2$ kips/bolt.

Bolt bearing on the angles controls over bolt shear for the end bolts.

Check bolt bearing on the column web

Tearout, the limit state captured by the first part of Equation J3-6a, is not applicable to the column because there is no edge through which the bolts can tear out.

LRFD	ASD
$\phi r_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(\tfrac{7}{8}\text{ in.})\left(\dfrac{0.360\text{ in.}}{2}\right)(65\text{ ksi})$ $= 18.4\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(\tfrac{7}{8}\text{ in.})\left(\dfrac{0.360\text{ in.}}{2}\right)(65\text{ ksi})}{2.00}$ $= 12.3\text{ kips/bolt}$

Bolt bearing on the column web controls over bolt shear. Dividing the web by 2 in the above calculation assumes that the same connection and load exist on both sides of the web. It is conservative for other cases.

Thus, the total available bolt strength, including the limit states of bolt shear and bolt bearing on the connection angles and the column web, is:

LRFD	ASD
$\phi R_n = 2(15.3\text{ kips}) + 4(18.4\text{ kips})$ $= 104\text{ kips} > 80.5\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2(10.2\text{ kips}) + 4(12.3\text{ kips})$ $= 69.6\text{ kips} > 53.7\text{ kips}$ **o.k.**

Check shear yielding on the angles

The gross area is:

$$\begin{aligned} A_{gv} &= 2(8\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.}) \\ &= 6.38\text{ in.}^2 \end{aligned}$$

The available shear yielding strength, from AISC *Specification* Section J4.2(a), Equation J4-3, is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(6.38 \text{ in.}^2)$ $= 138 \text{ kips} > 80.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(6.38 \text{ in.})}{1.50}$ $= 91.9 \text{ kips} > 53.7 \text{ kips}$

Check shear rupture on the angles

The net area is determined from AISC *Specification* Section B4.3:

$$
\begin{aligned}
A_{nv} &= A_g - 2(3t)(d_h + \tfrac{1}{16} \text{ in.}) \\
&= 6.38 \text{ in.}^2 - 2(3)(\tfrac{3}{8} \text{ in.})(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}) \\
&= 4.13 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section J4.2(b), Equation J4-4, the available shear rupture strength is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(4.13 \text{ in.}^2)$ $= 108 \text{ kips} > 80.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(4.13 \text{ in.}^2)}{2.00}$ $= 71.9 \text{ kips} > 53.7 \text{ kips}$ **o.k.**

Check block shear rupture on the angles

The controlling block shear failure path is assumed to start at the top of the angles and follows the vertical bolt lines and then runs perpendicular to the outer edge of each angle, as shown in Figure 5-18a. For the tension area calculated in the following, the net area is based on the length of the short-slotted hole dimension given in AISC *Specification* Table J3.3. For the shear area, the net area is based on the width of the short-slotted hole, which is the same as for a standard hole.

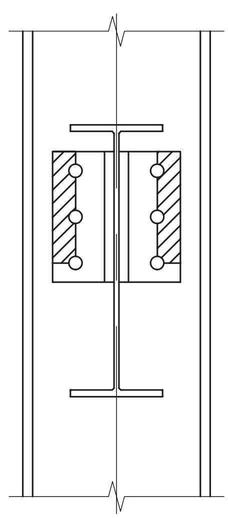


Fig. 5-18a. Block shear rupture failure path on double angles.

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

$$\begin{aligned} A_{gv} &= (7.25\text{ in.})(\tfrac{3}{8}\text{ in.})(2) \\ &= 5.44\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= 5.44\text{ in.}^2 - 2.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{8}\text{ in.})(2) \\ &= 3.57\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= \left[(4.00\text{ in.} + 4.00\text{ in.} + 0.300\text{ in.} - 5\tfrac{1}{2}\text{ in.})/2 - 0.5(1\tfrac{1}{8}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{3}{8}\text{ in.})(2) \\ &= 0.605\text{ in.}^2 \end{aligned}$$

U_{bs} is 1 from AISC *Specification* Section J4.3 because the bolts are uniformly loaded.

Shear yielding component:

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(36\text{ ksi})\left(5.44\text{ in.}^2\right) \\ &= 118\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(58\text{ ksi})\left(3.57\text{ in.}^2\right) \\ &= 124\text{ kips} \end{aligned}$$

Tension rupture component:

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(58\text{ ksi})\left(0.605\text{ in.}^2\right) \\ &= 35.1\text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 124\text{ kips} + 35.1\text{ kips} \\ &= 159\text{ kips} \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 118\text{ kips} + 35.1\text{ kips} \\ &= 153\text{ kips} \end{aligned}$$

Therefore, $R_n = 153$ kips.

LRFD	ASD
$\phi R_n = 0.75(153\text{ kips})$ $= 115\text{ kips} > 80.5\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{153\text{ kips}}{2.00}$ $= 76.5\text{ kips} > 53.7\text{ kips}$ **o.k.**

Design angle-to-beam web weld

From AISC *Manual* Table 10-2, with $L = 8\tfrac{1}{2}$ in. for Weld A, choose a $\tfrac{3}{16}$-in. fillet weld. Note that Table 10-2 is conservative for this example because it assumes an angle size of L4×3½, with the shorter leg attached to the beam. This example has an angle leg of 4 in. attached to the beam. The available weld strength of the double-angle connection is:

LRFD	ASD
$\phi R_n = 110\text{ kips} > 80.5\text{ kips}$	$\dfrac{R_n}{\Omega} = 73.5\text{ kips} > 53.7\text{ kips}$

Note: This is the case when the minimum web thickness is 0.286 in.

$t_w = 0.300\text{ in.} > 0.286\text{ in.}$ **o.k.**

The minimum web thickness of Table 10-2 is calculated from:

$$t_{min} = \frac{6.19D}{F_u}$$
$$= \frac{6.19(3)}{65\text{ ksi}}$$
$$= 0.286\text{ in.}$$

The completed design is shown in Figure 5-18.

Example 5.9—Chevron Brace Connection

Given:

The general arrangement of a chevron braced frame is shown in Figures 2-1(e), 2-5 and 3-2, and the theory is given in Figures 4-5 through 4-7. There are two types of chevron braced frames: V and inverted-V. Design the chevron heavy bracing connection for the W27×114 beam and the (2) HSS8×8×½ braces shown in Figure 5-19. Figure 5-19 is typical of chevron bracing of the inverted-V type.

The required axial strength of the bracing connections is:

LRFD	ASD
$P_u = \pm\ 289$ kips	$P_a = \pm\ 193$ kips

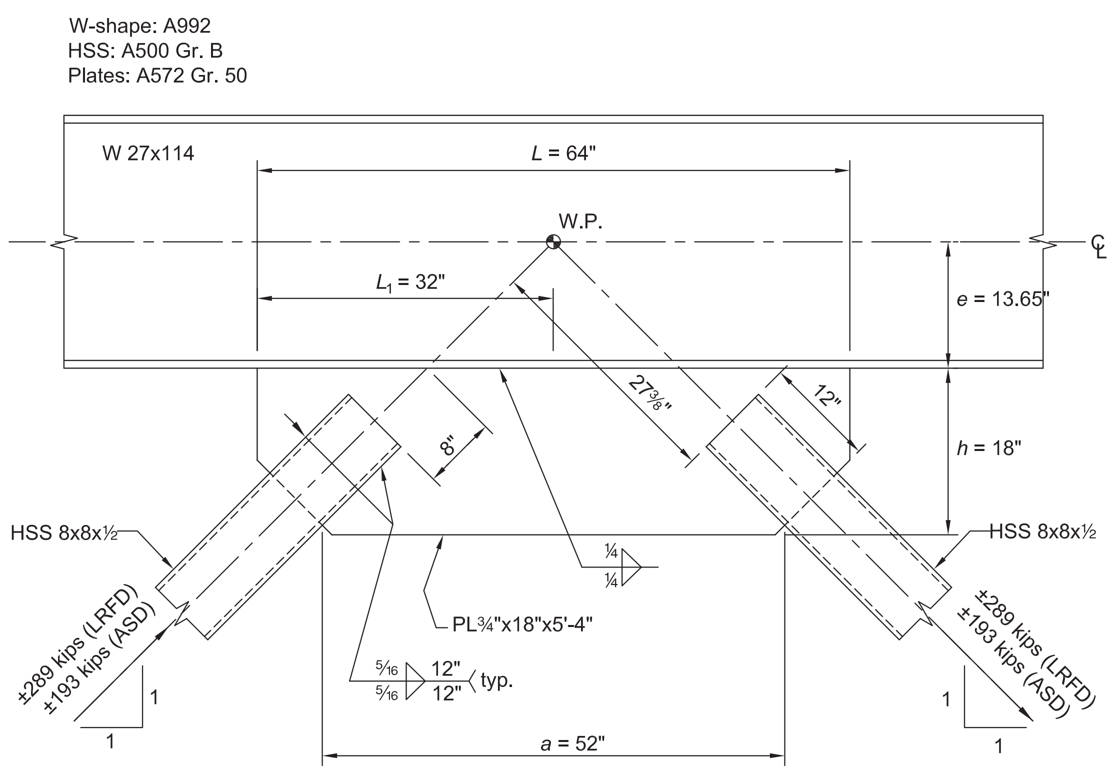


Fig. 5-19. Typical chevron brace connection.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A500 Grade B
$F_y = 46$ ksi $\quad F_u = 58$ ksi

ASTM A572 Grade 50
$F_y = 50$ ksi $\quad F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Table 1-12, the member geometric properties are as follows:

Beam: W27×114
$t_w = 0.570$ in. $\quad t_f = 0.930$ in. $\quad d = 27.3$ in. $\quad b_f = 10.1$ in. $\quad k_{des} = 1.53$ in.

Braces: HSS8×8×½
$A_g = 13.5$ in.^{2} $\quad r = 3.04$ in. $\quad t_{des} = 0.465$ in.

The geometry of this connection is shown in Figure 5-19. The work point is at the concentric location at the beam gravity axis, $e = 13.65$ in. from the outside face of the bottom flange. The brace bevels are equal and the brace loads are equal at ±289 kips (LRFD) or ±193 kips (ASD). Thus the gusset will be symmetrical and $\Delta = 0$, using the notation defined in Figure 4-5.

Because the brace forces are reversible, it is possible that both are tension or both are compression, but the most common case is for one to be tension and the other to be compression, as will be assumed here. Figure 5-20 shows the free body diagram (admissible force fields) for this case.

Using the general notation given in Figure 4-5:

$$\Delta = \frac{1}{2}\left(L_2 - L_1\right) = 0$$
$$L_1 = 32 \text{ in.}$$
$$L_2 = 32 \text{ in.}$$

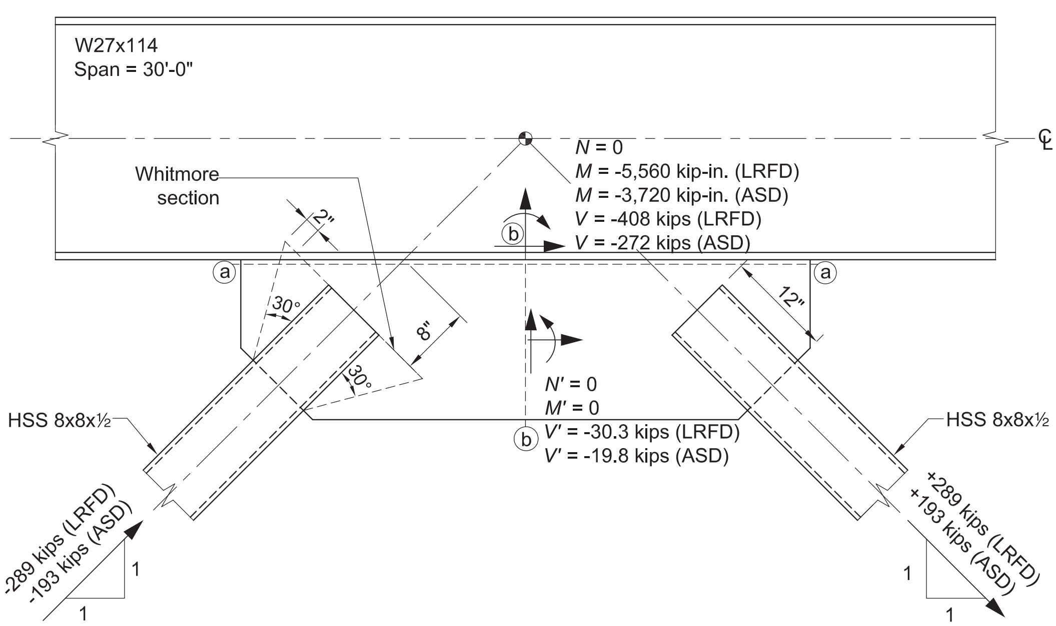


Fig. 5-20. Admissible force fields.

$L = 64$ in.
$e = 13.65$ in.
$h = 18$ in.

In the following calculations, the subscript "1" denotes Brace 1, on the left side of Figure 5-19, and the subscript "2" denotes Brace 2 on the right side. Axial force, shear force and moment acting on Section b-b have a "prime" symbol to distinguish them from the axial force, shear force and moment acting on Section a-a.

The horizontal and vertical components of the brace forces are:

LRFD	ASD
$P_{u1} = -289$ kips $H_{u1} = (-289\text{ kips})\cos 45°$ $= -204$ kips $V_{u1} = (-289\text{ kips})\sin 45°$ $= -204$ kips $P_{u2} = 289$ kips $H_{u2} = (289\text{ kips})\cos 45°$ $= 204$ kips $V_{u2} = (289\text{ kips})\sin 45°$ $= 204$ kips	$P_{a1} = -193$ kips $H_{a1} = (-193\text{ kips})\cos 45°$ $= -136$ kips $V_{a1} = (-193\text{ kips})\sin 45°$ $= -136$ kips $P_{a2} = 193$ kips $H_{a2} = (193\text{ kips})\cos 45°$ $= 136$ kips $V_{a2} = (193\text{ kips})\sin 45°$ $= 136$ kips

Using the equations presented in Figure 4-5, the moments are:

LRFD	ASD
$M_{u1} = H_{u1}e + V_{u1}\Delta$ $= (-204\text{ kips})(13.65\text{ in.}) + (-204\text{ kips})(0)$ $= -2,780$ kip-in. $M_{u2} = H_{u2}e - V_{u2}\Delta$ $= (204\text{ kips})(13.65\text{ in.}) - (204\text{ kips})(0)$ $= 2,780$ kip-in. $M'_{u1} = \frac{1}{8}V_{u1}L - \frac{1}{4}H_{u1}h - \frac{1}{2}M_{u1}$ $= \frac{1}{8}(-204\text{ kips})(64.0\text{ in.})$ $\quad -\frac{1}{4}(-204\text{ kips})(18.0\text{ in.}) - \frac{1}{2}(-2,780\text{ kips})$ $= 676$ kip-in. $M'_{u2} = \frac{1}{8}V_{u2}L - \frac{1}{4}H_{u2}h - \frac{1}{2}M_{u2}$ $= \frac{1}{8}(204\text{ kips})(64.0\text{ in.})$ $\quad -\frac{1}{4}(204\text{ kips})(18.0\text{ in.}) - \frac{1}{2}(2,780\text{ kips})$ $= -676$ kip-in.	$M_{a1} = H_{a1}e + V_{a1}\Delta$ $= (-136\text{ kips})(13.65\text{ in.}) + (-136\text{ kips})(0)$ $= -1,860$ kip-in. $M_{a2} = H_{a2}e - V_{a2}\Delta$ $= (136\text{ kips})(13.65\text{ in.}) - (136\text{ kips})(0)$ $= 1,860$ kip-in. $M'_{a1} = \frac{1}{8}V_{a1}L - \frac{1}{4}H_{a1}h - \frac{1}{2}M_{a1}$ $= \frac{1}{8}(-136\text{ kips})(64.0\text{ in.})$ $\quad -\frac{1}{4}(-136\text{ kips})(18.0\text{ in.}) - \frac{1}{2}(-1,860\text{ kips})$ $= 454$ kip-in $M'_{a2} = \frac{1}{8}V_{a2}L - \frac{1}{4}H_{a2}h - \frac{1}{2}M_{a2}$ $= \frac{1}{8}(136\text{ kips})(64.0\text{ in.})$ $\quad -\frac{1}{4}(136\text{ kips})(18.0\text{ in.}) - \frac{1}{2}(1,860\text{ kips})$ $= -454$ kip-in

Note that the signs on the various forces and equations are important.

Calculating the forces on Section a-a—the gusset-to-beam interface (using equations based on Figure 4-6):

LRFD	ASD
Axial $N_u = V_{u1} + V_{u2}$ $= -204 \text{ kips} + 204 \text{ kips}$ $= 0 \text{ kips}$ Shear $V_u = H_{u1} - H_{u2}$ $= -204 \text{ kips} - 204 \text{ kips}$ $= -408 \text{ kips}$ Moment $M_u = M_{u1} - M_{u2}$ $= -2{,}780 \text{ kip-in.} - 2{,}780 \text{ kip-in.}$ $= -5{,}560 \text{ kip-in.}$	Axial $N_a = V_{a1} + V_{a2}$ $= -136 \text{ kips} + 136 \text{ kips}$ $= 0 \text{ kips}$ Shear $V_a = H_{a1} - H_{a2}$ $= -136 \text{ kips} - 136 \text{ kips}$ $= -272 \text{ kips}$ Moment $M_a = M_{a1} - M_{a2}$ $= -1{,}860 \text{ kip-in.} - 1{,}860 \text{ kip-in.}$ $= -3{,}720 \text{ kip-in.}$

The negative signs on the shear and moment indicate that these forces act opposite to the positive directions assumed in Figure 4-6. An important location in the gusset is Section b-b shown in Figure 4-7. The forces on this section are:

LRFD	ASD
Axial $N'_u = \tfrac{1}{2}(H_{u1} + H_{u2})$ $= \tfrac{1}{2}(-204 \text{ kips} + 204 \text{ kips})$ $= 0 \text{ kips}$ Shear $V'_u = \tfrac{1}{2}(V_{u1} - V_{u2}) - \dfrac{2M_u}{L}$ $= \tfrac{1}{2}(-204 \text{ kips} - 204 \text{ kips}) - \dfrac{2(-5{,}560 \text{ kip-in.})}{64.0 \text{ in.}}$ $= -30.3 \text{ kips}$ Moment $M'_u = M'_{u1} + M'_{u2}$ $= 676 \text{ kip-in.} + (-676 \text{ kip-in.})$ $= 0$	Axial $N'_a = \tfrac{1}{2}(H_{a1} + H_{a2})$ $= \tfrac{1}{2}(-136 \text{ kips} + 136 \text{ kips})$ $= 0 \text{ kips}$ Shear $V'_a = \tfrac{1}{2}(V_{a1} - V_{a2}) - \dfrac{2M_a}{L}$ $= \tfrac{1}{2}(-136 \text{ kips} - 136 \text{ kips}) - \dfrac{2(-3{,}720 \text{ kip-in.})}{64.0 \text{ in.}}$ $= -19.8 \text{ kips}$ Moment $M'_a = M'_{a1} + M'_{a2}$ $= 454 \text{ kip-in.} + (-454 \text{ kip-in.})$ $= 0$

These forces are shown in Figure 5-20 acting on the left half of the gusset. V_u' is acting opposite to the direction shown, as indicated by the minus sign. With all interface (Section a-a) and internal (Section b-b) forces known, the connection can now be designed.

Brace-to-Gusset Connection

This part of the connection should be designed first because it will give a minimum required size of the gusset plate.

Check tension yielding on the brace

The available tensile yielding strength is determined from AISC *Specification* Section J4.1(a), Equation J4-1:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(46\text{ ksi})(13.5\text{ in.}^2)$ $= 559\text{ kips} > 289\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(46\text{ ksi})(13.5\text{ in.}^2)}{1.67}$ $= 372\text{ kips} > 193\text{ kips}$ **o.k.**

Check tensile rupture on the brace

From AISC *Specification* Equation J4-4, determine the minimum weld length required to provide adequate available shear rupture strength of the brace material. Choose a connection length between the brace and the gusset plate based on the limit state of shear rupture in the brace wall:

LRFD	ASD
$P_u = 0.60\left[\phi F_u tl(4)\right]$ $289\text{ kips} = 0.60(0.75)(58\text{ ksi})(0.465\text{ in.})l(4)$ Therefore, $l = 5.95$ in.	$P_a = 0.60\left[\dfrac{F_a}{\Omega} tl(4)\right]$ $193\text{ kips} = 0.60\left(\dfrac{58\text{ ksi}}{2.00}\right)(0.465\text{ in.})l(4)$ Therefore, $l = 5.96$ in.

If a 5⁄16-in. fillet weld is assumed with four lines of welds, AISC *Specification* Equation J2-4 and Table J2.5 give:

LRFD	ASD
$\phi P_n = \phi 0.60 F_{EXX}\left(\dfrac{1}{\sqrt{2}}\right)w(4l) \geq P_u$ $l \geq \dfrac{P_u}{\phi 0.60 F_{EXX}\left(\dfrac{1}{\sqrt{2}}\right)w(4)}$ $\geq \dfrac{289\text{ kips}}{0.75(0.60)(70\text{ ksi})\left(\dfrac{1}{\sqrt{2}}\right)(\tfrac{5}{16}\text{ in.})(4)}$ ≥ 10.4 in.	$\dfrac{P_n}{\Omega} = \dfrac{0.60 F_{EXX}\left(\dfrac{1}{\sqrt{2}}\right)w(4l)}{\Omega} \geq P_a$ $l \geq \dfrac{\Omega P_a}{0.60 F_{EXX}\left(\dfrac{1}{\sqrt{2}}\right)w(4)}$ $\geq \dfrac{2.00(193\text{ kips})}{0.60(70\text{ ksi})\left(\dfrac{1}{\sqrt{2}}\right)(\tfrac{5}{16}\text{ in.})(4)}$ ≥ 10.4 in.

Note that in this application, the actual weld size used would be 5⁄16 in. because of the slot gap; that is, with ¼-in. fillet welds called out on the drawing, the fabricator would increase the weld size to 5⁄16 in. in order to compensate for the gap between the brace and the gusset (see AWS D1.1 clause 5.22.1). This is a single pass fillet whereas the specified 5⁄16-in. fillet weld will be a multi-pass ⅜-in. fillet weld.

From the AISC *Manual* Part 8, Equations 8-2a and 8-2b, this calculation could more simply be written:

LRFD	ASD
$\phi P_n = 1.392Dl$ $289 \text{ kips} = 1.392(5)(4l)$ $l = 10.4 \text{ in.}$	$\dfrac{P_n}{\Omega} = 0.928Dl$ $193 \text{ kips} = 0.928(5)(4l)$ $l = 10.4 \text{ in.}$

Use a 12-in.-long 5⁄16-in. fillet weld. With l = 12.0 in., determine the shear lag factor, U.

The symbol, d_{slot}, used in the following is the width of the slot in the HSS brace, which is taken as the gusset thickness plus 1⁄16 in. on either side. A ¾-in.-thick gusset plate is assumed and later checked.

$$
\begin{aligned}
A_g &= 13.5 \text{ in.}^2 \\
A_n &= A_g - 2td_{slot} \quad (d_{slot} = \textit{width of slot}) \\
&= 13.5 \text{ in.}^2 - 2(0.465 \text{ in.})(\tfrac{3}{4} \text{ in.} + \tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}) \\
&= 12.7 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Table D3.1, Case 6 with a single concentric gusset plate:

$$
\begin{aligned}
\bar{x} &= \frac{B^2 + 2BH}{4(B+H)} \qquad (B = H = 8.00 \text{ in.}) \\
&= \frac{(8.00 \text{ in.})^2 + 2(8.00 \text{ in.})(8.00 \text{ in.})}{4(8.00 \text{ in.} + 8.00 \text{ in.})} \\
&= 3.00 \text{ in.} \\
U &= 1 - \left(\frac{\bar{x}}{l}\right) \\
&= 1 - \left(\frac{3.00 \text{ in.}}{12.0 \text{ in.}}\right) \\
&= 0.750
\end{aligned}
$$

From AISC *Specification* Equation D3-1:

$$
\begin{aligned}
A_e &= A_n U \qquad (\textit{Spec.} \text{ Eq. D3-1}) \\
&= (12.7 \text{ in.}^2)(0.750) \\
&= 9.53 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the brace is:

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75(58.0 \text{ ksi})(9.53 \text{ in.}^2)$ $= 415 \text{ kips} > 289 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{2.00}$ $= \dfrac{(58.0 \text{ ksi})(9.53 \text{ in.}^2)}{2.00}$ $= 276 \text{ kips} > 193 \text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

Assume a gusset plate thickness of ¾ in. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= A_{nv} \\ &= 2t_g l \\ &= (2)(3/4\text{ in.})(12.0\text{ in.}) \\ &= 18.0\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_y A_{gv} &= 0.60(50\text{ ksi})\left(18.0\text{ in.}^2\right) \\ &= 540\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} 0.60F_u A_{nv} &= 0.60(65\text{ ksi})\left(18.0\text{ in.}^2\right) \\ &= 702\text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{gt} &= A_{nt} \\ &= t_g B \\ &= (3/4\text{ in.})(8.00\text{ in.}) \\ &= 6.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_u A_{nt} &= 1(65\text{ ksi})\left(6.00\text{ in.}^2\right) \\ &= 390\text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned} 0.60F_u A_{nv} + U_{bs}F_u A_{nt} &= 702\text{ kips} + 390\text{ kips} \\ &= 1{,}090\text{ kips} \end{aligned}$$

$$\begin{aligned} 0.60F_y A_{gv} + U_{bs}F_u A_{nt} &= 540\text{ kips} + 390\text{ kips} \\ &= 930\text{ kips} \end{aligned}$$

Therefore, $R_n = 930$ kips.

LRFD	ASD
$\phi R_n = 0.75(930\text{ kips})$ $= 698\text{ kips} > 289\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{930\text{ kips}}{2.00}$ $= 465\text{ kips} > 193\text{ kips}$ **o.k.**

If the braces had different loads, block shear would need to be checked for each brace.

Check the gusset plate for tensile yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section is:

$$\begin{aligned} l_w &= B + 2l\tan 30^\circ \\ &= 8.00\text{ in.} + 2(12.0\text{ in.})\tan 30^\circ \\ &= 21.9\text{ in.} \end{aligned}$$

It can be seen from Figure 5-20 that about 2.00 in. of the Whitmore section is in the beam web. The web thickness, $t_w = 0.570$ in. < $t_g = \frac{3}{4}$ in.; therefore, the portion of the Whitmore area in the web is accounted for.

$$
\begin{aligned}
A_w &= (21.9 \text{ in.} - 2.00 \text{ in.})(\tfrac{3}{4} \text{ in.}) + (2.00 \text{ in.})(0.570 \text{ in.}) \\
&= 16.1 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section J4.1(a):

LRFD	ASD
$\begin{aligned}\phi R_n &= 0.90 F_y A_w \\ &= 0.90(50 \text{ ksi})(16.1 \text{ in.}^2) \\ &= 725 \text{ kips} > 289 \text{ kips} \quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= \frac{F_y A_w}{1.67} \\ &= \frac{(50 \text{ ksi})(16.1 \text{ in.}^2)}{1.67} \\ &= 482 \text{ kips} > 193 \text{ kips} \quad \textbf{o.k.}\end{aligned}$

Check the gusset plate for buckling on the Whitmore section

The available compressive strength of the gusset plate based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4, using an effective length factor, K, of 0.65 according to Dowswell (2006, 2009, 2012). As shown in Figure 5-20, the unbraced length of the gusset plate is 8.00 in.

$$
\begin{aligned}
\frac{KL}{r} &= \frac{0.65(8.00 \text{ in.})}{\left(\dfrac{\frac{3}{4} \text{ in.}}{\sqrt{12}}\right)} \\
&= 24.0
\end{aligned}
$$

Because $\dfrac{KL}{r} < 25.0$, according to AISC *Specification* Section J4.4, the strength in compression is the same as that for tension yielding, which was previously checked.

Gusset-to-Beam Connection (Section a-a)

The required strengths are:

LRFD	ASD
Required shear strength, $V_u = 408$ kips Required axial strength, $N_u = 0$ kips Required flexural strength, $M_u = 5{,}560$ kip-in.	Required shear strength, $V_a = 272$ kips Required axial strength, $N_a = 0$ kips Required flexural strength, $M_a = 3{,}720$ kip-in.

Check the gusset plate for tensile yielding and shear yielding along the beam flange

In this example, to allow combining the required normal strength and flexural strength, stresses will be used to check shear yielding and tensile yielding limit states on the gusset plate. Shear yielding is checked using AISC *Specification* Section J4.2 and tensile yielding is checked using Section J4.1:

LRFD	ASD
Shear yielding stress: $$\begin{aligned} f_{uv} &= \frac{V_u}{A_g} \\ &= \frac{408 \text{ kips}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})} \\ &= 8.50 \text{ ksi} < 1.00(0.60)(50 \text{ ksi}) = 30 \text{ ksi} \quad \textbf{o.k.} \end{aligned}$$ Tensile yielding stress: $$\begin{aligned} f_{ua} &= \frac{N_u}{A_g} \\ &= \frac{0 \text{ kips}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})} \\ &= 0 \text{ ksi} \\ f_{ub} &= \frac{M_u}{Z} \\ &= \frac{5{,}560 \text{ kip-in.}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})^2 / 4} \\ &= 7.24 \text{ ksi} \end{aligned}$$ The total normal stress = f_{un} $$\begin{aligned} f_{un} &= f_{ua} + f_{ub} \\ &= 0 \text{ ksi} + 7.24 \text{ ksi} \\ &= 7.24 \text{ ksi} < 0.90(50 \text{ ksi}) = 45.0 \text{ ksi} \quad \textbf{o.k.} \end{aligned}$$	Shear yielding stress: $$\begin{aligned} f_{av} &= \frac{V_a}{A_g} \\ &= \frac{272 \text{ kips}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})} \\ &= 5.67 \text{ ksi} < \frac{0.60(50 \text{ ksi})}{1.50} = 20 \text{ ksi} \quad \textbf{o.k.} \end{aligned}$$ Tensile yielding stress: $$\begin{aligned} f_{aa} &= \frac{N_a}{A_g} \\ &= \frac{0 \text{ kips}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})} \\ &= 0 \text{ ksi} \\ f_{ab} &= \frac{M_a}{Z} \\ &= \frac{3{,}720 \text{ kip-in.}}{(\tfrac{3}{4} \text{ in.})(64.0 \text{ in.})^2 / 4} \\ &= 4.84 \text{ ksi} \end{aligned}$$ The total normal stress = f_{an} $$\begin{aligned} f_{an} &= f_{aa} + f_{ab} \\ &= 0 \text{ ksi} + 4.84 \text{ ksi} \\ &= 4.84 \text{ ksi} < \frac{50 \text{ ksi}}{1.67} = 29.9 \text{ ksi} \quad \textbf{o.k.} \end{aligned}$$

Design weld at gusset-to-beam flange connection

The 408-kip (LRFD) or 272-kip (ASD) shear force is to be delivered to the center of the beam. This is a function of the required flexural strength, 5,560 kip-in (LRFD) or 3,720 kip-in. (ASD). The effective eccentricity of the shear force is:

LRFD	ASD
$$\begin{aligned} e &= \frac{M_u}{V_u} \\ &= \frac{5{,}560 \text{ kip-in.}}{408 \text{ kips}} \\ &= 13.6 \text{ in.} \end{aligned}$$	$$\begin{aligned} e &= \frac{M_a}{V_a} \\ &= \frac{3{,}720 \text{ kip-in.}}{272 \text{ kips}} \\ &= 13.7 \text{ in.} \end{aligned}$$

Therefore, from AISC *Manual* Table 8-4 for Angle = 0, $k = 0$, $a = 13.6 \text{ in.}/64.0 \text{ in.} = 0.213$, by interpolation $C = 3.46$. With $C_1 = 1.0$, the number of sixteenths of weld required with a ductility factor of 1.25 (as discussed in AISC *Manual* Part 13) is determined from AISC *Manual* Equation 8-13:

LRFD	ASD
$D_{req'd} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{(408\text{ kips})(1.25)}{(0.75)(3.46)(1.0)(64.0\text{ in.})}$ $= 3.07$	$D_{req'd} = \dfrac{\Omega P_a}{C C_1 l}$ $= \dfrac{2.00(272\text{ kips})(1.25)}{(3.46)(1.0)(64.0\text{ in.})}$ $= 3.07$

Use a ¼-in. double-sided fillet weld.

An alternative method to determine the required weld size that does not require the use of an AISC *Manual* table follows. See Appendix B for a discussion of the method. This method is similar to the method of Hewitt and Thornton (2004) except that it allows the use of the increased strength of fillet welds loaded at an angle relative to the weld longitudinal axis.

The required shear strength is $V_u = 408$ kips (LRFD) or $V_a = 272$ kips (ASD). The normal force and the moment can be combined to generate a maximum equivalent normal force (see Appendix B, Figure B-1):

LRFD	ASD
$N_{max} = \lvert N_u \rvert + \left\lvert \dfrac{2M_u}{L/2} \right\rvert$ $= \lvert N_u \rvert + \left\lvert \dfrac{4M_u}{L} \right\rvert$ $= 0\text{ kips} + \left\lvert \dfrac{4(-5{,}560\text{ kip-in.})}{64\text{ in.}} \right\rvert$ $= 348\text{ kips}$ and a minimum equivalent normal force: $N_{min} = \left\lvert \lvert N_u \rvert - \left\lvert \dfrac{4M_u}{L} \right\rvert \right\rvert$ $= \left\lvert \lvert 0\text{ kips} \rvert - \left\lvert \dfrac{4(-5{,}560\text{ kip-in.})}{64\text{ in.}} \right\rvert \right\rvert$ $= 348\text{ kips}$ The peak weld resultant force: $R_{peak} = \sqrt{V_u^2 + N_{max}^2}$ $= \sqrt{(408\text{ kips})^2 + (348\text{ kips})^2}$ $= 536\text{ kips}$ The average weld resultant force: $R_{avg} = \sqrt{V_u^2 + \left(\dfrac{N_{max} + N_{min}}{2}\right)^2}$ $= \sqrt{(408\text{ kips})^2 + \left(\dfrac{348\text{ kips} + 348\text{ kips}}{2}\right)^2}$ $= 536\text{ kips}$	$N_{max} = \lvert N_a \rvert + \left\lvert \dfrac{2M_a}{L/2} \right\rvert$ $= \lvert N_a \rvert + \left\lvert \dfrac{4M_a}{L} \right\rvert$ $= 0\text{ kips} + \left\lvert \dfrac{4(-3{,}720\text{ kip-in.})}{64.0\text{ in.}} \right\rvert$ $= 233\text{ kips}$ and a minimum equivalent normal force: $N_{min} = \left\lvert \lvert N_a \rvert - \left\lvert \dfrac{4M_a}{L} \right\rvert \right\rvert$ $= \left\lvert \lvert 0\text{ kips} \rvert - \left\lvert \dfrac{4(-3{,}720\text{ kip-in.})}{64.0\text{ in.}} \right\rvert \right\rvert$ $= 233\text{ kips}$ The peak weld resultant force: $R_{peak} = \sqrt{V_a^2 + N_{max}^2}$ $= \sqrt{(272\text{ kips})^2 + (233\text{ kips})^2}$ $= 358\text{ kips}$ The average weld resultant force: $R_{avg} = \sqrt{V_a^2 + \left(\dfrac{N_{max} + N_{min}}{2}\right)^2}$ $= \sqrt{(272\text{ kips})^2 + \left(\dfrac{233\text{ kips} + 233\text{ kips}}{2}\right)^2}$ $= 358\text{ kips}$

Because $1.25R_{avg} > R_{peak}$, use $1.25R_{avg}$ to size the weld. From AISC *Manual* Equation 8-2, including the directional strength increase of the fillet weld provided in AISC *Specification* Equation J2-4:

LRFD	ASD
$\theta = \tan^{-1}\left(\frac{N_{max}}{V_u}\right)$ $= \tan^{-1}\left(\frac{348 \text{ kips}}{408 \text{ kips}}\right)$ $= 40.5°$	$\theta = \tan^{-1}\left(\frac{N_{max}}{V_a}\right)$ $= \tan^{-1}\left(\frac{233 \text{ kips}}{272 \text{ kips}}\right)$ $= 40.6°$

LRFD	ASD
$D_{req'd} = \dfrac{1.25R_{avg}}{1.392\left(1.0+0.50\sin^{1.5}\theta\right)\left(2L\right)}$ $= \dfrac{1.25\left(536 \text{ kips}\right)}{1.392\left(1.0+0.50\sin^{1.5}40.5°\right)\left[2\left(64.0 \text{ in.}\right)\right]}$ $= 2.98$ % difference between methods $\left[\dfrac{\left(2.98-3.07\right)}{3.07}\right]\times 100\% = 2.93\%$	$D_{req'd} = \dfrac{1.25R_{avg}}{0.928\left(1.0+0.50\sin^{1.5}\theta\right)\left(2L\right)}$ $= \dfrac{1.25\left(358 \text{ kips}\right)}{0.928\left(1.0+0.50\sin^{1.5}40.6°\right)\left[2\left(64.0 \text{ in.}\right)\right]}$ $= 2.98$ % difference between methods $\left[\dfrac{\left(2.98-3.07\right)}{3.07}\right]\times 100\% = 2.93\%$

Note that the directional strength increase of $1.0 + 0.50 \sin^{1.5} 40.5° = 1.26$ indicates a 26% increase over the strength of a longitudinally loaded fillet weld.

This alternative method produces a result that is 2 to 3% smaller than the result of using the AISC *Manual* Table 8-4 method, which is essentially the same answer. The required weld size is ¼ in., which is the AISC minimum weld size from AISC *Specification* Table J2-4. A discussion on the alternative method is given in Appendix B.

Check gusset internal strength (Section b-b)

As mentioned previously, the prime symbol after the force quantity indicates that it acts on Section b-b through the gusset plate. The control section for this chevron gusset is the horizontal Section a-a. Section b-b, which is an internal gusset section, plus the beam cross section above the gusset Section b-b, must be able to transfer the brace vertical components, V_1 and V_2, across this section. The forces H', N' and M' are a direct result of the forces H, N and M assumed on the control Section a-a. See Figures 4-5, 4-6 and 4-7.

The required strengths are:

LRFD	ASD
Required shear strength, $V'_u = -30.3$ kips Required axial strength, $N'_u = 0$ kips Required flexural strength, $M'_u = 0$ kip-in.	Required shear strength, $V'_a = -19.8$ kips Required axial strength, $N'_a = 0$ kips Required flexural strength, $M'_a = 0$ kip-in.

Check the gusset plate for shear yielding at Section b-b

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(\tfrac{3}{4} \text{ in.})(18.0 \text{ in.})$ $= 405 \text{ kips} > 30.3 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(\tfrac{3}{4} \text{ in.})(18.0 \text{ in.})}{1.50}$ $= 270 \text{ kips} > 19.8 \text{ kips}$ **o.k.**

Because N' and M' are zero, there are no further checks.

If N' and M' were not zero, it is possible for a compressive stress to exist on the gusset free edge at Section b-b. In this case, the gusset should be checked for buckling under this stress. The procedure given in the AISC *Manual* on pages 9-8 and 9-9 for buckling of double-coped beams can be used. This *Manual* procedure is developed into a gusset free-edge buckling method in Appendix C, Section C.4, where it is referred to as the admissible force maintenance method (AFMM).

Check gusset stresses on Section b-b for a hypothetical load case

As an example, consider that both braces of Figure 5-19 are in compression simultaneously. Then $P_1 = P_2$, and from Figure 4-5:

LRFD	ASD
$M_{u1} = (-204 \text{ kips})(13.65 \text{ in.})$ $= -2{,}780 \text{ kip-in.}$ $M_{u2} = (-204 \text{ kips})(13.65 \text{ in.})$ $= -2{,}780 \text{ kip-in.}$ $M'_{u1} = \tfrac{1}{8}(-204 \text{ kips})(64.0 \text{ in.}) - \tfrac{1}{4}(-204 \text{ kips})(18.0 \text{ in.})$ $- \tfrac{1}{2}(-2{,}780 \text{ kip-in.})$ $= 676 \text{ kip-in.}$ $M'_{u2} = \tfrac{1}{8}(-204 \text{ kips})(64.0 \text{ in.}) - \tfrac{1}{4}(-204 \text{ kips})(18.0 \text{ in.})$ $- \tfrac{1}{2}(-2{,}780 \text{ kip-in.})$ $= 676 \text{ kip-in.}$	$M_{a1} = (-136 \text{ kips})(13.65 \text{ in.})$ $= -1{,}860 \text{ kip-in.}$ $M_{a2} = (-136 \text{ kips})(13.65 \text{ in.})$ $= -1{,}860 \text{ kip-in.}$ $M'_{a1} = \tfrac{1}{8}(-136 \text{ kips})(64.0 \text{ in.})$ $- \tfrac{1}{4}(-136 \text{ kips})(18.0 \text{ in.}) - \tfrac{1}{2}(-1{,}860 \text{ kip-in.})$ $= 454 \text{ kip-in.}$ $M'_{a2} = \tfrac{1}{8}(-136 \text{ kips})(64.0 \text{ in.})$ $- \tfrac{1}{4}(-136 \text{ kips})(18.0 \text{ in.}) - \tfrac{1}{2}(-1{,}860 \text{ kip-in.})$ $= 454 \text{ kip-in.}$

From Figure 4-6, the forces on Section a-a are:

LRFD	ASD
$N_u = -204 \text{ kips} + (-204 \text{ kips})$ $= -408 \text{ kips}$ $V_u = -204 \text{ kips} - (-204 \text{ kips})$ $= 0 \text{ kips}$ $M_u = -2{,}780 \text{ kip-in.} - (-2{,}780 \text{ kip-in.})$ $= 0 \text{ kip-in.}$	$N_a = -136 \text{ kips} + (-136 \text{ kips})$ $= -272 \text{ kips}$ $V_a = -136 \text{ kips} - (-136 \text{ kips})$ $= 0 \text{ kips}$ $M_a = -1{,}860 \text{ kip-in.} - (-1{,}860 \text{ kip-in.})$ $= 0 \text{ kip-in.}$

And those on Section b-b from Figure 4-7 are:

LRFD	ASD
$N' = \frac{1}{2}\left[-204\text{ kips} + \left(-204\text{ kips}\right)\right]$ $= -204\text{ kips}$ $V' = \frac{1}{2}\left[-204\text{ kips} - \left(-204\text{ kips}\right)\right] - \frac{2}{64\text{ in.}}(0)$ $= 0\text{ kips}$ $M' = 676\text{ kip-in.} + 676\text{ kip-in.}$ $= 1{,}350\text{ kip-in.}$	$N' = \frac{1}{2}\left[-136\text{ kips} + \left(-136\text{ kips}\right)\right]$ $= -136\text{ kips}$ $V' = \frac{1}{2}\left[-136\text{ kips} - \left(-136\text{ kips}\right)\right] - \frac{2}{64\text{ in.}}(0)$ $= 0\text{ kips}$ $M' = 454\text{ kip-in.} + 454\text{ kip-in.}$ $= 908\text{ kip-in.}$

The entire Section b-b is under a uniform compression of N' and an additional compressive force due to M'. The worst case equivalent normal force (compression) can be calculated (see Appendix B, Figure B-1) as:

LRFD	ASD
$N_{ue} = 204\text{ kips} + \left(\frac{1{,}350\text{ kip-in.}}{9.00\text{ in.}}\right)(2)$ $= 504\text{ kips}$	$N_{ae} = 136\text{ kips} + \left(\frac{908\text{ kip-in.}}{9.00\text{ in.}}\right)(2)$ $= 338\text{ kips}$

Note that this equivalent normal force will produce the correct gusset maximum normal stress but is conservative for buckling because the compressive portion of the stress due to M' acts over the upper half of gusset Section b-b, because M' is greater than zero. The dimensions a and b, scaled or calculated from Figure 5-19, are $a = 52$ in. and $b = 18$ in. From Appendix C, Section C.4, λ and Q can be determined as follows:

$$\lambda = \frac{\left(b/t_w\right)\sqrt{F_y}}{5\sqrt{475 + \dfrac{1{,}120}{\left(a/b\right)^2}}} \tag{C-7}$$

$$= \frac{\left(18.0\text{ in.}/\tfrac{3}{4}\text{ in.}\right)\sqrt{50\text{ ksi}}}{5\sqrt{475 + \dfrac{1{,}120}{\left(52.0\text{ in.}/18.0\text{ in.}\right)^2}}}$$

$$= 1.38$$

Since $0.70 < \lambda \le 1.41$:

$$Q = 1.34 - 0.486\lambda$$
$$= 1.34 - 0.486(1.38)$$
$$= 0.670$$

LRFD	ASD
$\phi F_{cr} = 0.90(0.670)(50\text{ ksi})$ $= 30.2\text{ ksi}$	$\frac{F_{cr}}{\Omega} = \frac{(0.670)(50\text{ ksi})}{1.67}$ $= 20.1\text{ ksi}$

The actual (worst case) compressive stress is:

LRFD	ASD
$f_{ua} = \dfrac{504 \text{ kips}}{(\frac{3}{4} \text{ in.})(18.0 \text{ in.})}$ $= 37.3 \text{ ksi} > 30.2 \text{ ksi}$ **n.g.**	$f_{aa} = \dfrac{338 \text{ kips}}{(\frac{3}{4} \text{ in.})(18.0 \text{ in.})}$ $= 25.0 \text{ ksi} > 20.1 \text{ ksi}$ **n.g.**

Because gusset buckling does control, an edge stiffener could be added, or a thicker plate could be used. The latter is usually the more economical solution. A ⅞-in.-thick plate will give:

$$\lambda = \frac{(18.0 \text{ in.}/\tfrac{7}{8} \text{ in.})\sqrt{50 \text{ ksi}}}{5\sqrt{475 + \dfrac{1{,}120}{(52.0 \text{ in.}/18.0 \text{ in.})^2}}}$$
$$= 1.18$$
$$Q = 1.34 - 0.486(1.18)$$
$$= 0.767$$

LRFD	ASD
$\phi F_{cr} = 0.90(0.767)(50 \text{ ksi})$ $= 34.5 \text{ ksi}$ $f_{ua} = \dfrac{504 \text{ kips}}{(\frac{7}{8} \text{ in.})(18.0 \text{ in.})}$ $= 32.0 \text{ ksi} < 34.5 \text{ ksi}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = \dfrac{(0.767)(50 \text{ ksi})}{1.67}$ $= 23.0 \text{ ksi}$ $f_{aa} = \dfrac{338 \text{ kips}}{(\frac{7}{8} \text{ in.})(18.0 \text{ in.})}$ $= 21.5 \text{ ksi} < 23.0 \text{ ksi}$ **o.k.**

A less conservative method for calculating the maximum compressive stress is based on the fact that the moment M' is a free vector and does not need to be applied at the half depth of Section b-b of the gusset. A discussion of this matter can be found in Tamboli (2010). If this method is used, it will be found that $f_u = 29.9$ ksi for LRFD and $f_a = 19.9$ ksi for ASD, and the compression zone occurs at the free edge when both braces are in tension. To calculate λ, $b = 13.6$ in. and use a ¾-in.-thick gusset plate:

$$\lambda = \frac{(13.6 \text{ in.}/\tfrac{3}{4} \text{ in.})\sqrt{50 \text{ ksi}}}{5\sqrt{475 + \dfrac{1{,}120}{(52.0 \text{ in.}/13.6 \text{ in.})^2}}}$$
$$= 1.09$$
$$Q = 1.34 - 0.486(1.09)$$
$$= 0.810$$

LRFD	ASD
$\phi F_{cr} = 0.90(0.810)(50 \text{ ksi})$ $= 36.5 \text{ ksi} > 29.9 \text{ ksi}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = \dfrac{(0.810)(50 \text{ ksi})}{1.67}$ $= 24.3 \text{ ksi} > 19.9 \text{ ksi}$ **o.k.**

Thus, the more complex and less conservative method verifies that the ¾-in.-thick gusset plate is satisfactory. The design checks will continue with the ¾-in.-thick gusset plate.

Check the gusset plate for buckling on the Whitmore section

Taking a conservative estimate of the buckling length as the vertical distance from the ends of the braces at their centerlines to the beam flange and $K = 1.2$:

$$\frac{KL}{r} = \frac{1.2\left(\dfrac{8.00 \text{ in.}}{\sqrt{2}}\right)}{\left(\dfrac{3/4 \text{ in.}}{\sqrt{12}}\right)}$$

$$= 31.4$$

As previously determined, $l_w = 21.9$ in. and $A_w = 16.1$ in.^{2}

From AISC *Specification* Section J4.4, because $KL/r > 25$, AISC *Specification* Chapter E applies. From AISC *Manual* Table 4-22, interpolating for $KL/r = 31.4$, and using AISC *Specification* Section E3, the available compressive strength is determined as follows:

LRFD	ASD
$\phi_c F_{cr} = 41.9$ ksi $\phi_c P_n = \phi_c F_{cr} A_w$ $= (41.9 \text{ ksi})(16.1 \text{ in.}^2)$ $= 675 \text{ kips} > 289 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 27.9$ ksi $\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= (27.9 \text{ ksi})(16.1 \text{ in.}^2)$ $= 449 \text{ kips} > 193 \text{ kips}$ **o.k.**

Check the gusset plate for sidesway buckling

For this alternate load case, with both braces in compression, consider gusset sidesway buckling under the compressive force of 408 kips (LRFD) or 272 kips (ASD) on Section a-a.

From AISC *Specification* Section J4.4, because $KL/r > 25$, AISC *Specification* Chapter E applies. From AISC *Manual* Table 4-22 and using AISC *Specification* Section E3, the available compressive strength is determined as follows:

LRFD	ASD
$\phi_c F_{cr} = 41.9$ ksi $\phi R_n = \phi F_{cr} t L$ $= (41.9 \text{ ksi})(3/4 \text{ in.})(64 \text{ in.})$ $= 2{,}010 \text{ kips} > 408 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 27.9$ ksi $R_n = \dfrac{F_{cr} t L}{\Omega}$ $= (27.9 \text{ ksi})(3/4 \text{ in.})(64 \text{ in.})$ $= 1{,}340 \text{ kips} > 272 \text{ kips}$ **o.k.**

In this buckling calculation it is assumed that the beam web is supported laterally by in-fill beams that frame orthogonally to the W27×114 and are of a similar depth. If the infill beams are shallow or nonexistent, a beam bottom flange bracing strut may be necessary.

Returning now to the original problem, check the beam for web local yielding and local crippling.

Check beam web local yielding

The force used is the maximum normal force, N_{max}. The normal force is applied near the midspan of the beam, which is at a distance from the end of the beam greater than the beam depth. Therefore, use AISC *Specification* Equation J10-2.

From AISC *Specification* Equation J10-2:

LRFD	ASD
$N_{max} = 348 \text{ kips}$ $\phi R_n = \phi F_y t_w (5k + l_b)$ $= 1.00(50 \text{ ksi})(0.570 \text{ in.})[5(1.53 \text{ in.}) + 64.0 \text{ in.}]$ $= 2{,}040 \text{ kips} > 348 \text{ kips}$ **o.k.**	$N_{max} = 233 \text{ kips}$ $\dfrac{R_n}{\Omega} = \dfrac{F_y t_w}{\Omega}(5k + l_b)$ $= \dfrac{(50 \text{ ksi})(0.570 \text{ in.})}{1.50}[5(1.53 \text{ in.}) + 64.0 \text{ in.}]$ $= 1{,}360 \text{ kips} > 233 \text{ kips}$ **o.k.**

Check beam web shear yielding

The maximum required shear strength is V_u = 408 kips (LRFD) and V_a = 272 kips (ASD). From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= \phi 0.60 F_y t_w l$ $= 1.00(0.60)(50 \text{ ksi})(0.570 \text{ in.})(64.0 \text{ in.})$ $= 1{,}090 \text{ kips} > 408 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.6 F_y A_{gv}}{\Omega}$ $= \dfrac{0.6 F_y t_w l}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(0.570 \text{ in.})(64.0 \text{ in.})}{1.50}$ $= 730 \text{ kips} > 272 \text{ kips}$ **o.k.**

The gusset length, l, is used here, but any length up to about half the beam length can be used.

Check beam web local crippling

The normal force is applied near the midspan of the beam, which is greater than a distance of $d/2$ from the beam end. Therefore, AISC *Specification* Equation J10-4 is applicable:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_{yw} t_f}{t_w}}$ $= (0.75)(0.80)(0.570 \text{ in.})^2 \left[1 + 3\left(\dfrac{64.0 \text{ in.}}{27.3 \text{ in.}}\right)\left(\dfrac{0.570 \text{ in.}}{0.930 \text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.930 \text{ in.})}{0.570 \text{ in.}}}$ $= 1{,}310 \text{ kips} > 348 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2}{\Omega}\left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_{yw} t_f}{t_w}}$ $= \dfrac{(0.80)(0.570 \text{ in.})^2}{2.00}\left[1 + 3\left(\dfrac{64.0 \text{ in.}}{27.3 \text{ in.}}\right)\left(\dfrac{0.570 \text{ in.}}{0.930 \text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.930 \text{ in.})}{0.570 \text{ in.}}}$ $= 874 \text{ kips} > 233 \text{ kips}$ **o.k.**

Check transverse section web yielding

The transverse shear induced in the beam at the centerline of the gusset (gusset Section b-b) is:

LRFD	ASD
$V_u = 204 \text{ kips} - 30.3 \text{ kips}$ $= 174 \text{ kips}$ From AISC *Specification* Section G2.1, the available shear yielding strength is: $\phi V_n = \phi 0.6 F_y A_w C_v$ $= 1.00(0.6)(50 \text{ ksi})(0.570 \text{ in.})(27.3 \text{ in.})(1.0)$ $= 467 \text{ kips} > 174 \text{ kips}$ **o.k.**	$V_a = 136 \text{ kips} - 19.8 \text{ kips}$ $= 116 \text{ kips}$ From AISC *Specification* Section G2.1, the available shear yielding strength is: $\dfrac{V_n}{\Omega} = \dfrac{0.6 F_y t_w d C_v}{\Omega}$ $= \dfrac{(0.6)(50 \text{ ksi})(0.570 \text{ in.})(27.3 \text{ in.})(1.0)}{1.50}$ $= 311 \text{ kips} > 116 \text{ kips}$ **o.k.**

This completes the calculations supporting the connection design shown in Figure 5-19.

Example 5.10—Nonorthogonal Bracing Connection

Given:

Building columns must sometimes be designed for changing floor arrangements from floor to floor. Building facades are sometimes arranged with a slope, resulting in nonorthogonal beam-to-column connections and bracing connections. This topic is discussed in Section 4.2.5. The theory is shown in Figure 4-17, and Figure 5-21 shows the connection designed in this example.

The required strengths of the brace (and bracing connection) and the beam are:

LRFD	ASD
Brace required strength, $P_u = \pm 525 \text{ kips}$ Beam required shear strength, $V_u = 30.0 \text{ kips}$	Brace required strength, $P_a = \pm 350 \text{ kips}$ Beam required shear strength, $V_a = 20.0 \text{ kips}$

Use 1-in.-diameter ASTM A490-X bolts in the beam-to-column and gusset-to-column connections.

Solution:

From AISC *Manual* Tables 1-1, 1-7 and 1-12, the geometric properties are as follows:

Beam
W18×50
$t_w = 0.355$ in. $\quad d = 18.0$ in. $\quad b_f = 7.50$ in. $\quad t_f = 0.570$ in. $\quad k_{des} = 0.972$ in.

Column
W14×342
$t_w = 1.54$ in. $\quad d = 17.5$ in. $\quad b_f = 16.4$ in. $\quad t_f = 2.47$ in. $\quad h/t_w = 7.41$ in.

Connection Angles: 2L5×5×½ and 2L8×6×⅝ (6 in. leg trimmed to 5 in.)

Brace
HSS10×10×½
$A_g = 17.2 \text{ in.}^2 \quad t_{des} = 0.465$ in.

From AISC *Manual* Table 2-4 and 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A500 Grade B
$F_y = 46$ ksi $\quad F_u = 58$ ksi

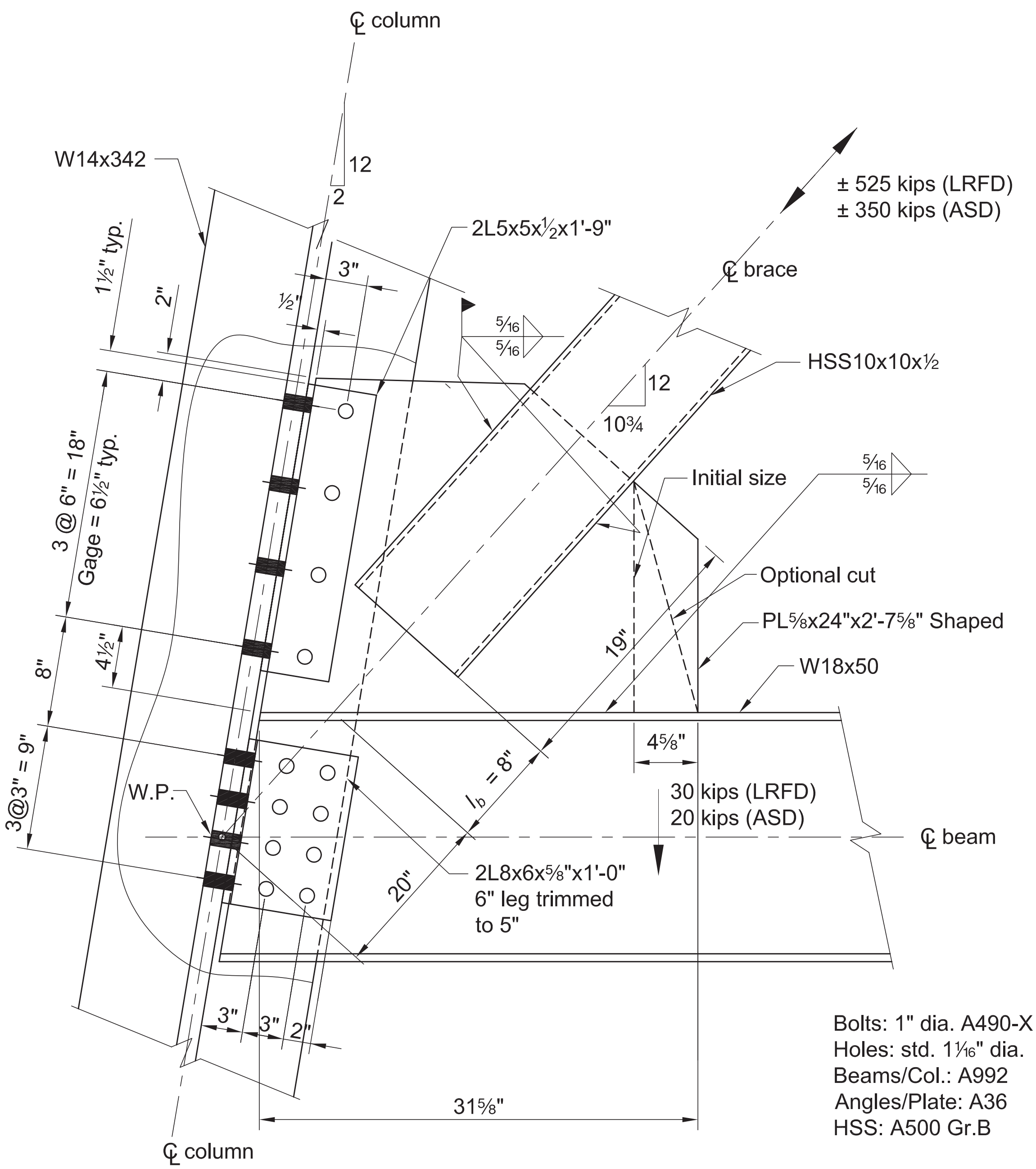


Fig. 5-21. Nonorthogonal bracing connection.

From AISC *Manual* Tables 7-1 and 7-2 the available bolt strength for the 1-in.-diameter ASTM A490-X bolts is:

LRFD	ASD
$\phi r_{nv} = 49.5$ kips single shear	$\dfrac{r_{nv}}{\Omega} = 33.0$ kips single shear
$\phi r_{nt} = 66.6$ kips tension	$\dfrac{r_{nt}}{\Omega} = 44.4$ kips tension

Brace-to-Gusset Connection

This part of the connection should be designed first to arrive at a preliminary gusset plate size.

Check tension yielding on the brace

The available tensile yielding strength is determined from AISC *Specification* Section D2, Equation D2-1. Alternatively, AISC *Manual* Table 5-5 may be used.

LRFD	ASD
$\phi_t P_n = \phi_t F_y A_g$ $= 0.90(46 \text{ ksi})(17.2 \text{ in.}^2)$ $= 712 \text{ kips} > 525 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = \dfrac{F_y A_g}{\Omega_t}$ $= \dfrac{(46 \text{ ksi})(17.2 \text{ in.}^2)}{1.67}$ $= 474 \text{ kips} > 350 \text{ kips}$ **o.k.**

Check tensile rupture on the brace

For preliminary calculations, assume a ⅝-in.-thick gusset plate. Assume that the slot in the brace is ⅛ in. wider than the gusset plate. The effective net area is:

$$A_e = A_n U \qquad (\textit{Spec.}\ \text{Eq. D3-1})$$

where

$$\begin{aligned} A_n &= A_g - 2(t_g + \tfrac{1}{8} \text{ in.})\, t_{des} \\ &= 17.2 \text{ in.}^2 - 2(\tfrac{5}{8} \text{ in.} + \tfrac{1}{8} \text{ in.})(0.465 \text{ in.}) \\ &= 16.5 \text{ in.}^2 \end{aligned}$$

The shear lag factor, U, is determined from AISC *Specification* Table D3.1, Case 6, with a single concentric gusset plate, assuming $l \geq H$:

$$\begin{aligned} \bar{x} &= \frac{B^2 + 2BH}{4(B+H)} \qquad (B = H = 10.0 \text{ in.}) \\ &= \frac{(10.0 \text{ in.})^2 + 2(10.0 \text{ in.})(10.0 \text{ in.})}{4(10.0 \text{ in.} + 10.0 \text{ in.})} \\ &= 3.75 \text{ in.} \end{aligned}$$

To proceed, the length, l, of the HSS-to-gusset weld is needed, which is based on the brace-to-gusset lap length. This is determined by checking the limit state of shear rupture in the brace wall. From AISC *Specification* Section J4.2:

$$R_n = 0.60F_uA_{nv} \qquad \text{(Spec. Eq. J4-4)}$$

In this calculation, the net cross-sectional area of the brace is taken as the four walls of the brace, $A_{nv} = 4lt_{des}$. Solving for the lap length or weld length, l:

LRFD	ASD
$l \geq \dfrac{P_u}{\phi 0.60F_ut_{des}4}$ $\geq \dfrac{525 \text{ kips}}{0.75(0.60)(58 \text{ ksi})(0.465 \text{ in.})(4)}$ $\geq 10.8 \text{ in.}$	$l \geq \dfrac{\Omega P_a}{0.60F_ut_{des}4}$ $\geq \dfrac{2.00(350 \text{ kips})}{0.60(58 \text{ ksi})(0.465 \text{ in.})(4)}$ $\geq 10.8 \text{ in.}$

Therefore, the minimum lap length is 10.8 in. Determine the length in the following based on assumed weld size.

Design the weld between the brace and the gusset

The strength of fillet welds defined in AISC *Specification* Section J2 can be simplified, as explained in Part 8 of the AISC *Manual*, to AISC *Manual* Equations 8-2a and 8-2b:

LRFD	ASD
$\phi R_n = 1.392Dl$	$\dfrac{R_n}{\Omega} = 0.928Dl$

Try 5⁄16-in. fillet welds for the four lines of weld, which can be made in a single pass, and solve for l:

LRFD	ASD
$4(1.392)Dl \geq P_u$ $l \geq \dfrac{525 \text{ kips}}{4(1.392)(5 \text{ sixteenths})}$ $\geq 18.9 \text{ in.}$	$4(0.928)Dl \geq P_a$ $l \geq \dfrac{350 \text{ kips}}{4(0.928)(5 \text{ sixteenths})}$ $\geq 18.9 \text{ in.}$

Use 19.0-in.-long, 5⁄16-in. fillet welds for the brace-to-gusset connection. The minimum required weld size according to AISC *Specification* Table J2.4 for ½-in.-thick material is 3⁄16 in.

Check that there are no limitations on the fillet welds according to AISC *Specification* Section J2.2b. For end-loaded fillet welds with a length up to 100 times the weld size, it is permitted to take the effective length equal to the actual length:

$$(\tfrac{5}{16} \text{ in.})(100) = 31.3 \text{ in.} > 19.0 \text{ in.} \qquad \textbf{o.k.}$$

Returning to the tensile rupture check on the brace, the equation in AISC *Specification* Table D3.1, Case 6, with a single concentric gusset plate can be used because the length of weld, l, exceeds the wall of the HSS, H:

$$\begin{aligned} U &= 1 - \left(\frac{\bar{x}}{l}\right) \\ &= 1 - \left(\frac{3.75 \text{ in.}}{19.0 \text{ in.}}\right) \\ &= 0.803 \end{aligned}$$

Therefore, the effective net area is:

$$A_e = A_n U \qquad \text{(\textit{Spec}. Eq. D3-1)}$$
$$= \left(16.5 \text{ in.}^2\right)(0.803)$$
$$= 13.2 \text{ in.}^2$$

From AISC *Specification* Section D2(b), Equation D2-2, the available strength for the limit state of tensile rupture is:

LRFD	ASD
$\phi_t R_n = \phi_t F_u A_e$ $= 0.75(58 \text{ ksi})\left(13.2 \text{ in.}^2\right)$ $= 574 \text{ kips} > 525 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_t} = \dfrac{F_u A_e}{\Omega_t}$ $= \dfrac{(58 \text{ ksi})\left(13.2 \text{ in.}^2\right)}{2.00}$ $= 383 \text{ kips} > 350 \text{ kips}$ **o.k.**

The gusset size can now be approximated as shown in Figure 5-21 as 24 in. high and 27 in. wide. This is a preliminary size. The 27 in. width is shown as the initial size in Figure 5-21.

Check the gusset plate for tensile yielding on the Whitmore section

The gusset has been assumed to be ⅝ in. thick. From AISC *Manual* Part 9, the width of the Whitmore section is:

$$l_w = 10.0 \text{ in.} + 2(19.0 \text{ in.})\tan 30^\circ$$
$$= 31.9 \text{ in.}$$

This section penetrates the beam web by 6 in. and passes out into "space" past the column web by 2 in. To keep the Whitmore section symmetric, subtract 2 in. off of each side so that l_w = 31.9 in. − 4.00 in. = 27.9 in. The beam web is thinner than the gusset, so the effective Whitmore area is:

$$A_w = (l_w - 4.00 \text{ in.})(t_g) + (4.00 \text{ in.})(t_w)$$
$$= (27.9 \text{ in.} - 4.00 \text{ in.})(\tfrac{5}{8} \text{ in.}) + (4.00 \text{ in.})(0.355 \text{ in.})$$
$$= 16.4 \text{ in.}^2$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate on the Whitmore section is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(36 \text{ ksi})\left(16.4 \text{ in.}^2\right)$ $= 531 \text{ kips} > 525 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{(36 \text{ ksi})\left(16.4 \text{ in.}^2\right)}{1.67}$ $= 354 \text{ kips} > 350 \text{ kips}$ **o.k.**

Check the gusset plate for compression buckling on the Whitmore section

From Dowswell (2006), this is a compact gusset and buckling is not a limit state. However, if buckling is checked, the available compressive strength based on the limit state of flexural buckling is determined from AISC *Specification* Section J4.4.

Whitmore buckling is based on the gusset free length, l_b = 8 in., in Figure 5-21.

Check block shear rupture on the gusset plate

The block shear rupture failure path is assumed to follow the overlap of the brace on the gusset plate. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= (19.0 \text{ in.})(2)(\tfrac{5}{8} \text{ in.}) \\ &= 23.8 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(36 \text{ ksi})(23.8 \text{ in.}^2) \\ &= 514 \text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= A_{gv} \\ &= 23.8 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(58 \text{ ksi})(23.8 \text{ in.}^2) \\ &= 828 \text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned} A_{nt} &= (10.0 \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 6.25 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(58 \text{ ksi})(6.25 \text{ in.}^2) \\ &= 363 \text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 828 \text{ kips} + 363 \text{ kips} \\ &= 1{,}190 \text{ kips} \\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 514 \text{ kips} + 363 \text{ kips} \\ &= 877 \text{ kips} \end{aligned}$$

Therefore, $R_n = 877$ kips.

LRFD	ASD
$\phi R_n = 0.75(877 \text{ kips})$ $= 658 \text{ kips} > 525 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{877 \text{ kips}}{2.00}$ $= 439 \text{ kips} > 350 \text{ kips}$ **o.k.**

Interface Forces

Following the nonorthogonal UFM as presented in general in Figure 4-17, with the connection to a column web:

$$e_c = 0$$
$$e_b = 9.00 \text{ in.}$$
$$\gamma = \tan^{-1}\left(\frac{2}{12}\right) = 9.46°$$
$$\theta = \tan^{-1}\left(\frac{10¾}{12}\right) = 41.9°$$

Taking $\beta = \bar{\beta} = 4½ \text{ in.} + \dfrac{18.0 \text{ in.}}{2} = 13.5 \text{ in.}$ (the distance to the centroid of the gusset-to-column connection), the distance to the centroid of the gusset-to-beam connection is:

$$\begin{aligned}\alpha &= e_b(\tan\theta - \tan\gamma) - \frac{e_c}{\cos\gamma} + \beta(\cos\gamma\tan\theta - \sin\gamma)\\ &= (9.00 \text{ in.})(\tan 41.9° - \tan 9.46°) - \frac{0}{\cos 9.46°} + (13.5 \text{ in.})[\cos 9.46° \tan 41.9° - \sin 9.46°]\\ &= 16.3 \text{ in.}\end{aligned}$$

To get $\alpha = \bar{\alpha}$, set:

$$16.3 \text{ in.} = \frac{l_g}{2} + \frac{1}{2} \text{ and solve for } l_g$$
$$l_g = 31.6 \text{ in.}$$

Set the horizontal length of the gusset, l_g, equal to 31⅝ in., as shown in Figure 5-21. From Figure 4-17:

$$\begin{aligned}r &= \sqrt{\left(\alpha + e_b\tan\gamma + \beta\sin\gamma + \frac{e_c}{\cos\gamma}\right)^2 + (e_b + \beta\cos\gamma)^2}\\ &= \sqrt{[16.3 \text{ in.} + (9.00 \text{ in.})\tan 9.46° + (13.5 \text{ in.})\sin 9.46° + 0]^2 + [9.00 \text{ in.} + (13.5 \text{ in.})\cos 9.46°]^2}\\ &= 30.0 \text{ in.}\end{aligned}$$

LRFD	ASD
$\dfrac{P_u}{r} = \dfrac{525 \text{ kips}}{30.0 \text{ in.}} = 17.5 \text{ kip/in.}$	$\dfrac{P_a}{r} = \dfrac{350 \text{ kips}}{30.0 \text{ in.}} = 11.7 \text{ kip/in.}$
$V_{ub} = e_b \dfrac{P_u}{r} = (9.00 \text{ in.})(17.5 \text{ kip/in.}) = 158 \text{ kips}$	$V_{ab} = e_b \dfrac{P_a}{r} = (9.00 \text{ in.})(11.7 \text{ kip/in.}) = 105 \text{ kips}$
$V_{uc} = \beta\cos\gamma\dfrac{P_u}{r} = (13.5 \text{ in.})\cos 9.46° (17.5 \text{ kip/in.}) = 233 \text{ kips}$	$V_{ac} = \beta\cos\gamma\dfrac{P_a}{r} = (13.5 \text{ in.})\cos 9.46° (11.7 \text{ kip/in.}) = 156 \text{ kips}$

LRFD	ASD
$\Sigma(V_{ub}+V_{uc}) = 158\text{ kips}+233\text{ kips}$ $= 391\text{ kips}$ $= (525\text{ kips})\cos 41.9°$ $= 391\text{ kips}$ **o.k.** $H_{ub} = (\alpha + e_b\tan\gamma)\dfrac{P_u}{r}$ $= \left[16.3\text{ in.} + (9.00\text{ in.})\tan 9.46°\right](17.5\text{ kip/in.})$ $= 312\text{ kips}$ $H_{uc} = \left(\beta\sin\gamma + \dfrac{e_c}{\cos\gamma}\right)\dfrac{P_u}{r}$ $= \left[(13.5\text{ in.})\sin 9.46° + 0\right](17.5\text{ kip/in.})$ $= 38.7\text{ kips}$ $\Sigma(H_{uc}+H_{ub}) = 38.7\text{ kips}+312\text{ kips}$ $= 351\text{ kips}$ $= (525\text{ kips})(\sin 41.9°)$ $= 351\text{ kips}$ **o.k.** $Q_u = H_{uc} - P_u\cos\theta\tan\gamma$ $= 38.7\text{ kips} - (525\text{ kips})\cos 41.9°\tan 9.46°$ $= -26.6\text{ kips}$	$\Sigma(V_{ab}+V_{ac}) = 105\text{ kips}+156\text{ kips}$ $= 261\text{ kips}$ $= (350\text{ kips})\cos 41.9°$ $= 261\text{ kips}$ **o.k.** $H_{ab} = (\alpha + e_b\tan\gamma)\dfrac{P_a}{r}$ $= \left[16.3\text{ in.} + (9.00\text{ in.})\tan 9.46°\right](11.7\text{ kip/in.})$ $= 208\text{ kips}$ $H_{ac} = \left(\beta\sin\gamma + \dfrac{e_c}{\cos\gamma}\right)\dfrac{P_a}{r}$ $= \left[(13.5\text{ in.})\sin 9.46° + 0\right](11.7\text{ kip/in.})$ $= 25.9\text{ kips}$ $\Sigma(H_{ac}+H_{ab}) = 208\text{ kips}+25.9\text{ kips}$ $= 234\text{ kips}$ $= (350\text{ kips})\sin 41.9°$ $= 234\text{ kips}$ **o.k.** $Q_a = H_{ac} - P_a\cos\theta\tan\gamma$ $= 25.9\text{ kips} - (350\text{ kips})\cos 41.9°\tan 9.46°$ $= -17.6\text{ kips}$

The admissible force field is shown in Figure 5-22. The negative sign on Q_u and Q_a indicates that it is opposite in direction to that shown in Figure 5-22. Figure 5-22 and Figure 4-17 are referred to as positive diagrams; all forces are shown in positive directions. The angle γ is positive when the angle between the beam and column is less than 90°, and negative when the angle is greater than 90°.

Gusset-to-Column Connection

The forces H_{uc}, H_{ac}, V_{uc} and V_{ac} need to be converted into normal and tangential forces, oriented relative to the column's longitudinal axis, for connection design.

For the normal force:

LRFD	ASD
$N_{uc} = H_{uc}\cos\gamma - V_{uc}\sin\gamma$ $= (38.7\text{ kips})\cos 9.46° - (233\text{ kips})\sin 9.46°$ $= -0.0538\text{ kips} \approx 0\text{ kips}$	$N_{ac} = H_{ac}\cos\gamma - V_{ac}\sin\gamma$ $= (25.9\text{ kips})\cos 9.46° - (156\text{ kips})\sin 9.46°$ $= -0.0466\text{ kips} \approx 0\text{ kips}$

The small normal force is due to round off, and is taken as zero. The normal force is in tension if positive and compression if negative. For the tangential force:

LRFD	ASD
$T_{uc} = V_{uc}\cos\gamma + H_{uc}\sin\gamma$ $= (233\text{ kips})\cos 9.46° + (38.7\text{ kips})\sin 9.46°$ $= 236\text{ kips}$	$T_{ac} = V_{ac}\cos\gamma + H_{ac}\sin\gamma$ $= (156\text{ kips})\cos 9.46° + (25.9\text{ kips})\sin 9.46°$ $= 158\text{ kips}$

The tangential force is positive when acting down on the gusset. Since the tangential force is the shear, its sign is irrelevant. The connection required strengths are:

LRFD	ASD
Required axial strength, $N_u = 0$ kips Required shear strength, $V_u = 236$ kips	Required axial strength, $N_a = 0$ kips Required shear strength, $V_a = 158$ kips

Design bolts at gusset-to-column web connection

From the gusset geometry, a maximum of seven rows of bolts will fit.

At the column web double-angle connection, from AISC *Manual* Table 7-1, the available shear strength of 1-in.-diameter ASTM A490-X bolts in single shear is:

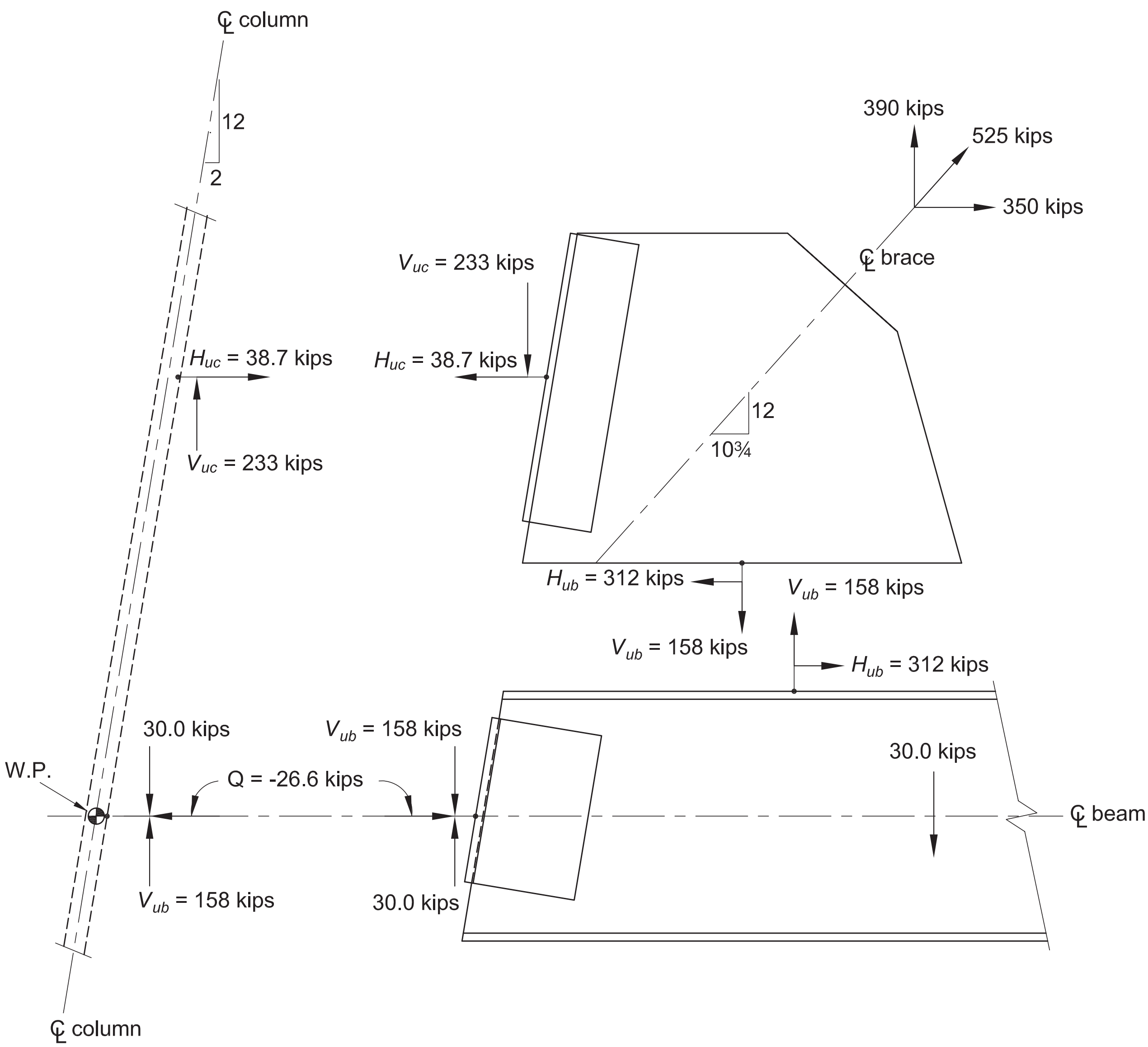


Fig. 5-22a. Admissible force fields for nonorthogonal connection in Example 5.10—LRFD.

LRFD	ASD
$\phi R_{nv} = (49.5 \text{ kips/bolt})(7 \text{ bolts})(2)$ $= 693 \text{ kips} > 236 \text{ kips}$ **o.k.** Since the available strength is almost three times what is required, try four bolts with 6 in. spacing: $\phi R_{nv} = (49.5 \text{ kips/bolt})(4 \text{ bolts})(2)$ $= 396 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_{nv}}{\Omega} = (33.0 \text{ kips/bolt})(7 \text{ bolts})(2)$ $= 462 \text{ kips} > 158 \text{ kips}$ **o.k.** Since the available strength is almost three times what is required, try four bolts with 6 in. spacing: $\dfrac{R_{nv}}{\Omega} = (33.0 \text{ kips/bolt})(4 \text{ bolts})(2)$ $= 264 \text{ kips} > 158 \text{ kips}$ **o.k.**

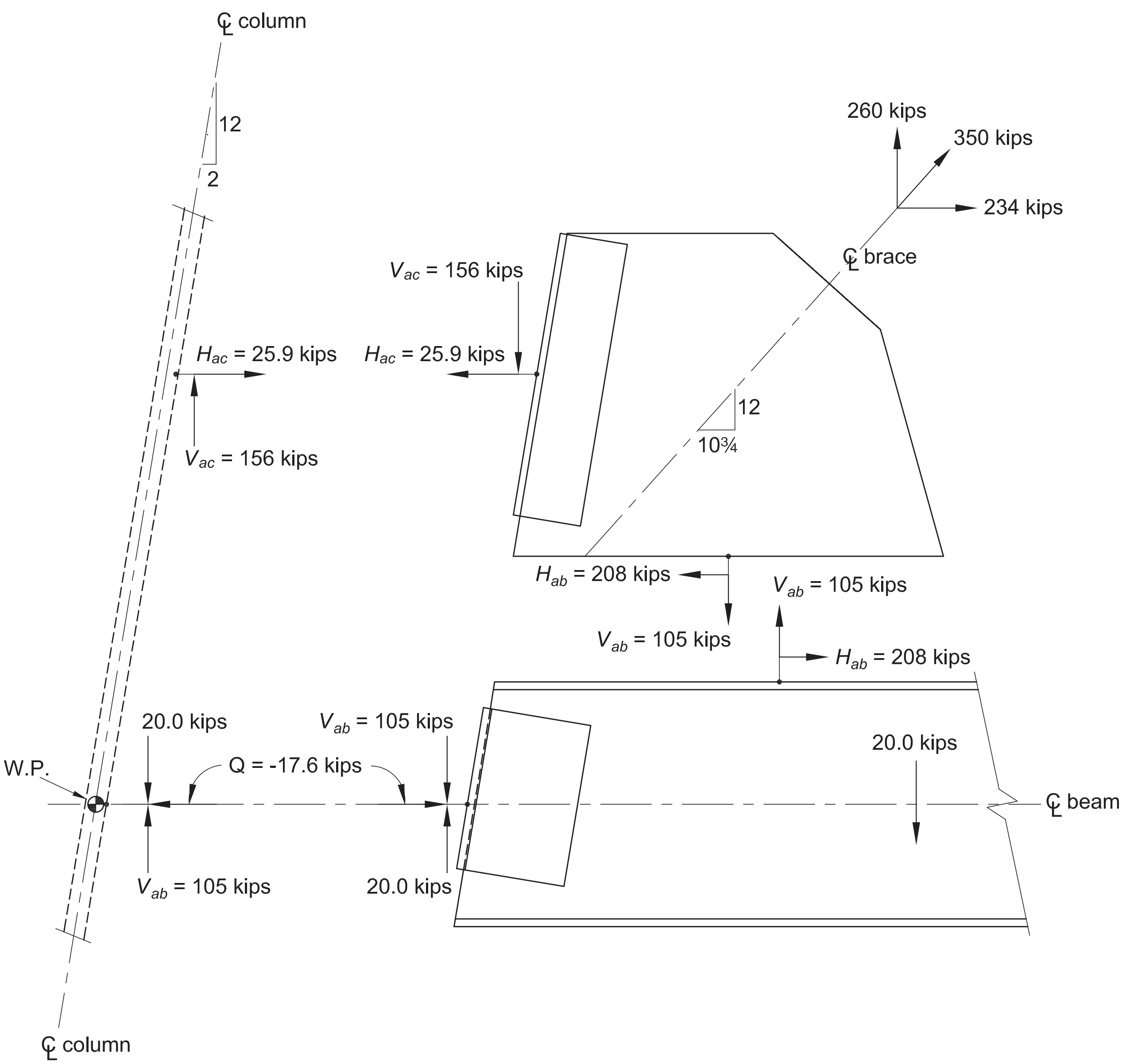


Fig. 5-22b. Admissible force fields for nonorthogonal connection in Example 5.10—ASD.

Figure 5-21 shows four rows of bolts at 6-in. spacing. Continue with this arrangement.

At the gusset side of the double-angle connection, the bolts have an eccentricity of 3 in. As stipulated in the discussion of double-angle connections in AISC *Manual* Part 10, for a single column of bolts, eccentricity is generally ignored up to about 3 in. From AISC *Manual* Table 7-1, the available shear strength of the four 1-in.-diameter ASTM A490-X bolts in double shear is:

LRFD	ASD
$\phi R_{nv} = (98.9 \text{ kips/bolt})(4)$ $= 396 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_{nv}}{\Omega} = (65.9 \text{ kips/bolt})(4)$ $= 264 \text{ kips} > 158 \text{ kips}$ **o.k.**

Use four bolts at 6-in. spacing.

Check shear yielding on double angles

Start with the angle size given in Figure 5-21, 2L5×5×½.

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the double angles is determined as follows:

$$A_{gv} = (\tfrac{1}{2} \text{ in.})(21.0 \text{ in.})(2) \qquad \text{(multiply by two for two angles)}$$
$$= 21.0 \text{ in.}^2$$

LRFD	ASD
$\phi R_{gv} = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(21.0 \text{ in.}^2)$ $= 454 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_{gv}}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(21.0 \text{ in.}^2)}{1.50}$ $= 302 \text{ kips} > 158 \text{ kips}$ **o.k.**

Check shear rupture on double angles

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the double angles is determined as follows, with the standard hole size for a 1-in.-diameter bolt equal to 1$\tfrac{1}{16}$ in. from AISC *Specification* Table J3.3:

$$A_{nv} = 21.0 \text{ in.}^2 - 4(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})(2)$$
$$= 16.5 \text{ in.}^2$$

LRFD	ASD
$\phi R_{nv} = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(16.5 \text{ in.}^2)$ $= 431 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_{nv}}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(16.5 \text{ in.}^2)}{2.00}$ $= 287 \text{ kips} > 158 \text{ kips}$ **o.k.**

Check block shear rupture on double angles

The block shear rupture failure path is assumed to begin at the top of each angle and follow the bolt line, and then turn perpendicular to the outside free edge of each angle. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= (1\tfrac{1}{2}\text{ in.} + 18.0\text{ in.})(\tfrac{1}{2}\text{ in.})(2) \\ &= 19.5\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_y A_{gv} &= 0.60(36\text{ ksi})(19.5\text{ in.}^2) \\ &= 421\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 19.5\text{ in.}^2 - [3.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.})](2) \\ &= 15.6\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_u A_{nv} &= 0.60(58\text{ ksi})(15.6\text{ in.}^2) \\ &= 543\text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

L_e, the horizontal edge distance to the center of the bolt hole is 2.00 in. on the gusset side and 2.06 in. on the column side. Use $L_e = 2.00$ in.

$$\begin{aligned} A_{nt} &= [2.00\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})](\tfrac{1}{2}\text{ in.})(2) \\ &= 1.44\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs} F_u A_{nt} &= 1(58\text{ ksi})(1.44\text{ in.}^2) \\ &= 83.5\text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is:

$$\begin{aligned} 0.60F_u A_{nv} + U_{bs} F_u A_{nt} &= 543\text{ kips} + 83.5\text{ kips} \\ &= 627\text{ kips} \\ 0.60F_y A_{gv} + U_{bs} F_u A_{nt} &= 421\text{ kips} + 83.5\text{ kips} \\ &= 505\text{ kips} \end{aligned}$$

Therefore, $R_n = 505$ kips.

LRFD	ASD
$\phi R_n = 0.75(505\text{ kips})$ $= 379\text{ kips} > 236\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{505\text{ kips}}{2.00}$ $= 253\text{ kips} > 158\text{ kips}$ **o.k.**

Check block shear rupture on the gusset plate

The block shear rupture failure path on the gusset plate at the gusset-to-column connection is assumed to run from the top of the gusset plate through the bolt line and then on a perpendicular path to the closest free edge of the gusset. From Figure 5-21, the gusset top (measured parallel to the column) edge distance is 2 in.

Shear yielding component:

$$\begin{aligned} A_{gv} &= (2.00 \text{ in.} + 18.0 \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 12.5 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(36 \text{ ksi})(12.5 \text{ in.}^2) \\ &= 270 \text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= (12.5 \text{ in.}^2) - (3.5)(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 10.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(58 \text{ ksi})(10.0 \text{ in.}^2) \\ &= 348 \text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned} A_{nt} &= [2.50 \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})](\tfrac{5}{8} \text{ in.}) \\ &= 1.21 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(58 \text{ ksi})(1.21 \text{ in.}^2) \\ &= 70.2 \text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 348 \text{ kips} + 70.2 \text{ kips} \\ &= 418 \text{ kips} \\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 270 \text{ kips} + 70.2 \text{ kips} \\ &= 340 \text{ kips} \end{aligned}$$

Therefore, $R_n = 340$ kips.

LRFD	ASD
$\phi R_n = 0.75(340 \text{ kips})$ $= 255 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{340 \text{ kips}}{2.00}$ $= 170 \text{ kips} > 158 \text{ kips}$ **o.k.**

Check bolt bearing on the double angles and the gusset plate

Using the ½-in.-thick double angles shown in Figure 5-21 and $d_h = 1\frac{1}{16}$ in., check bolt bearing on the angle material and the gusset plate at the angle-to-gusset connection. According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength on the double angles or on the gusset plate. Bearing strength is limited by bearing

deformation of the hole or by tearout of the material. From AISC *Specification* Section J3.10, when deformation at the bolt hole is a design consideration, the available strength for the limit state of bearing at bolt holes is determined as follows:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(Spec. Eq. J3-6a)}$$

Determine the effective strength of the top bolt in the double angle. The edge distance is:

$$\begin{aligned} l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5d_h \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5\left(1\tfrac{1}{16}\text{ in.}\right) \\ &= 0.969\text{ in.} \end{aligned}$$

For the top bolt, using AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.969\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})(2)$ $= 50.6\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})(2)$ $= 104\text{ kips/bolt}$ Therefore, $\phi r_n = 50.6$ kips/bolt < 98.9 kips/bolt (bolt shear)	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{(1.2)(0.969\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})(2)}{2.00}$ $= 33.7\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{(2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})(2)}{2.00}$ $= 69.6\text{ kips/bolt}$ Therefore $r_n/\Omega = 33.7$ kips/bolt < 65.9 kips/bolt (bolt shear)

Bearing at the top bolt hole in the double angles controls over bolt shear strength at the angle-to-gusset connection.

Determine the effective strength of the top bolt in the gusset plate. The edge distance is:

$$\begin{aligned} l_c &= 2.00\text{ in.} - 0.5d_h \\ &= 2.00\text{ in.} - 0.5\left(1\tfrac{1}{16}\text{ in.}\right) \\ &= 1.47\text{ in.} \end{aligned}$$

For the top bolt, using AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.47\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 48.0\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 65.3\text{ kips/bolt}$ Therefore, $\phi r_n = 48.0$ kips/bolt < 98.9 kips/bolt (bolt shear)	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{(1.2)(1.47\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 32.0\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{(2.4)(1.00\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 43.5\text{ kips/bolt}$ Therefore $r_n/\Omega = 32.0$ kips/bolt < 65.9 kips/bolt (bolt shear)

The bearing strength on the gusset plate controls the strength of the top bolt.

Determine the bearing strength at bolt holes on the gusset plate for the inner bolts. The edge distance is:

$$\begin{aligned} l_c &= 6.00\text{ in.} - d_h \\ &= 6.00\text{ in.} - 1\tfrac{1}{16}\text{ in.} \\ &= 4.94\text{ in.} \end{aligned}$$

For the inner bolts using AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(4.94 \text{ in.})(5\!/\!8 \text{ in.})(58 \text{ ksi})$ $= 161 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(5\!/\!8 \text{ in.})(58 \text{ ksi})$ $= 65.3 \text{ kips/bolt}$ Therefore, $\phi r_n = 65.3$ kips/bolt < 98.9 kips/bolt (bolt shear)	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{(1.2)(4.94 \text{ in.})(5\!/\!8 \text{ in.})(58 \text{ ksi})}{2.00}$ $= 107 \text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{(2.4)(1.00 \text{ in.})(5\!/\!8 \text{ in.})(58 \text{ ksi})}{2.00}$ $= 43.5 \text{ kips/bolt}$ Therefore $r_n/\Omega = 43.5$ kips/bolt < 65.9 kips/bolt (bolt shear)

By inspection, the bolt bearing strength on the angles need not be checked because the strength of the gusset will control. The bolt spacing (6 in.) is the same for both, and the thickness of the two angles is greater than the thickness of the gusset plate.

Considering the limit states of bolt shear, gusset bearing and tearout, and angle bearing and tearout, the available strength is:

LRFD	ASD
$\phi R_n = 48.3 \text{ kips} + 3(65.3 \text{ kips})$ $= 244 \text{ kips} > 236 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 32.0 \text{ kips} + 3(43.5 \text{ kips})$ $= 163 \text{ kips} > 158 \text{ kips}$ **o.k.**

Check bolt bearing on the column web

The column web needs to be checked for bolt bearing. The right side of AISC *Specification* Equation J3-6a will control. With $t_w = 1.54$ in.:

LRFD	ASD
$\phi R_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(1.00 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})$ $= 180 \text{ kips} > \dfrac{236}{8 \text{ bolts}} = 29.5 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 120 \text{ kips} > \dfrac{158 \text{ kips}}{8 \text{ bolts}} = 19.8 \text{ kips}$ **o.k.**

Gusset-to-Beam Connection

From Figure 5-22, the required strengths at the gusset-to-beam interface are:

LRFD	ASD
Required shear strength, $H_u = 312$ kips Required axial strength, $V_u = 158$ kips	Required shear strength, $H_a = 208$ kips Required axial strength, $V_a = 105$ kips

Check gusset plate for shear yielding and tension yielding along the beam flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(5/8 \text{ in.})(31\,5/8 \text{ in.})$ $= 427 \text{ kips} > 312 \text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(5/8 \text{ in.})(31\,5/8 \text{ in.})$ $= 640 \text{ kips} > 158 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(5/8 \text{ in.})(31\,5/8 \text{ in.})}{1.50}$ $= 285 \text{ kips} > 208 \text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(36 \text{ ksi})(5/8 \text{ in.})(31\,5/8 \text{ in.})}{1.67}$ $= 426 \text{ kips} > 105 \text{ kips}$ **o.k.**

Design weld at gusset-to-beam flange connection

As discussed previously in Example 5.1, and in AISC *Manual* Part 13, to allow for redistribution of force in welded gusset connections, the weld is designed for an increased strength of 1.25R. The strength of the fillet weld can be determined from AISC *Manual* Equation 8-2, incorporating the increased strength permitted from AISC *Specification* Equation J2-5, as follows:

LRFD	ASD
The resultant load is: $R_u = \sqrt{(312 \text{ kips})^2 + (158 \text{ kips})^2}$ $= 350 \text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{158 \text{ kips}}{312 \text{ kips}}\right)$ $= 26.8°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{1.25 R_u}{2(l)(1.392 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5}\theta)}$ $= \dfrac{1.25(350 \text{ kips})}{2(31\,5/8 \text{ in.})(1.392 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5} 26.8°)}$ $= 4.32$ sixteenths	The resultant load is: $R_a = \sqrt{(208 \text{ kips})^2 + (105 \text{ kips})^2}$ $= 233 \text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{105 \text{ kips}}{208 \text{ kips}}\right)$ $= 26.8°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{1.25 R_a}{2(l)(0.928 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5}\theta)}$ $= \dfrac{1.25(233 \text{ kips})}{2(31\,5/8 \text{ in.})(0.928 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5} 26.8°)}$ $= 4.31$ sixteenths

Use a double-sided 5/16-in. fillet weld. This also satisfies the 1/4-in. minimum fillet weld required by AISC *Specification* Table J2.4.

Check beam web local yielding

The normal force is applied at 31⅝ in./2 = 15.8 in. from the beam end, which is less than the beam depth of 18 in. Therefore, use AISC *Specification* Equation J10-3, with $k = k_{des}$:

LRFD	ASD
$\phi R_n = \phi F_y t_w \left(2.5k_{des} + l_b\right)$ $= 1.00(50 \text{ ksi})(0.355 \text{ in.})\left[2.5(0.972 \text{ in.}) + 31\tfrac{5}{8} \text{ in.}\right]$ $= 604 \text{ kips} > 158 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w \left(2.5k_{des} + l_b\right)}{\Omega}$ $= \dfrac{(50 \text{ ksi})(0.355 \text{ in.})\left[2.5(0.972 \text{ in.}) + 31\tfrac{5}{8} \text{ in.}\right]}{1.50}$ $= 403 \text{ kips} > 105 \text{ kips}$ **o.k.**

Check beam web shear yielding and shear buckling

This check is used to guard against excessive horizontal web shear in the beam. This can be a concern when a thick gusset is welded to a beam with a thin web. The effective length of web is not well defined, but one half of the beam length is a reasonable maximum. The beam length for this problem is 10 ft, therefore:

$$\begin{aligned} l_{eff} &= \tfrac{1}{2}(10 \text{ ft})\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \\ &= 60.0 \text{ in.} \end{aligned}$$

The available shear strength is determined from AISC *Specification* Section G2.1, where $C_v = 1.0$:

LRFD	ASD
$\phi_v R_n = \phi_v 0.6 F_y A_w C_v$ $= 1.00(0.6)(50 \text{ ksi})(0.355 \text{ in.})(60.0 \text{ in.})(1.0)$ $= 639 \text{ kips} > 312 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_v} = \dfrac{0.6 F_y A_w C_v}{\Omega_v}$ $= \dfrac{0.6(50 \text{ ksi})(0.355 \text{ in.})(60.0 \text{ in.})(1.0)}{1.50}$ $= 426 \text{ kips} > 208 \text{ kips}$ **o.k.**

Check beam web local crippling

The normal force is applied at 15.8 in. from the beam end, which is greater than $d/2$. Therefore, AISC *Specification* Equation J10-4 is applicable:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_{yw} t_f}{t_w}}$ $= 0.75(0.80)(0.355 \text{ in.})^2 \left[1 + 3\left(\dfrac{31\tfrac{5}{8} \text{ in.}}{18.0 \text{ in.}}\right)\left(\dfrac{0.355 \text{ in.}}{0.570 \text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.570 \text{ in.})}{0.355 \text{ in.}}}$ $= 414 \text{ kips} > 158 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2}{\Omega}\left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{E F_y t_f}{t_w}}$ $= \dfrac{0.80(0.355 \text{ in.})^2}{2.00}\left[1 + 3\left(\dfrac{31\tfrac{5}{8} \text{ in.}}{18.0 \text{ in.}}\right)\left(\dfrac{0.355 \text{ in.}}{0.570 \text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.570 \text{ in.})}{0.355 \text{ in.}}}$ $= 276 \text{ kips} > 105 \text{ kips}$ **o.k.**

Beam-to-Column Connection

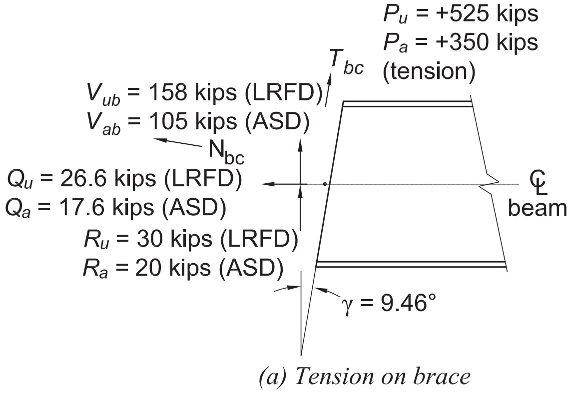
The forces required are the tangential and normal forces. The beam shear reaction, R_u = 30 kips (LRFD) or R_a = 20 kips (ASD) does not reverse, but V_{ub} (LRFD) or V_{ab} (ASD), and Q_u (LRFD) or Q_a (ASD) do when P_u (LRFD) or P_a (ASD) goes from tension to compression. Figure 5-23 shows this load case on the beam end.

In Figure 5-22, the force $Q_u = -26.6$ kips and $Q_a = -17.6$ kips, are negative by the sign convention of Figure 4-17. This force is shown in Figure 5-23(a) and 5-23(b) in its actual direction. T_{bc} and N_{bc} are also shown in their directions in Figures 5-23(a) and 5-23(b) to avoid the confusion of having negative quantities (i.e., quantities acting in directions opposite to the directions shown).

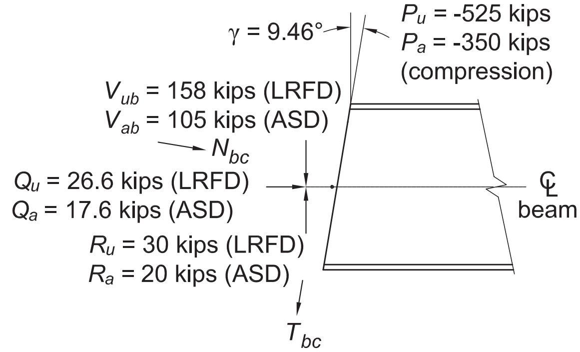
LRFD	ASD
From Figure 5-23(a): $$\begin{aligned} T_{ubc} &= (158\text{ kips} + 30\text{ kips})\cos 9.46° - (26.6\text{ kips})\sin 9.46° \\ &= 181\text{ kips (shear)} \\ N_{ubc} &= (158\text{ kips} + 30\text{ kips})\sin 9.46° + (26.6\text{ kips})\cos 9.46° \\ &= 57.1\text{ kips (tension)} \end{aligned}$$	From Figure 5-23(a): $$\begin{aligned} T_{abc} &= (105\text{ kips} + 20\text{ kips})\cos 9.46° - (17.6\text{ kips})\sin 9.46° \\ &= 120\text{ kips (shear)} \\ N_{abc} &= (105\text{ kips} + 20\text{ kips})\sin 9.46° + (17.6\text{ kips})\cos 9.46° \\ &= 37.9\text{ kips (tension)} \end{aligned}$$
From Figure 5-23(b): $$\begin{aligned} T_{ubc} &= (158\text{ kips} - 30\text{ kips})\cos 9.46° - (26.6\text{ kips})\sin 9.46° \\ &= 122\text{ kips} \\ N_{ubc} &= (158\text{ kips} - 30\text{ kips})\sin 9.46° + (26.6\text{ kips})\cos 9.46° \\ &= 47.2\text{ kips (compression)} \end{aligned}$$	From Figure 5-23(b): $$\begin{aligned} T_{abc} &= (105\text{ kips} - 20\text{ kips})\cos 9.46° - (17.6\text{ kips})\sin 9.46° \\ &= 80.9\text{ kips} \\ N_{abc} &= (105\text{ kips} - 20\text{ kips})\sin 9.46° + (17.6\text{ kips})\cos 9.46° \\ &= 31.3\text{ kips (compression)} \end{aligned}$$

From the preceding analysis, the critical load case is when the brace is in tension. The beam-to-column interface required strengths are:

LRFD	ASD
Required shear, $T_{ubc} = 181$ kips Required axial, $N_{ubc} = 57.1$ kips (tension)	Required shear, $T_{abc} = 120$ kips Required axial, $N_{abc} = 37.9$ kips (tension)



(a) Tension on brace



(b) Compression on brace

Figure 5-23. Beam end forces.

Determine bolt shear strength at beam web

The load angle is:

LRFD	ASD
$\text{angle} = \tan^{-1}\left(\dfrac{57.1 \text{ kips}}{181 \text{ kips}}\right)$ $= 17.5°$	$\text{angle} = \tan^{-1}\left(\dfrac{37.9 \text{ kips}}{120 \text{ kips}}\right)$ $= 17.5°$

With $e_x = 4.50$ in., $n = 4$, $s = 3.00$ in., Angle = 15.0°, use AISC *Manual* Table 7-7 to determine $C = 4.53$.

From AISC *Manual* Table 7-1, the available bolt shear strength for 1-in.-diameter bolts in double shear is:

LRFD	ASD
$\phi r_n = 98.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 65.9 \text{ kips/bolt}$

From AISC *Manual* Chapter 7, Equation 7-19:

LRFD	ASD
The resultant load is: $R_u = \sqrt{(181 \text{ kips})^2 + (57.1 \text{ kips})^2}$ $= 190 \text{ kips}$ $\phi r_{nv} = \dfrac{\phi r_{nv}\, C}{n}$ $= \dfrac{(98.9 \text{ kips/bolt})(4.53 \text{ bolts})}{8 \text{ bolts}}$ $= 56.0 \text{ kips/bolt}$	The resultant load is: $R_a = \sqrt{(120 \text{ kips})^2 + (37.9 \text{ kips})^2}$ $= 126 \text{ kips}$ $\dfrac{r_{nv}}{\Omega} = \left(\dfrac{r_{nv}}{\Omega}\right)\dfrac{C}{n}$ $= \dfrac{(65.9 \text{ kips/bolt})(4.53 \text{ bolts})}{8 \text{ bolts}}$ $= 37.3 \text{ kips/bolt}$

Check bolt bearing on the double angles and the beam web

Initially assume there is one vertical line of four bolts in the beam web.

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(\textit{Spec}. Eq. J3-6a)}$$

Assuming conservatively that the horizontal edge distance is 2.50 in., for a bolt hole size of 1$\frac{1}{16}$ in., the distance to the nearest edge for all of the bolts in the beam web is:

$$\begin{aligned} l_c &= 2.50 \text{ in.} - 0.5\left(1\tfrac{1}{16} \text{ in.}\right) \\ &= 1.97 \text{ in.} \end{aligned}$$

LRFD	ASD
The required shear strength per bolt is: $r_{uv} = \frac{190 \text{ kips}}{4} = 47.5 \text{ kips/bolt}$ The available bearing strengths of the beam web are: $\phi r_n = \phi 1.2 l_c t F_u = 0.75(1.2)(1.97 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) = 40.9 \text{ kips/bolt}$ $\phi r_n = \phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) = 41.5 \text{ kips/bolt}$ Therefore, $\phi r_n = 40.9$ kips/bolt < 47.5 kips/bolt **n.g.**	The required shear strength per bolt is: $r_{av} = \frac{126 \text{ kips}}{4} = 31.5 \text{ kips}$ The available bearing strengths of the beam web are: $\frac{r_n}{\Omega} = \frac{1.2 l_c t F_u}{\Omega} = \frac{1.2(1.97 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})}{2.00} = 27.3 \text{ kips}$ $\frac{r_n}{\Omega} = \frac{2.4 d t F_u}{\Omega} = \frac{2.4(1.00 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})}{2.00} = 27.7 \text{ kips}$ Therefore $r_n/\Omega = 27.3$ kips/bolt < 31.5 kips/bolt **n.g.**

Because the available bearing strength for one bolt is less than the required shear strength per bolt, a four-bolt connection fails.

Try two vertical lines of four bolts spaced 3 in. apart at the beam web connection as shown in Figure 5-21. For the four bolts nearest to the beam web edge, the available bearing strength is as determined previously. For the four additional bolts furthest from the edge, the bearing strength is determined as follows, where $l_c = 3.00 \text{ in.} - 1\frac{1}{16} \text{ in.} = 1.94 \text{ in.}$:

LRFD	ASD
The required shear strength per bolt is: $r_{uv} = \frac{190 \text{ kips}}{8} = 23.8 \text{ kips/bolt}$ The available tearout strength of the beam web is: $\phi r_n = \phi 1.2 l_c t F_u = 0.75(1.2)(1.94 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) = 40.3 \text{ kips/bolt}$ The available bearing strength of the beam web is: $\phi r_n = \phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) = 41.5 \text{ kips/bolt}$ Therefore, the tearout strength controls and $\phi r_n = 40.3$ kips/bolt.	The required shear strength per bolt is: $r_{av} = \frac{126 \text{ kips}}{8} = 15.8 \text{ kips/bolt}$ The available tearout strength of the beam web is: $\frac{r_n}{\Omega} = \frac{1.2 l_c t F_u}{\Omega} = \frac{1.2(1.94 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})}{2.00} = 26.9 \text{ kips}$ The available bearing strength of the beam web is: $\frac{r_n}{\Omega} = \frac{2.4 d t F_u}{\Omega} = \frac{2.4(1.00 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})}{2.00} = 27.7 \text{ kips/bolt}$ Therefore, the tearout strength controls and $r_n/\Omega = 26.9$ kips/bolt.

Check bolt bearing on double angle—beam side

Try 2L8×6×½ with the 6-in. leg trimmed to 5 in. Check bearing on the double angle at the beam web side. The clear distance to the top edge of the angle, based on a 1¹⁄₁₆-in.-diameter hole, is:

$$\begin{aligned} l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5\left(1\tfrac{1}{16}\text{ in.}\right) \\ &= 0.969\text{ in.} \end{aligned}$$

For the top and bottom bolts, using AISC *Specification* Equation J3-6a:

LRFD	ASD
The available tearout strength of the double angle at the beam web is: $\phi r_n = \phi 1.2 l_c t F_u$ $= 0.75(1.2)(0.969\text{ in.})(\tfrac{1}{2}\text{ in.})(2\text{ angles})(58\text{ ksi})$ $= 50.6\text{ kips/bolt}$ The available bearing strength of the double angle at the beam web is: $\phi r_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(2)(58\text{ ksi})$ $= 104\text{ kips/bolt} > 56.0\text{ kips/bolt}$ Therefore, bolt shear controls the strength of the top and bottom bolts compared to bearing or tearout on the angles.	The available tearout strength of the double angle at the beam web is: $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega}$ $= \dfrac{1.2(0.969\text{ in.})(\tfrac{1}{2}\text{ in.})(2)(58\text{ ksi})}{2.00}$ $= 33.7\text{ kips/bolt}$ The available bearing strength of the double angle at the beam web is: $\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00\text{ in.})(\tfrac{1}{2}\text{ in.})(2)(58\text{ ksi})}{2.00}$ $= 69.6\text{ kips/bolt} > 37.3\text{ kips/bolt}$ Therefore, bolt shear controls the strength of the top and bottom bolts compared to bearing or tearout on the angles.

For the inner bolts bearing on the double angle at the beam web connection, where $l_c = 3.00\text{ in.} - 1\tfrac{1}{16}\text{ in.} = 1.94\text{ in.}$:

LRFD	ASD
The available bearing strength of the double angle at the beam web is: $\phi r_n = \phi 1.2 l_c t F_u$ $= 0.75(1.2)(1.94\text{ in.})(\tfrac{1}{2}\text{ in.})(2\text{ angles})(58\text{ ksi})$ $= 101\text{ kips/bolt}$ The available bearing strength is 104 kips/bolt as determined previously.	The available bearing strength of the double angle at the beam web is: $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega}$ $= \dfrac{1.2(1.94\text{ in.})(\tfrac{1}{2}\text{ in.})(2\text{ angles})(58\text{ ksi})}{2.00}$ $= 67.5\text{ kips/bolt}$ The available bearing strength is 69.6 kips/bolt as determined previously.

Bolt shear strength controls the strength of the bolts bearing on the double angle. Edge tearout controls the bolts bearing on the beam web; therefore, $\phi r_n = 40.3$ kips/bolt and $r_n/\Omega = 26.9$ kips/bolt.

The effective strength of the bolt group at the double angle-to-beam web connection is based on the controlling available strength based on bolt shear, and bearing and tearout of the double angle and beam web:

LRFD	ASD
$\phi R_n = 4.53(40.3 \text{ kips})$ $= 183 \text{ kips} < 190 \text{ kips}$ **n.g.**	$\dfrac{R_n}{\Omega} = 4(26.9 \text{ kips})$ $= 108 \text{ kips} < 126 \text{ kips}$ **n.g.**

Based on this simplified bearing check, the beam web has insufficient bearing strength. However, a more rigorous approach which considers the actual magnitude and direction of the load on each bolt will prove the web to be sufficient. The more rigorous approach, using the same instantaneous center of rotation method used to generate the C-value of 4.53, provides $\phi R_n = 190$ kips, which is exactly equal to $R_u = 190$ kips and $R_n/\Omega = 126$ kips, which is exactly equal to $R_a = 126$ kips.

Check bolt shear strength at column web

From AISC *Manual* Table 7-1, the available bolt shear strength for 1-in.-diameter bolts in single shear is:

LRFD	ASD
$\phi r_n = 49.5 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 33.0 \text{ kips/bolt}$

The required bolt shear at the double angle-to-column web connection is:

LRFD	ASD
On the angle legs attached to the column web side, the maximum shear per bolt is: $\dfrac{V_u}{n} = \dfrac{181 \text{ kips}}{8}$ $= 22.6 \text{ kips/bolt} < 49.5 \text{ kips/bolt}$ **o.k.**	On the leg attached to the column web side, the maximum shear per bolt is: $\dfrac{V_a}{n} = \dfrac{120 \text{ kips}}{8}$ $= 15.0 \text{ kips/bolt} < 33.0 \text{ kips/bolt}$ **o.k.**

Check bolt bearing on double angle and column web

With a bolt spacing of 3.00 in. ($l_c = 3.00$ in.–1$\frac{1}{16}$ in. = 1.94 in.), the available bearing and tearout strengths of the column web are:

LRFD	ASD
The available tearout strength of the column web is: $\phi r_n = \phi 1.2 l_c t F_u$ $= 0.75(1.2)(1.94 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})$ $= 175 \text{ kips/bolt} > 22.6 \text{ kips/bolt}$ **o.k.** The available bearing strength of the column web is: $\phi r_n = \phi 2.4 d t F_u$ $= 0.75\,(2.4)(1.00 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})$ $= 180 \text{ kips/bolt} > 22.6 \text{ kips/bolt}$ **o.k.**	The available tearout strength of the column web is: $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega}$ $= \dfrac{1.2(1.94 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 117 \text{ kips/bolt} > 15.0 \text{ kips/bolt}$ **o.k.** The available bearing strength of the column web is: $\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00 \text{ in.})(1.54 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 120 \text{ kips/bolt} > 15.0 \text{ kips/bolt}$ **o.k.**

Bolt shear controls the strength of the bolt group in the column web.

For bearing of the outer bolts on the double angle at the column web connection, with 1½-in. edge distance, from the previous calculation, the available bearing strength per bolt is:

LRFD	ASD
$\phi r_n = 50.6 \text{ kips} > 22.6 \text{ kips/bolt}$ **o.k.** The available bolt shear of 49.5 kips/bolt controls the strength of the outer bolts.	$\dfrac{R_n}{\Omega} = 33.7 \text{ kips/bolt} > 15.0 \text{ kips/bolt}$ **o.k.** The available bolt shear of 33.0 kips/bolt controls the strength of the outer bolts.

For the inner bolts in the double angle at the column web connection, where $l_c = 3.00 \text{ in.} - 1\tfrac{1}{16} \text{ in.} = 1.94 \text{ in.}$:

LRFD	ASD
The available tearout strength of the double angle is: $\phi r_n = \phi 1.2 l_c t F_u$ $= 0.75(1.2)(1.94 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 50.6 \text{ kips/bolt} > 22.6 \text{ kips/bolt}$ **o.k.** The available bearing strength of the double angle is: $\phi r_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 52.2 \text{ kips/bolt} > 22.6 \text{ kips/bolt}$ **o.k.** The available bolt shear of 49.5 kips/bolt controls the strength of the inner bolts.	The available tearout strength of the double angle is: $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega}$ $= \dfrac{1.2(1.94 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 33.8 \text{ kips/bolt} > 15.0 \text{ kips/bolt}$ **o.k.** The available bearing strength of the double angle is: $\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 34.8 \text{ kips/bolt} > 15.0 \text{ kips/bolt}$ **o.k.** The available bolt shear of 33.0 kips/bolt controls the strength of the inner bolts.

The effective strength of the bolt group at the double angle-to-column web connection is based on the controlling available strength due to bolt shear and bearing and tearout of the double angle and column web. Bolt shear controls.

LRFD	ASD
$\phi R_n = 8(49.5 \text{ kips})$ $= 396 \text{ kips} > 181 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 8(33.0 \text{ kips})$ $= 264 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check gross shear on double angle—column side

The gross area of the double angle is:

$$\begin{aligned} A_{gv} &= (12.0 \text{ in.})(\tfrac{1}{2} \text{ in.})(2) \\ &= 12.0 \text{ in.}^2 \end{aligned}$$

From AISC *Specification* Equation J4-3, the available shear yielding strength of the double angle is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(12.0\text{ in.}^2)$ $= 259\text{ kips} > 181\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12.0\text{ in.}^2)}{1.50}$ $= 173\text{ kips} > 120\text{ kips}$ **o.k.**

Check net shear on double angle

Based on the required hole size for a 1-in.-diameter bolt in standard holes from AISC *Specification* Table J3.3 and the $\frac{1}{16}$ in. increase required by AISC *Specification* Section B4.3b, the net area is:

$$\begin{aligned} A_{nv} &= 12.0\text{ in.}^2 - 4(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.})(2) \\ &= 7.50\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Equation J4-4, the available shear rupture strength of the double angle is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(7.50\text{ in.}^2)$ $= 196\text{ kips} > 181\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(7.50\text{ in.}^2)}{2.00}$ $= 131\text{ kips} > 120\text{ kips}$ **o.k.**

Check block shear rupture on angles—beam side

The controlling block shear rupture failure path is assumed to be the path shown in Figure 5-23c. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Shear yielding component:

$$\begin{aligned} A_{gv} &= 2(10.5\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 10.5\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_y A_{gv} &= 0.60(36\text{ ksi})(10.5\text{ in.}^2) \\ &= 227\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 10.5\text{ in.}^2 - 3.5(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.})(2) \\ &= 6.56\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60 F_u A_{nv} &= 0.60(58\text{ ksi})(6.56\text{ in.}^2) \\ &= 228\text{ kips} \end{aligned}$$

Tension rupture component:

$$A_{nt} = \left[5.00 \text{ in.} - 1.5\left(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{1}{2} \text{ in.}\right)(2)$$
$$= 3.31 \text{ in.}^2$$

In this case, because of the double column of bolts, $U_{bs} = 0.5$.

$$U_{bs}F_uA_{nt} = 0.5(58 \text{ ksi})\left(3.31 \text{ in.}^2\right)$$
$$= 96.0 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 228 \text{ kips} + 96.0 \text{ kips}$$
$$= 324 \text{ kips}$$
$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 227 \text{ kips} + 96.0 \text{ kips}$$
$$= 323 \text{ kips}$$

Therefore, $R_n = 323$ kips.

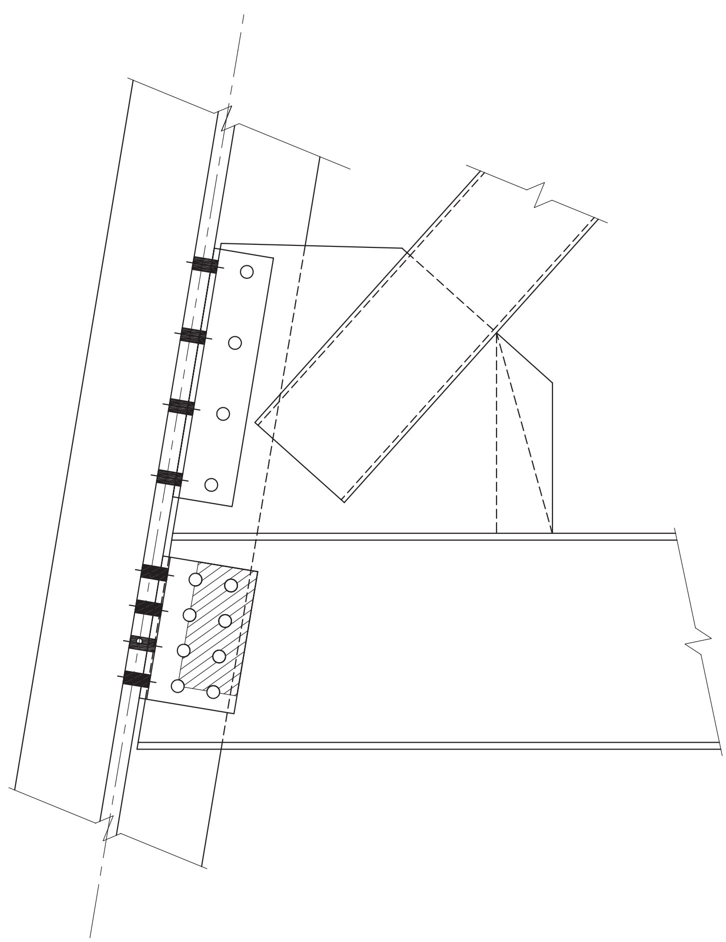


Fig. 5-23c. Block shear rupture failure path on double angle in beam web.

LRFD	ASD
$\phi R_n = 0.75(323 \text{ kips})$ $= 242 \text{ kips} > 181 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{323 \text{ kips}}{2.00}$ $= 162 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check block shear rupture on angles—column side

The controlling block shear failure path is similar to that shown in Figure 5-10a.

Shear yielding component (same as for beam side):

$$A_{gv} = 10.5 \text{ in.}^2$$
$$0.60F_yA_{gv} = 227 \text{ kips}$$

Shear rupture component (same as for beam side):

$$A_{nv} = 6.56 \text{ in.}^2$$
$$0.60F_uA_{nv} = 228 \text{ kips}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{gt} &= (10.0 \text{ in.} + 0.355 \text{ in.} - 6\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 1.93 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= 1.93 \text{ in.}^2 - 1(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 1.37 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(58 \text{ ksi})(1.37 \text{ in.}^2) \\ &= 79.5 \text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 228 \text{ kips} + 79.5 \text{ kips} \\ &= 308 \text{ kips} \\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 227 \text{ kips} + 79.5 \text{ kips} \\ &= 307 \text{ kips} \end{aligned}$$

Therefore, $R_n = 307$ kips.

LRFD	ASD
$\phi R_n = 0.75(307 \text{ kips})$ $= 230 \text{ kips} > 181 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{307 \text{ kips}}{2.00}$ $= 154 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check block shear rupture on beam web due to tensile force

The controlling block shear failure path forms a rectangle bounded by the beam end, the top and bottom row of bolts, and the inner vertical line of bolts.

Shear yielding component:

$$\begin{aligned} A_{gv} &= 2(5.50\text{ in.})(0.355\text{ in.}) \\ &= 3.91\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(50\text{ ksi})\left(3.91\text{ in.}^2\right) \\ &= 117\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 3.91\text{ in.}^2 - 1.5\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(0.355\text{ in.})(2) \\ &= 2.71\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(65\text{ ksi})\left(2.71\text{ in.}^2\right) \\ &= 106\text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$\begin{aligned} A_{nt} &= \left[9.00\text{ in.} - 3\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.355\text{ in.}) \\ &= 2.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(65\text{ ksi})\left(2.00\text{ in.}^2\right) \\ &= 130\text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 106\text{ kips} + 130\text{ kips} \\ &= 236\text{ kips} \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 117\text{ kips} + 130\text{ kips} \\ &= 247\text{ kips} \end{aligned}$$

Therefore, $R_n = 236$ kips.

LRFD	ASD
$\phi R_n = 0.75(236\text{ kips})$ $= 177\text{ kips} > 57.1\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{236\text{ kips}}{2.00}$ $= 118\text{ kips} > 37.9\text{ kips}$ **o.k.**

Check interaction of shear and tension on the bolts in the column web

The interaction of shear and tension on a bolt is checked using AISC *Specification* Section J3.7:

LRFD	ASD
The required shear per bolt is: $r_{uv} = \dfrac{T_{ubc}}{N_b}$ $= \dfrac{181 \text{ kips}}{8 \text{ bolts}}$ $= 22.6 \text{ kips/bolt}$ and the required tension per bolt is: $r_{ut} = \dfrac{N_{ubc}}{N_b}$ $= \dfrac{57.1 \text{ kips}}{8 \text{ bolts}}$ $= 7.14 \text{ kips/bolt}$ $F_{nt}' = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ $f_{rv} = \dfrac{r_{uv}}{A_b}$ $F_{nt} = 113$ ksi from AISC *Specification* Table J3.2 $F_{nv} = 84$ ksi from AISC *Specification* Table J3.2 $F_{nt}' = 1.3(113 \text{ ksi}) - \dfrac{(113 \text{ ksi})}{(0.75)(84 \text{ ksi})}\left(\dfrac{22.6 \text{ kips/bolt}}{0.785 \text{ in.}^2}\right)$ $= 95.3 \text{ ksi} < 113 \text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\phi r_{nt} = \phi F_{nt}' A_b$ $= 0.75(95.3 \text{ kips})(0.785 \text{ in.}^2)$ $= 56.1 \text{ kips/bolt} > 7.14 \text{ kips/bolt}$ **o.k.**	The required shear per bolt is: $r_{av} = \dfrac{T_{abc}}{N_b}$ $= \dfrac{120 \text{ kips}}{8 \text{ bolts}}$ $= 15.0 \text{ kips/bolt}$ and the required tension per bolt is: $r_{at} = \dfrac{N_{abc}}{N_b}$ $= \dfrac{37.9 \text{ kips}}{8 \text{ bolts}}$ $= 4.74 \text{ kips/bolt}$ $F_{nt}' = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ $f_{rv} = \dfrac{r_{av}}{A_b}$ $F_{nt} = 113$ ksi from AISC *Specification* Table J3.2 $F_{nv} = 84$ ksi from AISC *Specification* Table J3.2 $F_{nt}' = 1.3(113 \text{ ksi}) - \dfrac{2.00(113 \text{ ksi})}{(84 \text{ ksi})}\left(\dfrac{15.0 \text{ kips/bolt}}{0.785 \text{ in.}^2}\right)$ $= 95.5 \text{ ksi} < 113 \text{ ksi}$ **o.k.** The reduced available tensile force per bolt due to combined forces is: $\dfrac{r_{nt}}{\Omega} = \dfrac{F_{nt}' A_b}{\Omega}$ $= \dfrac{(95.5 \text{ kips})(0.785 \text{ in.}^2)}{2.00}$ $= 37.5 \text{ kips/bolt} > 4.74 \text{ kips/bolt}$ **o.k.**

Check prying action on bolts and double angle

From AISC *Manual* Figure 9-4b:

$$b = \frac{(g - t_{wb})}{2} - \frac{t_{angle}}{2}$$
$$= \frac{6\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.}}{2} - \frac{\tfrac{1}{2} \text{ in.}}{2}$$
$$= 2.82 \text{ in.}$$

$$a = \frac{2(angle\ leg\ width) + t_{wb} - g}{2}$$

$$= \frac{2(5.00\text{ in.}) + 0.355\text{ in.} - 6\tfrac{1}{2}\text{ in.}}{2}$$

$$= 1.93\text{ in.}$$

$$a' = a + \frac{d_b}{2} \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual}\text{ Eq. 9-27})$$

$$1.25b = 1.25(2.82\text{ in.})$$

$$= 3.53\text{ in.}$$

$1.93\text{ in.} < 3.53\text{ in.}$ **o.k.**

Use $a = 1.93$ in.

$$a' = a + \frac{d_b}{2}$$

$$= 1.93\text{ in.} + \frac{1.00\text{ in.}}{2}$$

$$= 2.43\text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad (\textit{Manual}\text{ Eq. 9-21})$$

$$= 2.82\text{ in.} - \frac{1.00\text{ in.}}{2}$$

$$= 2.32\text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad (\textit{Manual}\text{ Eq. 9-26})$$

$$= \frac{2.32\text{ in.}}{2.43\text{ in.}}$$

$$= 0.955$$

p = pitch of bolts

$= 3.00$ in.

$$\delta = 1 - \frac{d'}{p} \qquad (\textit{Manual}\text{ Eq. 9-24})$$

$$= 1 - \frac{(1\tfrac{1}{16}\text{ in.})}{3.00\text{ in.}}$$

$$= 0.646$$

From AISC *Manual* Part 9:

LRFD	ASD
From AISC *Manual* Equation 9-30a: $t_c = \sqrt{\dfrac{4Bb'}{\phi p F_u}}$ $= \sqrt{\dfrac{4(56.1\text{ kips})(2.32\text{ in.})}{0.90(3.00\text{ in.})(58\text{ ksi})}}$ $= 1.82$ in.	From AISC *Manual* Equation 9-30b: $t_c = \sqrt{\dfrac{4\Omega Bb'}{p F_u}}$ $= \sqrt{\dfrac{4(1.67)(37.5\text{ kips})(2.32\text{ in.})}{(3.00\text{ in.})(58\text{ ksi})}}$ $= 1.83$ in.

Note that t_c is the angle thickness required to eliminate prying action, i.e., it is the angle thickness required to develop the strength of the bolt with no prying force. It is, therefore, an upper bound on angle thickness. Any angle thicker than t_c would provide no additional strength. The optimal ratio of the moment in the angle leg at the bolt line to that at the center of the other leg is α'. From AISC *Specification* Equation 9-35:

LRFD	ASD
$$\begin{aligned}\alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2-1\right]\\ &= \frac{1}{0.646(1+0.955)}\left[\left(\frac{1.82\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2-1\right]\\ &= 9.70\text{ in.}\end{aligned}$$ Because $\alpha' > 1$ the angle leg will control and the prying action effect is determined from AISC *Manual* Equation 9-34: $$\begin{aligned}Q &= \left(\frac{t}{t_c}\right)^2(1+\delta)\\ &= \left(\frac{\tfrac{1}{2}\text{ in.}}{1.82\text{ in.}}\right)^2(1+0.646)\\ &= 0.124\end{aligned}$$ $$\begin{aligned}T_{avail} &= BQ\\ &= (56.1\text{ kips})(0.124)\\ &= 6.96\text{ kips} < 7.14\text{ kips}\quad \textbf{n.g.}\end{aligned}$$	$$\begin{aligned}\alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2-1\right]\\ &= \frac{1}{0.646(1+0.955)}\left[\left(\frac{1.83\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2-1\right]\\ &= 9.81\text{ in.}\end{aligned}$$ Because $\alpha' > 1$ the angle leg will control and the prying action effect is determined from AISC *Manual* Equation 9-34: $$\begin{aligned}Q &= \left(\frac{t}{t_c}\right)^2(1+\delta)\\ &= \left(\frac{\tfrac{1}{2}\text{ in.}}{1.83\text{ in.}}\right)^2(1+0.646)\\ &= 0.123\end{aligned}$$ $$\begin{aligned}T_{avail} &= BQ\\ &= (37.5\text{ kips})(0.123)\\ &= 4.61\text{ kips} < 4.74\text{ kips}\quad \textbf{n.g.}\end{aligned}$$

The 2L8×6×½ angles are inadequate. Try 2L8×6×⅝ angles. From AISC *Manual* Figure 9-4b:

$$\begin{aligned}b &= \frac{(g-t_{wb})}{2}-\frac{t_{angle}}{2}\\ &= \frac{6\tfrac{1}{2}\text{ in.}-0.355\text{ in.}}{2}-\frac{\tfrac{5}{8}\text{ in.}}{2}\\ &= 2.76\text{ in.}\end{aligned}$$

$a = 1.93$ in. as determined previously

$$a' = a+\frac{d_b}{2} \le \left(1.25b+\frac{d_b}{2}\right) \qquad (\textit{Manual}\text{ Eq. 9-27})$$

$$\begin{aligned}1.25b &= 1.25(2.76\text{ in.})\\ &= 3.45\text{ in.}\end{aligned}$$

$1.93\text{ in.} < 3.45\text{ in.}$

Use $a = 1.93$ in.

$$\begin{aligned}a' &= a+\frac{d_b}{2}\\ &= 1.93\text{ in.}+\frac{1.00\text{ in.}}{2}\\ &= 2.43\text{ in.}\end{aligned}$$

$$b' = b - \frac{d_b}{2}$$
$$= 2.76 \text{ in.} - \frac{1.00 \text{ in.}}{2}$$
$$= 2.26 \text{ in.}$$

$$\rho = \frac{b'}{a'}$$
$$= \frac{2.26 \text{ in.}}{2.43 \text{ in.}}$$
$$= 0.930$$

$$p = 3.00 \text{ in.}$$

$$\delta = 0.646 \text{ as determined previously}$$

LRFD	ASD
From AISC *Manual* Equation 9-30a: $t_c = \sqrt{\frac{4Bb'}{\phi p F_u}}$ $= \sqrt{\frac{4(56.1 \text{ kips})(2.26 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})}}$ $= 1.80 \text{ in.}$ $\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$ $= \frac{1}{0.646(1+0.930)}\left[\left(\frac{1.80 \text{ in.}}{\frac{5}{8} \text{ in.}}\right)^2 - 1\right]$ $= 5.85 \text{ in.}$ Because $\alpha' > 1$, the angle leg will control and the prying action effect is determined from AISC *Manual* Equation 9-34: $Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$ $= \left(\frac{\frac{5}{8} \text{ in.}}{1.80 \text{ in.}}\right)^2 (1+0.646)$ $= 0.198$ $T_{avail} = BQ$ $= (56.1 \text{ kips})(0.198)$ $= 11.1 \text{ kips} > 7.14 \text{ kips}$ **o.k.**	From AISC *Manual* Equation 9-30b: $t_c = \sqrt{\frac{4\Omega Bb'}{p F_u}}$ $= \sqrt{\frac{4(1.67)(37.5 \text{ kips})(2.26 \text{ in.})}{(3.00 \text{ in.})(58 \text{ ksi})}}$ $= 1.80 \text{ in.}$ $\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$ $= \frac{1}{0.646(1+0.930)}\left[\left(\frac{1.80 \text{ in.}}{\frac{5}{8} \text{ in.}}\right)^2 - 1\right]$ $= 5.85 \text{ in.}$ Because $\alpha' > 1$, the angle leg will control and the prying action effect is determined from AISC *Manual* Equation 9-34: $Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$ $= \left(\frac{\frac{5}{8} \text{ in.}}{1.80 \text{ in.}}\right)^2 (1+0.646)$ $= 0.198$ $T_{avail} = BQ$ $= (37.5 \text{ kips})(0.198)$ $= 7.43 \text{ kips} > 4.74 \text{ kips}$ **o.k.**

Note that the prying action design did not require the actual calculation of the prying force q. If this force is needed for analysis as is sometimes the case with fatigue design, it can be calculated as follows. Calculate α from AISC *Manual* Equation 9-29:

LRFD	ASD
$\alpha = \dfrac{1}{\delta}\left[\dfrac{T}{B}\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.646}\left[\left(\dfrac{7.14\text{ kips}}{56.1\text{ kips}}\right)\left(\dfrac{1.80\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right]$ $= 0.0861$	$\alpha = \dfrac{1}{\delta}\left[\dfrac{T}{B}\left(\dfrac{t_c}{t}\right)^2 - 1\right]$ $= \dfrac{1}{0.646}\left[\left(\dfrac{4.74\text{ kips}}{37.5\text{ kips}}\right)\left(\dfrac{1.80\text{ in.}}{5/8\text{ in.}}\right)^2 - 1\right]$ $= 0.0749$

Note that $\alpha \neq \alpha'$. α is the actual ratio of bolt line to leg line moment, while α' is the optimal value (see Thornton, 1985). Since α is an actual value, it is limited to the range $0 \le \alpha \le 1$. From AISC *Manual* Equation 9-28:

LRFD	ASD
$q = B\left[\delta\alpha\rho\left(\dfrac{t}{t_c}\right)^2\right]$ $= (56.1\text{ kips})\left[(0.646)(0.0861)(0.930)\left(\dfrac{5/8\text{ in.}}{1.80\text{ in.}}\right)^2\right]$ $= 0.350\text{ kips}$ The actual maximum force per bolt is $T + q$ $T + q = 7.14\text{ kips} + 0.350\text{ kips}$ $= 7.49\text{ kips}$ $< B = 56.1\text{ kips}$ **o.k.**	$q = B\left[\delta\alpha\rho\left(\dfrac{t}{t_c}\right)^2\right]$ $= (37.5\text{ kips})\left[(0.646)(0.0749)(0.930)\left(\dfrac{5/8\text{ in.}}{1.80\text{ in.}}\right)^2\right]$ $= 0.203\text{ kips}$ The actual maximum force per bolt is $T + q$ $T + q = 4.74\text{ kips} + 0.203\text{ kips}$ $= 4.94\text{ kips}$ $< B = 37.5\text{ kips}$ **o.k.**

This same conclusion was reached before with $T_{avail} > T$, without calculating q.

Check yielding in the column web

When the normal force, N_{bc}, is large and the column web is thin, web yielding can be a controlling limit state. Also, the prying action analysis of the beam-to-column connection angles assumes that the column web is sufficiently stiff to allow double curvature bending in the angle legs to develop, or the axial force of 57.1 kips (LRFD) or 37.9 kips (ASD), is picked up by matching connection angles on the other side of the web. Assume for the present example that the column web must carry the normal force of 57.1 kips (LRFD) or 37.9 kips (ASD) (i.e., there is no opposing matching connection).

A yield line for this case is assumed as given in Figure 5-13, and the theory is presented in Example 5.5. The column web transverse strength is given by:

$$P_n = 8m_p\left[\sqrt{\frac{2h}{h-g}} + \frac{l}{2(h-g)}\right]$$

where the terms are defined in Example 5.5. For the present case:

$$\begin{aligned} m_p &= \frac{F_y t_w^2}{4} \\ &= \frac{(50\text{ ksi})(1.54\text{ in.})^2}{4} \\ &= 29.6\text{ kip-in./in.} \end{aligned}$$

$$
\begin{aligned}
h &= \left(\frac{h}{t_w}\right)t_w \\
&= 7.41(1.54 \text{ in.}) \\
&= 11.4 \text{ in.} \\
l &= 9 \text{ in.} \\
g &= 6\tfrac{1}{2} \text{ in.} \\
P_n &= 8(29.6 \text{ kip-in./in.})\left[\sqrt{\frac{2(11.4 \text{ in.})}{(11.4 \text{ in.} - 6\tfrac{1}{2} \text{ in.})}} + \frac{9.00 \text{ in.}}{2(11.4 \text{ in.} - 6\tfrac{1}{2} \text{ in.})}\right] \\
&= 728 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\phi P_n = 0.90(728 \text{ kips})$ $= 655 \text{ kips} > 57.1 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{728 \text{ kips}}{1.67}$ $= 436 \text{ kips} > 37.9 \text{ kips}$ **o.k.**

Prying Action Validation

In order to allow the double curvature prying action to exist, the authors recommend that the strength of the column web in yielding be at least 10 times the required strength. The normal double curvature prying action formulation assumes that the support is rigid, i.e., infinitely stiff. This is the reason for the factor of 10 on strength.

LRFD	ASD
$\dfrac{\phi P_n}{10} = \dfrac{0.90(728 \text{ kips})}{10}$ $= 65.5 \text{ kips} > 57.1 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega(10)} = \dfrac{728 \text{ kips}}{1.67(10)}$ $= 43.6 \text{ kips} > 37.9 \text{ kips}$ **o.k.**

Check Beam Web Vertical Shear

Adjacent to the beam-to-column connection, the beam web will have a required shear strength and available shear strength from AISC *Manual* Table 3-6, as shown in the following:

LRFD	ASD
$V_u = 158 \text{ kips} + 30 \text{ kips}$ $= 188 \text{ kips}$ $\phi_v V_n = 192 \text{ kips} > 188 \text{ kips}$ **o.k.**	$V_a = 105 \text{ kips} + 20 \text{ kips}$ $= 125 \text{ kips}$ $\dfrac{V_n}{\Omega_v} = 128 \text{ kips} > 125 \text{ kips}$ **o.k.**

If this check failed, a web doubler plate could be used or some of the required shear strength could be transferred to the gusset-to-column connection via a Special Case 2 approach to the nonorthogonal problem. This would be similar to the ΔV_{ab} analysis of Section 4.2.3. The completed design is shown in Figure 5-21. The calculations for this problem show some of the adjustments that must be made as the design proceeds.

Example 5.11—Truss Connection

Given:

The versatility of the uniform force method will be demonstrated in a truss-to-truss connection as shown in Figure 5-24. Truss connections are similar to bracing connections, as bracing connections are part of a vertical truss that provides lateral support to the building.

Figure 5-24 shows a secondary truss, Truss B, connected to a primary truss, Truss D. The overall objective of this connection is to deliver the vertical component of the secondary truss diagonal, 902 kips for LRFD and 599 kips for ASD, to the centerline of the primary truss. The given properties of the connection are as shown in Figure 5-24. Use 1-in.-diameter ASTM A490-SC bolts, surface Class B, in standard holes in the members and oversized holes in the plates. Threads are included in the shear plane. Use 70-ksi electrodes.

LRFD	ASD
Brace force, $P_u = \pm 1{,}100$ kips Truss B, top chord, beam shear, $V_u = 61.5$ kips	Brace force, $P_a = \pm 730$ kips Truss B, top chord, beam shear, $V_a = 41$ kips

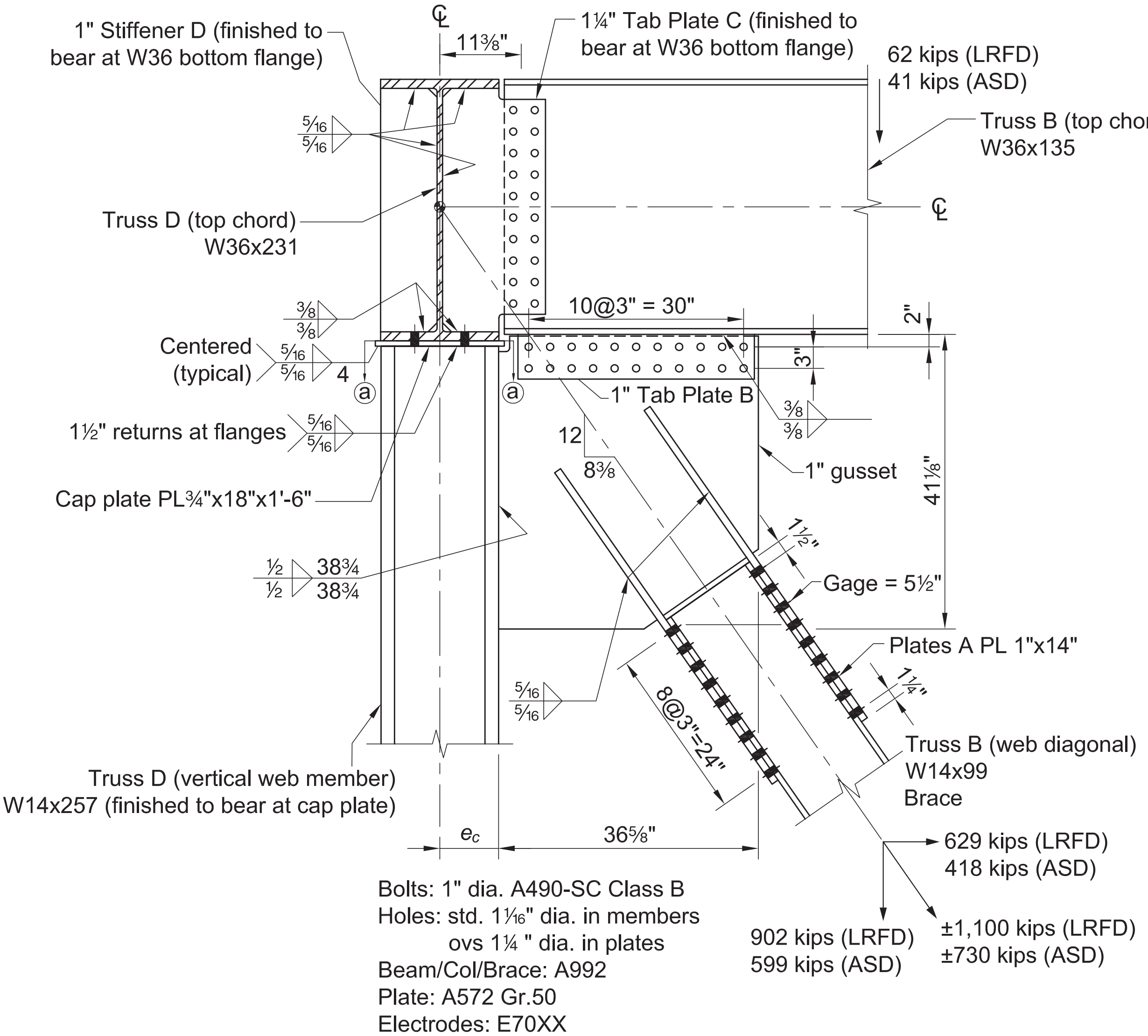


Fig. 5-24. Connection of Truss B to Truss D completed design.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A572 Grade 50
$F_y = 50$ ksi $\quad F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam (Truss D top chord)
W36×231
$d = 36.5$ in. $\quad t_w = 0.760$ in. $\quad b_f = 16.5$ in. $\quad t_f = 1.26$ in.

Beam (Truss B top chord)
W36×135
$d = 35.6$ in. $\quad t_w = 0.600$ in. $\quad b_f = 12.0$ in. $\quad t_f = 0.790$ in. $\quad k = 1.54$ in. $\quad h/t_w = 54.1$

Column (Truss D vertical web member)
W14×257
$d = 16.4$ in. $\quad t_w = 1.18$ in. $\quad b_f = 16.0$ in. $\quad t_f = 1.89$ in. $\quad k_1 = 1$ in.

Brace (Truss B web diagonal)
W14×99
$A_g = 29.1$ in.^{2} $\quad d = 14.2$ in. $\quad t_w = 0.485$ in. $\quad b_f = 14.6$ in. $\quad t_f = 0.780$ in.

Bolt Shear Strength

From AISC *Manual* Tables 7-2 and 7-3, the available tensile and shear strengths for 1-in.-diameter ASTM A490 bolts in slip-critical connections, Class B, are:

LRFD	ASD
$\phi r_{nt} = 66.6$ kips/bolt For standard holes: $\phi r_{nv} = (21.7 \text{ kips})(1.67)$ $= 36.2$ kips/bolt (single shear) $\phi r_{nv} = (43.4 \text{ kips})(1.67)$ $= 72.5$ kips/bolt (double shear) For oversized holes: $\phi r_{nv} = (18.4 \text{ kips})(1.67)$ $= 30.7$ kips/bolt (single shear) $\phi r_{nv} = (36.9 \text{ kips})(1.67)$ $= 61.6$ kips/bolt (double shear)	$\dfrac{r_{nt}}{\Omega} = 44.4$ kips/bolt For standard holes: $\dfrac{r_{nv}}{\Omega} = (14.5 \text{ kips})(1.67)$ $= 24.2$ kips/bolt (single shear) $\dfrac{r_{nv}}{\Omega} = (28.9 \text{ kips})(1.67)$ $= 48.3$ kips/bolt (double shear) For oversized holes: $\dfrac{r_{nv}}{\Omega} = (12.3 \text{ kips})(1.67)$ $= 20.5$ kips/bolt (single shear) $\dfrac{r_{nv}}{\Omega} = (24.7 \text{ kips})(1.67)$ $= 41.2$ kips/bolt (double shear)

AISC *Specification* Section J3.8 requires that slip-critical connections be designed for the limit states of bearing-type connections. From AISC *Manual* Table 7-1, the available bolt shear strength with threads included in the shear planes is:

LRFD	ASD
$\phi r_{nv} = 40.0 \text{ kips/bolt} > 30.7 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_{nv}}{\Omega} = 26.7 \text{ kips/bolt} > 20.5 \text{ kips/bolt}$

Brace-to-Gusset Connection

Check tension yielding on the brace

From AISC *Specification* Section D2(a):

LRFD	ASD
$\phi_t P_n = \phi_t F_y A_g$ $= 0.90(50 \text{ ksi})(29.1 \text{ in.}^2)$ $= 1{,}310 \text{ kips} > 1{,}100 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = \dfrac{F_y A_g}{\Omega_t}$ $= \dfrac{(50 \text{ ksi})(29.1 \text{ in.}^2)}{1.67}$ $= 871 \text{ kips} > 730 \text{ kips}$ **o.k.**

Check tensile rupture on the brace

The net area of the brace is determined from AISC *Specification* Section B4.3b:

$$
\begin{aligned}
A_n &= A_g - 4\left(d_h + \tfrac{1}{16} \text{ in.}\right)t_f \qquad (d_h = \text{ bolt hole diameter}) \\
&= 29.1 \text{ in.}^2 - 4\left(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)(0.780 \text{ in.}) \\
&= 25.6 \text{ in.}^2
\end{aligned}
$$

From AISC Specification Section D3 and Table D3.1, Case 7:

$$U = 0.90$$

Table D3.1, Case 7, says that the shear lag factor can also be calculated from Table D3.1, Case 2, and the larger value is permitted to be used. The tee shape that corresponds to a W14×99 is a WT7×49.5. From Table 1-8, use $\bar{x} = \bar{y} = 1.14$ in., to calculate U as follows:

$$
\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{1.14 \text{ in.}}{24.0 \text{ in.}} \\
&= 0.953
\end{aligned}
$$

Using $U = 0.953$, the effective net area is:

$$A_e = A_n U \qquad (Spec. \text{ Eq. D3-1})$$

$$
\begin{aligned}
&= (25.6 \text{ in.}^2)(0.953) \\
&= 24.4 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section D2(b), the available tensile rupture strength of the brace is:

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75(65\text{ ksi})(24.4\text{ in.}^2)$ $= 1{,}190\text{ kips} > 1{,}100\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(65\text{ ksi})(24.4\text{ in.}^2)}{2.00}$ $= 793\text{ kips} > 730\text{ kips}$ **o.k.**

Check bolt shear

The bolts in the brace flanges are in single shear. Using the controlling available bolt shear strength based on the oversized holes in the connection plates, the available bolt shear strength is:

LRFD	ASD
$\phi R_{nv} = n\phi r_{nv}$ $= (36)(30.7\text{ kips})$ $= 1{,}110\text{ kips} > 1{,}100\text{ kips}$ **o.k.**	$\dfrac{R_{nv}}{\Omega} = n\left(\dfrac{r_{nv}}{\Omega}\right)$ $= (36)(20.5\text{ kips})$ $= 738\text{ kips} > 730\text{ kips}$ **o.k.**

Check block shear rupture on the brace

Block shear rupture on the brace flanges will control over block shear rupture of the connection plates because the flanges are thinner. The controlling block shear rupture failure path is assumed to run through both lines of bolts and then on a perpendicular line to the edge of each beam flange. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = \left[1\tfrac{1}{2}\text{ in.} + 8(3.00\text{ in.})\right](0.780\text{ in.})(4)$$
$$= 79.6\text{ in.}^2$$

$$0.60F_y A_{gv} = 0.60(50\text{ ksi})(79.6\text{ in.}^2)$$
$$= 2{,}390\text{ kips}$$

Shear rupture component:

$$A_{nv} = 79.6\text{ in.}^2 - 8.5\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(0.780\text{ in.})(4)$$
$$= 49.8\text{ in.}^2$$

$$0.60F_u A_{nv} = 0.60(65\text{ ksi})(49.8\text{ in.}^2)$$
$$= 1{,}940\text{ kips}$$

Tension rupture component:

U_{bs} = 1 from AISC *Specification* Section J4.3 because the bolts are uniformly loaded

$$A_{nt} = \left[14.6 \text{ in.} - 5\tfrac{1}{2} \text{ in.} - 1\left(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(0.780 \text{ in.}\right)\left(2\right)$$
$$= 12.4 \text{ in.}^2$$
$$U_{bs}F_uA_{nt} = 1\left(65 \text{ ksi}\right)\left(12.4 \text{ in.}^2\right)$$
$$= 806 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 1{,}940 \text{ kips} + 806 \text{ kips}$$
$$= 2{,}750 \text{ kips}$$
$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 2{,}390 \text{ kips} + 806 \text{ kips}$$
$$= 3{,}200 \text{ kips}$$

Therefore, R_n = 2,750 kips.

LRFD	ASD
$\phi R_n = 0.75\left(2{,}750 \text{ kips}\right)$ $= 2{,}060 \text{ kips} > 1{,}100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{2{,}750 \text{ kips}}{2.00}$ $= 1{,}380 \text{ kips} > 730 \text{ kips}$ **o.k.**

Check bolt bearing on the brace

The available bearing and tearout strengths of the brace are determined as follows:

LRFD	ASD
$\phi r_n = \phi 1.2 l_c t F_u$ $l_c = 1\tfrac{1}{2} \text{ in.} - 0.5\left(1\tfrac{1}{16} \text{ in.}\right)$ $= 0.969 \text{ in.}$ $\phi r_n = 0.75\left(1.2\right)\left(0.969 \text{ in.}\right)\left(0.780 \text{ in.}\right)\left(65 \text{ ksi}\right)$ $= 44.2 \text{ kips/bolt}$ $\phi r_n = \phi 2.4 d t F_u$ $= 0.75\left(2.4\right)\left(1.00 \text{ in.}\right)\left(0.780 \text{ in.}\right)\left(65 \text{ ksi}\right)$ $= 91.3 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega}$ $l_c = 1\tfrac{1}{2} \text{ in.} - 0.5\left(1\tfrac{1}{16} \text{ in.}\right)$ $= 0.969 \text{ in.}$ $\dfrac{r_n}{\Omega} = \dfrac{1.2\left(0.969 \text{ in.}\right)\left(0.780 \text{ in.}\right)\left(65 \text{ ksi}\right)}{2.00}$ $= 29.5 \text{ kips/bolt}$ $\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4\left(1.00 \text{ in.}\right)\left(0.780 \text{ in.}\right)\left(65 \text{ ksi}\right)}{2.00}$ $= 60.8 \text{ kips/bolt}$

The bolt shear strength of 30.7 kips/bolt for LRFD and 20.5 kips/bolt for ASD controls the strength of the bolt group compared to bearing or tearout on the brace. Bolt shear for the bolt group was checked previously.

Check bolt bearing on the connection plate

Because t_p = 1.00 in. > t_f = 0.780 in., the plate is satisfactory for bearing.

For tearout:

$$l_c = 1\tfrac{1}{2}\text{ in.} - 0.5(1\tfrac{1}{4}\text{ in.})$$
$$= 0.875\text{ in.}$$

LRFD	ASD
$\phi r_n = 0.75(1.2)(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 51.2\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = \dfrac{1.2(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 34.1\text{ kips/bolt}$

The bolt shear strength of 30.7 kips/bolt for LRFD and 20.5 kips/bolt for ASD controls the strength of the bolt group compared to bearing or tearout on the plate. Bolt shear for the bolt group was checked previously.

Check tensile rupture of Plates A at gusset plate slot

The available tensile rupture strength of Plates A is determined from AISC *Specification* Section D2(b) assuming an ⅛-in. gap at the slot, where $A_e = A_nU$. The net area is:

$$A_n = (14.0\text{ in.} - 1\tfrac{1}{8}\text{ in.})(1.00\text{ in.})$$
$$= 12.9\text{ in.}^2$$
U = 1.0 from AISC *Specification* Table D3.1

LRFD	ASD
$\phi R_{nt} = \phi F_u A_n U\,(2)$ $= 0.75(65\text{ ksi})(12.9\text{ in.}^2)(1.0)(2)$ $= 1{,}260\text{ kips} > 1{,}100\text{ kips}$ **o.k.**	$\dfrac{R_{nt}}{\Omega} = \dfrac{F_u A_n U}{\Omega}(2)$ $= \left[\dfrac{65\text{ ksi}(12.9\text{ in.}^2)(1.0)}{2.00}\right](2)$ $= 839\text{ kips} > 730\text{ kips}$ **o.k.**

Check tensile yielding of Plates A

The available tensile yielding strength of Plates A is determined from AISC *Specification* D3(a) as follows. The gross area is:

$$A_{gt} = (14.0\text{ in.})(1.00\text{ in.})$$
$$= 14.0\text{ in.}^2$$

LRFD	ASD
$\phi R_n = \phi F_y A_{gt}\,(2)$ $= 0.90(50\text{ ksi})(14.0\text{ in.}^2)(2)$ $= 1{,}260\text{ kips} > 1{,}100\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_{gt}}{\Omega}(2)$ $= \left[\dfrac{(50\text{ ksi})(14.0\text{ in.}^2)}{1.67}\right](2)$ $= 838\text{ kips} > 730\text{ kips}$ **o.k.**

Check tensile rupture of Plates A at bolts

The available tensile rupture strength of Plates A is determined from AISC *Specification* D3(b) with an oversized hole diameter of 1¼ in. in the plates, and $A_e = A_n U$:

$$A_n = \left[14.0\text{ in.} - 2\left(1\tfrac{1}{4}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](1.00\text{ in.})$$
$$= 11.4\text{ in.}^2$$

U = 1.0 from AISC *Specification* Table D3.1

LRFD	ASD
$\phi R_n = \phi F_u A_e(2)$ $= 0.75(65\text{ ksi})(11.4\text{ in.}^2)(1.0)(2)$ $= 1{,}110\text{ kips} > 1{,}100\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}(2)$ $= \dfrac{(65\text{ ksi})(11.4\text{ in.}^2)(1.0)}{2.00}(2)$ $= 741\text{ kips} > 730\text{ kips}$ **o.k.**

Check weld of Plates A to gusset plate

The welds to the gusset are loaded eccentrically by 3.22 in. (See Figure 5-25). The load is applied at the center of the plate above and below the slot and is transferred at the face of the slot. Try a weld length of l = 25 in. Then al = 3.22, a = 0.129 and from AISC *Manual* Table 8-4, Angle = 0°, k = 0, C = 3.69.

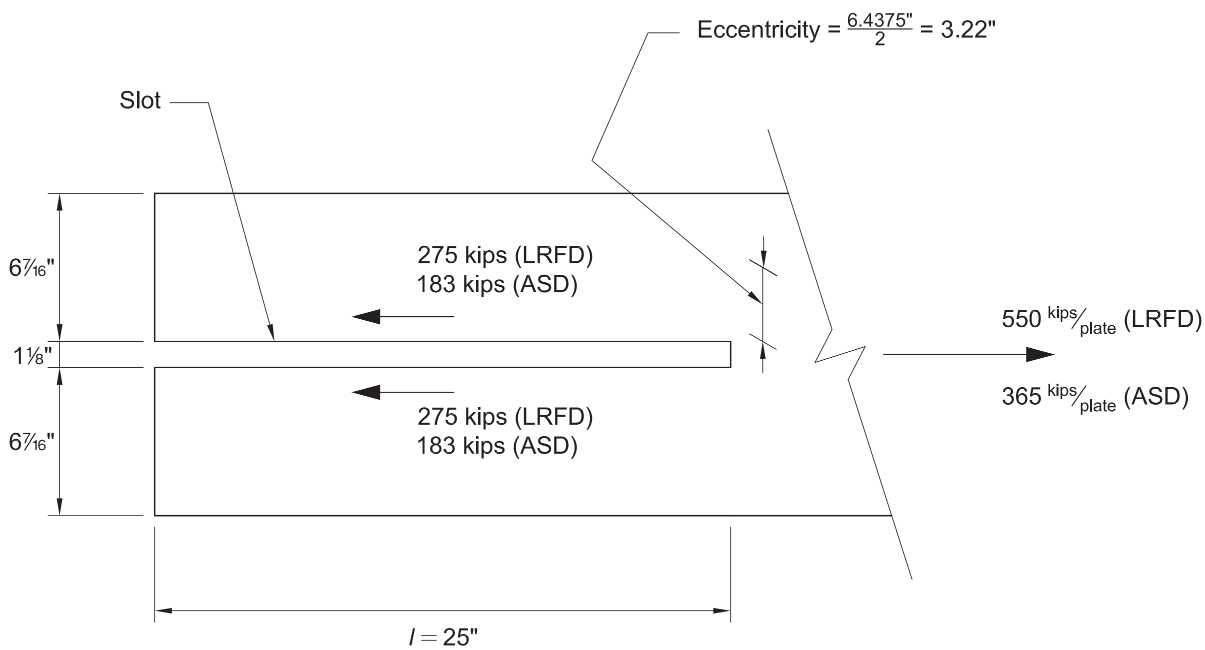


Fig. 5-25. Weld of plate to gusset.

LRFD	ASD
$D_{req'd} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{275 \text{ kips}}{(0.75)(3.69)(1.0)(25.0 \text{ in.})}$ $= 3.97$	$D_{req'd} = \dfrac{\Omega P_a}{C C_1 l}$ $= \dfrac{2.00(183 \text{ kips})}{(3.69)(1.0)(25.0 \text{ in.})}$ $= 3.97$

Use ¼-in. fillet welds.

Based on strength alone, ¼-in. fillet welds are indicated; however, AISC *Specification* Table J2.4 requires a minimum fillet weld size of 5⁄16 in. for material thicknesses over ¾ in. Therefore, use 5⁄16-in. fillet welds to connect the plate to the gusset.

Using AISC *Manual* Equation 9-3, the minimum base metal thickness required for the weld size determined from Table 8-4 previously is:

$$
\begin{aligned}
t_{min} &= \frac{6.19D}{F_u} \\
&= \frac{6.19(3.97)}{65 \text{ ksi}} \\
&= 0.380 \text{ in.}
\end{aligned}
$$

Because the gusset plate and the connection plates are both 1 in. thick, this criterion is satisfied. Note that the formula for t_{min} is valid for both LRFD and ASD designs.

Check the Whitmore section in the gusset plate

The width of the Whitmore section, as defined in AISC *Manual* Part 9, is:

$$
\begin{aligned}
l_w &= 14.2 \text{ in.} + 1.00 \text{ in.} + 1.00 \text{ in.} + 2(25.0 \text{ in.})\tan 30° \\
&= 45.1 \text{ in.}
\end{aligned}
$$

This section enters the column, which has a web thickness, t_w = 1.18 in. > gusset thickness, t_g. Therefore, the entire Whitmore section is effective, and:

$$
\begin{aligned}
A_w &= t_g l_w \\
&= (1.00 \text{ in.})(45.1 \text{ in.}) \\
&= 45.1 \text{ in.}^2
\end{aligned}
$$

The tensile yielding strength of the gusset plate is determined from AISC *Specification* Section D2(a), with $A_g = A_w$:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(50 \text{ ksi})(45.1 \text{ in.}^2)$ $= 2{,}030 \text{ kips} > 1{,}100 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{(50 \text{ ksi})(45.1 \text{ in.}^2)}{1.67}$ $= 1{,}350 \text{ kips} > 730 \text{ kips}$ **o.k.**

Because the brace is always in tension, there is no need to check gusset buckling. The 1-in.-thick gusset plate is adequate.

Force Distributions

Figure 5-26 shows the uniform force method control points determined as described in AISC *Manual* Part 9 or Section 4.1.1 of this Design Guide.

$$e_c = 8.2 \text{ in.}$$

$$\tan\theta = \frac{8\tfrac{3}{8} \text{ in.}}{12.0 \text{ in.}}$$

$$= 0.698$$

$$e_b = 17.8 \text{ in.}$$

$$\theta = 34.9°$$

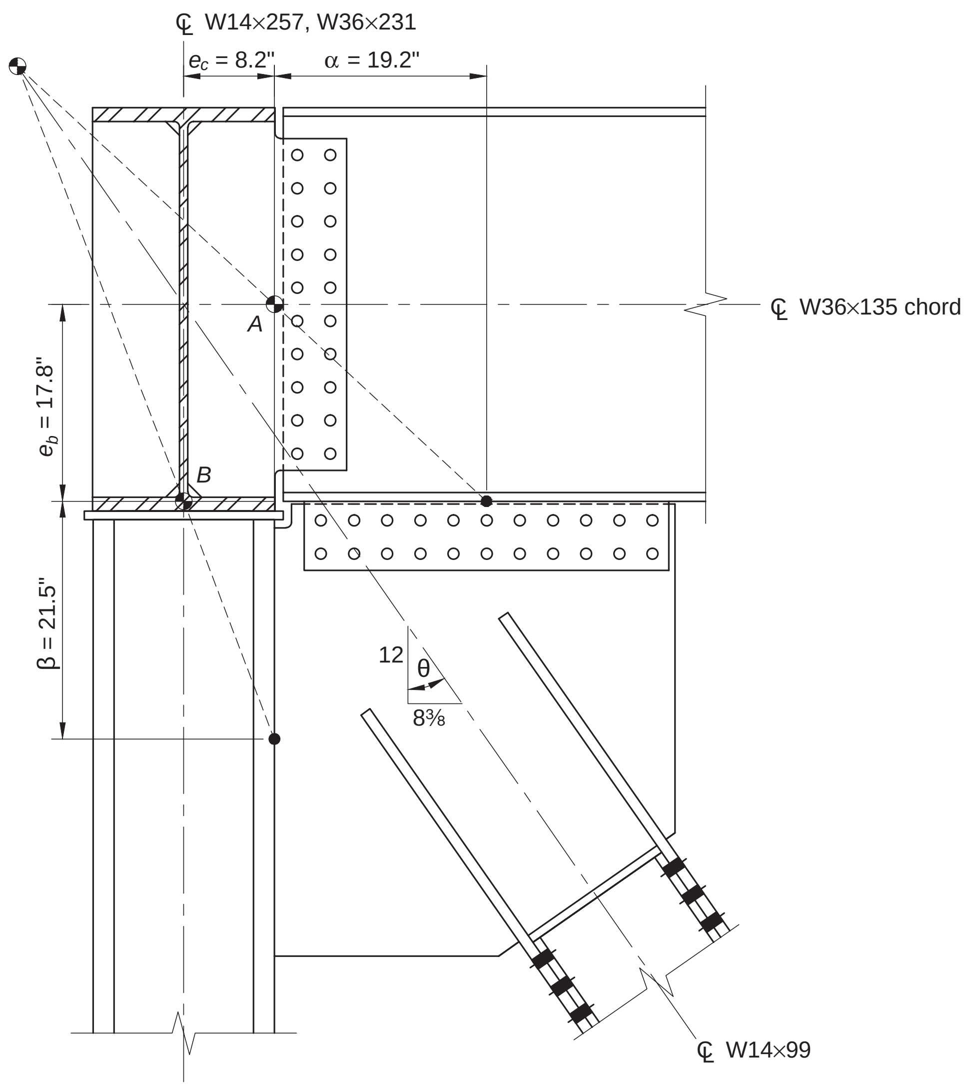


Fig. 5-26. Control points for the uniform force method.

From Figure 5-24:

$$\text{Set } \beta = \overline{\beta} = \frac{38\tfrac{3}{4}\text{ in.}}{2} + \text{distance from top end of gusset-to-truss vertical weld to bottom of secondary truss chord flange}$$

$$= \frac{38\tfrac{3}{4}\text{ in.}}{2} + \tfrac{1}{2}\text{ in.} + \tfrac{3}{4}\text{ in.} + \tfrac{7}{8}\text{ in.}$$

$$= 21.5\text{ in.}$$

With $\beta = \overline{\beta} = 21.5$, calculate α required to eliminate moments at the gusset interfaces:

$$\alpha = (\beta + e_b)\tan\theta - e_c \qquad \text{(from AISC } Manual \text{ Eq. 13-1)}$$

$$= (21.5\text{ in.} + 17.8\text{ in.})\left(\frac{8\tfrac{3}{8}\text{ in.}}{12.0\text{ in.}}\right) - 8.20\text{ in.}$$

$$= 19.2\text{ in.}$$

The gusset-to-secondary chord connection has been arranged so that $\overline{\alpha} = 19.0$ in., thus there is no moment on this interface:

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad \text{(AISC } Manual \text{ Eq. 13-6)}$$

$$= \sqrt{(19.2\text{ in.} + 8.20\text{ in.})^2 + (21.5\text{ in.} + 17.8\text{ in.})^2}$$

$$= 47.9\text{ in.}$$

Where ½ in. is the clearance between the cap plate and the gusset plate, ¾ in. is the cap plate thickness, and ⅞ in. is the difference between the Truss B top chord and the Truss D top chord.

LRFD	ASD
$\frac{P_u}{r} = \frac{1{,}100\text{ kips}}{47.9\text{ in.}}$ $= 23.0\text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ub} = \alpha\frac{P_u}{r}$ $= (19.2\text{ in.})(23.0\text{ kip/in.})$ $= 442\text{ kips}$ From AISC *Manual* Equation 13-3 $H_{uc} = e_c\frac{P_u}{r}$ $= (8.20\text{ in.})(23.0\text{ kip/in.})$ $= 189\text{ kips}$ $\Sigma(H_{uc} + H_{ub}) = 442\text{ kips} + 189\text{ kips}$ $= 631\text{ kips}$ $= (1{,}100\text{ kips})\sin 34.9°$ $= 629\text{ kips}$ **o.k.**	$\frac{P_a}{r} = \frac{730\text{ kips}}{47.9\text{ in.}}$ $= 15.2\text{ kip/in.}$ From AISC *Manual* Equation 13-5: $H_{ab} = \alpha\frac{P_a}{r}$ $= (19.2\text{ in.})(15.2\text{ kip/in.})$ $= 292\text{ kips}$ From AISC *Manual* Equation 13-3: $H_{ac} = e_c\frac{P_a}{r}$ $= (8.20\text{ in.})(15.2\text{ kip/in.})$ $= 125\text{ kips}$ $\Sigma(H_{ac} + H_{ab}) = 292\text{ kips} + 125\text{ kips}$ $= 417\text{ kips}$ $= (730\text{ kips})\sin 34.9°$ $= 418\text{ kips}$ **o.k.**

LRFD	ASD
From AISC *Manual* Equation 13-4: $$\begin{aligned} V_{ub} &= e_b \frac{P_u}{r} \\ &= (17.8 \text{ in.})(23.0 \text{ kip/in.}) \\ &= 409 \text{ kips} \end{aligned}$$ From AISC *Manual* Equation 13-2: $$\begin{aligned} V_{uc} &= \beta \frac{P_u}{r} \\ &= (21.5 \text{ in.})(23.0 \text{ kip/in.}) \\ &= 495 \text{ kips} \end{aligned}$$ $$\begin{aligned} \Sigma(V_{uc} + V_{ub}) &= 495 \text{ kips} + 409 \text{ kips} \\ &= 904 \text{ kips} \\ &= (1{,}100 \text{ kips})\cos 34.9° \\ &= 902 \text{ kips} \quad \textbf{o.k.} \end{aligned}$$	From AISC *Manual* Equation 13-4: $$\begin{aligned} V_{ab} &= e_b \frac{P_a}{r} \\ &= (17.8 \text{ in.})(15.2 \text{ kip/in.}) \\ &= 271 \text{ kips} \end{aligned}$$ From AISC *Manual* Equation 13-2: $$\begin{aligned} V_{ac} &= \beta \frac{P_a}{r} \\ &= (21.5 \text{ in.})(15.2 \text{ kip/in.}) \\ &= 327 \text{ kips} \end{aligned}$$ $$\begin{aligned} \Sigma(V_{ac} + V_{ab}) &= 327 \text{ kips} + 271 \text{ kips} \\ &= 598 \text{ kips} \\ &= (730 \text{ kips})\cos 34.9° \\ &= 599 \text{ kips} \quad \textbf{o.k.} \end{aligned}$$

This distribution of forces is shown in Figure 5-27.

Gusset-to-Column Connection (Primary Truss Vertical)

Design weld

LRFD	ASD
The resultant load is: $$\begin{aligned} R_u &= \sqrt{(495 \text{ kips})^2 + (189 \text{ kips})^2} \\ &= 530 \text{ kips} \end{aligned}$$ $$\begin{aligned} \theta &= \tan^{-1}\left(\frac{189 \text{ kips}}{495 \text{ kips}}\right) \\ &= 20.9° \end{aligned}$$ From AISC *Manual* Table 8-4 with Angle $= 15°$, $l = 38.75$ in., $a = k = 0$, $C = 3.96$: $$\begin{aligned} D_{req'd} &= \frac{P_u}{\phi C C_1 l}(ductility\ factor) \\ &= \frac{530 \text{ kips}}{0.75(3.96)(1.0)(38\tfrac{3}{4} \text{ in.})}(1.25) \\ &= 5.76 \text{ sixteenths} \end{aligned}$$	The resultant load is: $$\begin{aligned} R_a &= \sqrt{(327 \text{ kips})^2 + (125 \text{ kips})^2} \\ &= 350 \text{ kips} \end{aligned}$$ $$\begin{aligned} \theta &= \tan^{-1}\left(\frac{125 \text{ kips}}{327 \text{ kips}}\right) \\ &= 20.9° \end{aligned}$$ From AISC *Manual* Table 8-4 with Angle $= 15°$, $l = 38.75$ in., $a = k = 0$, $C = 3.96$: $$\begin{aligned} D_{req'd} &= \frac{P_a \Omega}{C C_1 l}(ductility\ factor) \\ &= \frac{(350 \text{ kips})(2.00)}{3.96(1.0)(38\tfrac{3}{4} \text{ in.})}(1.25) \\ &= 5.70 \text{ sixteenths} \end{aligned}$$

Use a ⅜-in. fillet weld. From AISC *Specification* Table J2.4, the minimum fillet weld required for a 1-in.-thick plate is 5⁄16 in.

The use of the ductility factor in this manner has been explained in previous examples.

Check gusset plate for shear yielding and tensile yielding along the column flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

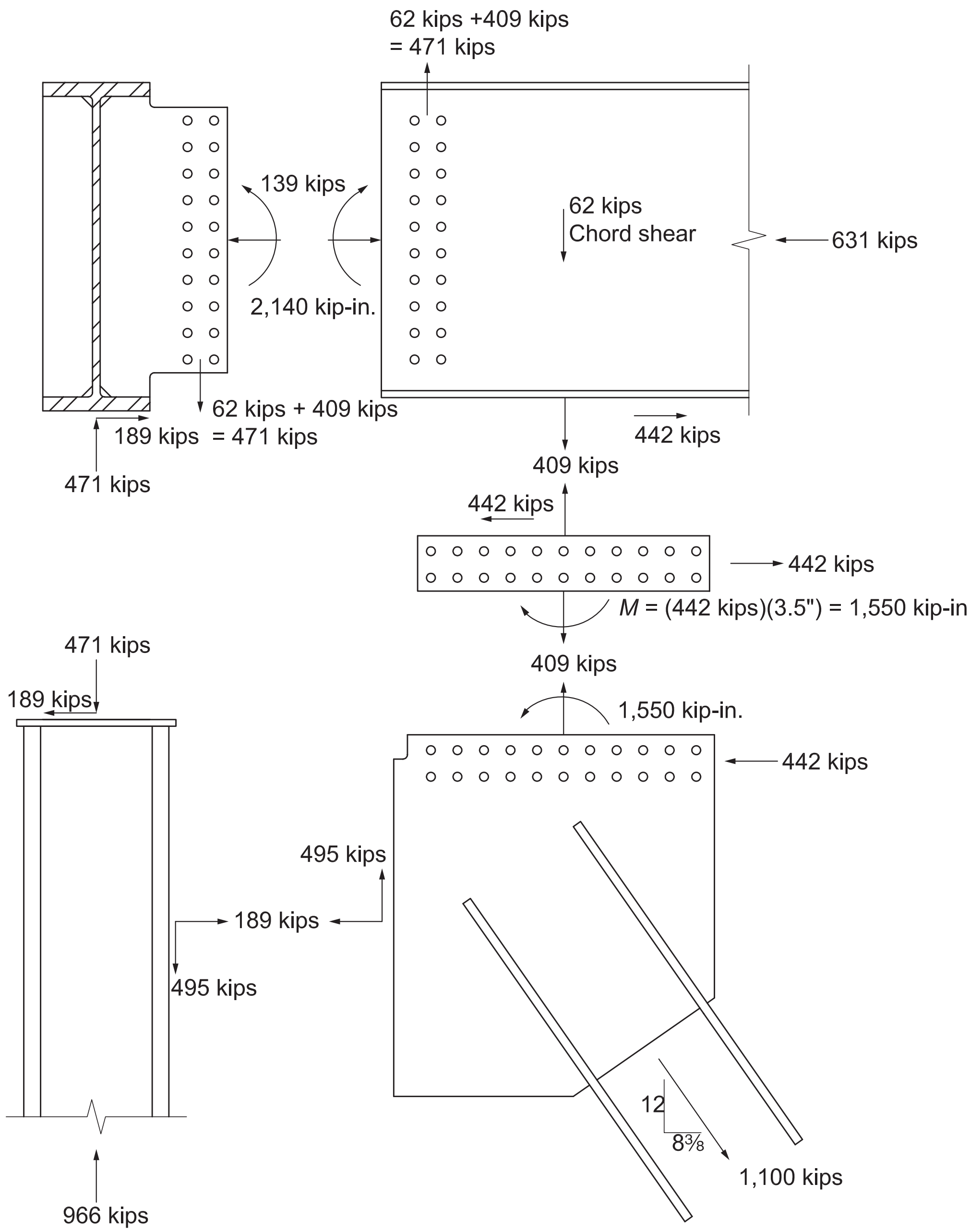


Fig. 5-27a. Admissible force fields for truss-to-truss example—LRFD.

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(1.00\text{ in.})(38\tfrac{3}{4}\text{ in.})$ $= 1{,}160\text{ kips} > 495\text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(50\text{ ksi})(1.00\text{ in.})(38\tfrac{3}{4}\text{ in.})$ $= 1{,}740\text{ kips} > 189\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})(1.00\text{ in.})(38\tfrac{3}{4}\text{ in.})}{1.50}$ $= 775\text{ kips} > 327\text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50\text{ ksi})(1.00\text{ in.})(38\tfrac{3}{4}\text{ in.})}{1.67}$ $= 1{,}160\text{ kips} > 125\text{ kips}$ **o.k.**

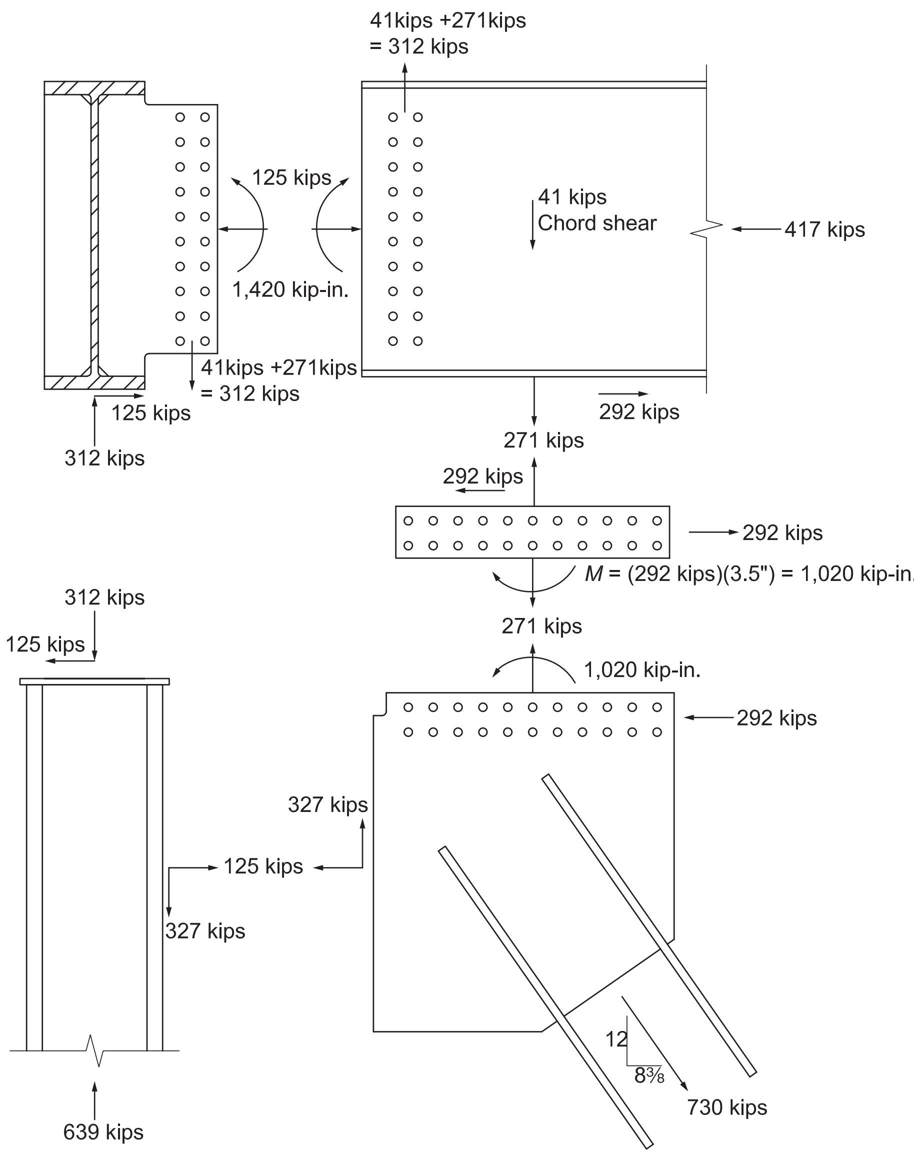


Fig. 5-27b. Admissible force fields for truss-to-truss example—ASD.

Gusset-to-Beam Connection (Secondary Truss Chord)

Design tab plate-to-chord weld

LRFD	ASD
The resultant load on the weld is: $$R_u = \sqrt{(409\text{ kips})^2 + (442\text{ kips})^2} = 602\text{ kips}$$ $$\theta = \tan^{-1}\left(\frac{409\text{ kips}}{442\text{ kips}}\right) = 42.8°$$ From AISC *Manual* Table 8-4 with Angle = 30°, a = k = 0, C = 4.37: $$D_{req'd} = \frac{P_u}{\phi C C_1 l} = \frac{602\text{ kips}}{0.75(4.37)(1.0)(33.0\text{ in.})} = 5.57\text{ sixteenths}$$	The resultant load on the weld is: $$R_a = \sqrt{(271\text{ kips})^2 + (292\text{ kips})^2} = 398\text{ kips}$$ $$\theta = \tan^{-1}\left(\frac{271\text{ kips}}{292\text{ kips}}\right) = 42.9°$$ From AISC *Manual* Table 8-4 with Angle = 30°, a = k = 0, C = 4.37: $$D_{req'd} = \frac{P_a \Omega}{C C_1 l} = \frac{(398\text{ kips})(2.00)}{(4.37)(1.0)(33.0\text{ in.})} = 5.52\text{ sixteenths}$$

Use a ⅜-in. fillet weld.

Note that a ductility factor is not used here because the slip in the bolted connection on the other side of the tab plate and bolt shear ductility will allow redistribution to occur.

Check web local yielding on secondary truss chord due to normal force

The normal force is applied at a distance from the member end that is less than the depth of the member, d = 35.6 in.; therefore, use AISC *Specification* Equation J10-3:

LRFD	ASD
$$\phi R_n = \phi F_{yw} t_w (2.5k + l_b) = 1.00(50\text{ ksi})(0.600\text{ in.})[2.5(1.54\text{ in.}) + 33.0\text{ in.}] = 1{,}110\text{ kips} > 409\text{ kips} \quad \textbf{o.k.}$$	$$\frac{R_n}{\Omega} = \frac{F_{yw} t_w (2.5k + l_b)}{\Omega} = \frac{(50\text{ ksi})(0.600\text{ in.})[2.5(1.54\text{ in.}) + 33.0\text{ in.}]}{1.50} = 737\text{ kips} > 271\text{ kips} \quad \textbf{o.k.}$$

Check beam web local crippling on secondary truss chord due to normal force

The normal force of 409 kips (LRFD) or 271 kips (ASD) is always tension, so this limit state does not exist.

Check bolts connecting tab plate to gusset plate

As determined previously, the required strength at the gusset-to-beam interface is:

LRFD	ASD
$R_u = 602$ kips	$R_a = 398$ kips

The angle of the resultant is θ = 42.9° as determined previously.

The load on the bolt group is eccentric, i.e., $e_x = 3.5$ in.

From AISC Manual Table 7-7 for Angle = 30°, $n = 11$, $e_x = 3.5$ in., $s = 3$ in., $C = 19.6$, and the available bolt shear strength values determined previously for oversized holes in single shear, the available strength of the bolt group is:

LRFD	ASD
$\begin{aligned}\phi R_{nv} &= C\phi r_{nv} \\ &= 19.6(30.7 \text{ kips/bolt}) \\ &= 602 \text{ kips} \geq 602 \text{ kips} \quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_{nv}}{\Omega} &= C\phi r_{nv} \\ &= 19.6(20.5 \text{ kips/bolt}) \\ &= 402 \text{ kips} > 398 \text{ kips} \quad \textbf{o.k.}\end{aligned}$

Check bolt bearing on the tab plate and the gusset plate

Because of the 3.5-in. eccentricity, the 22 bolts are equivalent to 19.6 directly loaded bolts ($C = 19.6$ as determined previously). Therefore, the maximum force per bolt is:

LRFD	ASD
$\begin{aligned}P_{max} &= \frac{R_u}{C} \\ &= \frac{602 \text{ kips}}{19.6 \text{ bolts}} \\ &= 30.7 \text{ kips/bolt}\end{aligned}$	$\begin{aligned}P_{max} &= \frac{R_a}{C} \\ &= \frac{398 \text{ kips}}{19.6 \text{ bolts}} \\ &= 20.3 \text{ kips/bolt}\end{aligned}$

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength on the tab plate or on the gusset plate. Bearing strength is limited by bearing deformation of the hole or by tearout of the material. From AISC *Specification* Section J3.10, when deformation at the bolt hole is a design consideration, the available strength for the limit state of bearing at bolt holes is determined as follows:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

Bearing and tearout will not control as long as they provide more than 30.7 kips per bolt for LRFD and 20.3 kips per bolt for ASD. Because the tab and the gusset have the same thickness, the following checks apply to both. The clear distance to the edge is determined as follows, with a bolt hole diameter of 1¼ in. and assuming the edge distance is 1½ in.:

$$\begin{aligned}l_c &= 1\tfrac{1}{2} \text{ in.} - 0.5(1\tfrac{1}{4} \text{ in.}) \\ &= 0.875 \text{ in.}\end{aligned}$$

For the outer bolt, using AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 51.2\text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 117\text{ kips}$ Therefore, $\phi r_n = 51.2$ kips/bolt > 30.7 kips/bolt (bolt shear)	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 34.1\text{ kips}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 78.0\text{ kips}$ Therefore, $r_n/\Omega = 34.1$ kips/bolt > 20.3 kips/bolt (bolt shear)

Bolt shear controls the strength of the connection and is o.k. as previously determined.

Check tab plate for shear yielding and tensile yielding

The available shear yielding strength of the tab plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(1.00\text{ in.})(33.0\text{ in.})$ $= 990\text{ kips} > 442\text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(50\text{ ksi})(1.00\text{ in.})(33.0\text{ in.})$ $= 1{,}490\text{ kips} > 409\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})(1.00\text{ in.})(33.0\text{ in.})}{1.50}$ $= 660\text{ kips} > 292\text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50\text{ ksi})(1.00\text{ in.})(33.0\text{ in.})}{1.67}$ $= 988\text{ kips} > 271\text{ kips}$ **o.k.**

Check tab plate for shear rupture and tensile rupture

The available shear rupture strength of the tab plate is determined from AISC *Specification* Equation J4-4, and the available tensile rupture strength is determined from AISC *Specification* Equation J4-2. The net area, with an oversized hole diameter of 1¼ in., is:

$$
\begin{aligned}
A_{nv} &= \left[33.0\text{ in.} - 11\left(1\tfrac{1}{4}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](1.00\text{ in.}) \\
&= 18.6\text{ in.}^2
\end{aligned}
$$

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(65 \text{ ksi})(18.6 \text{ in.}^2)$ $= 544 \text{ kips} > 442 \text{ kips}$ **o.k.** $\phi R_n = \phi F_u A_e$ $= 0.75(65 \text{ ksi})(18.6 \text{ in.}^2)$ $= 907 \text{ kips} > 409 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(65 \text{ ksi})(18.6 \text{ in.}^2)}{2.00}$ $= 363 \text{ kips} > 292 \text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(65 \text{ ksi})(18.6 \text{ in.}^2)}{2.00}$ $= 605 \text{ kips} > 271 \text{ kips}$ **o.k.**

Check combined shear, normal and flexural strength on the tab plate

The forces on the critical section are:

LRFD	ASD
$V_u = 442 \text{ kips}$ $N_u = 409 \text{ kips}$ $M_u = (442 \text{ kips})(3.50 \text{ in.})$ $= 1{,}550 \text{ kip-in.}$	$V_a = 292 \text{ kips}$ $N_a = 271 \text{ kips}$ $M_a = (292 \text{ kips})(3.50 \text{ in.})$ $= 1{,}020 \text{ kip-in.}$

An interaction equation that includes shear, normal and flexural stresses is the fourth order interaction equation proposed by Neal (1977) and derived based on plastic theory:

LRFD	ASD
$\dfrac{M_u}{\phi M_n} + \left(\dfrac{N_u}{\phi N_n}\right)^2 + \left(\dfrac{V_u}{\phi V_n}\right)^4 \le 1.0$	$\dfrac{\Omega M_a}{M_n} + \left(\dfrac{\Omega N_a}{N_n}\right)^2 + \left(\dfrac{\Omega V_a}{V_n}\right)^4 \le 1.0$

where M_n, N_n and V_n, as well as ϕ and Ω, vary depending on whether yield or rupture is being considered.

The plate gross plastic section modulus is:

$$
\begin{aligned}
Z_g &= \tfrac{1}{4}(1.00 \text{ in.})(33.0 \text{ in.})^2 \\
&= 272 \text{ in.}^3
\end{aligned}
$$

The plate net plastic section modulus is:

$$
\begin{aligned}
Z_n &= \tfrac{1}{4} t\left(s - d_h'\right)\left(n^2 s + d_h'\right) \qquad (\textit{Manual} \text{ Eq. 15-4}) \\
&= \tfrac{1}{4}(1.00 \text{ in.})\left[3.00 \text{ in.} - \left(1\tfrac{1}{4} \text{ in.} + 1\tfrac{1}{16} \text{ in.}\right)\right]\left[\left(11^2\right)(3.00 \text{ in.}) + \left(1\tfrac{1}{4} \text{ in.} + 1\tfrac{1}{16} \text{ in.}\right)\right] \\
&= 154 \text{ in.}^3
\end{aligned}
$$

The available flexural yield strength of the tab plate is determined from AISC *Manual* Equation 15-2, and the available flexural rupture strength is determined from AISC *Manual* Equation 15-3 as follows.

LRFD	ASD
$\phi M_n = 0.90 F_y Z_g$ $= 0.90(50\text{ ksi})(272\text{ in.}^3)$ $= 12{,}200\text{ kip-in.} > 1{,}550\text{ kip-in.}$ **o.k.** $\phi M_n = 0.75 F_u Z_n$ $= 0.75(65\text{ ksi})(154\text{ in.}^3)$ $= 7{,}510\text{ kip-in.} > 1{,}550\text{ kip-in.}$ **o.k.**	$\dfrac{M_n}{\Omega} = \dfrac{F_y Z_g}{\Omega}$ $= \dfrac{(50\text{ ksi})(272\text{ in.}^3)}{1.67}$ $= 8{,}140\text{ kip-in.} > 1{,}020\text{ kip-in.}$ **o.k.** $\dfrac{M_n}{\Omega} = \dfrac{F_u Z_n}{\Omega}$ $= \dfrac{(65\text{ ksi})(154\text{ in.}^3)}{2.00}$ $= 5{,}010\text{ kip-in.} > 1{,}020\text{ kip-in.}$ **o.k.**

From Neal (1977), for yielding:

LRFD	ASD
$\dfrac{1{,}550\text{ kip-in.}}{12{,}200\text{ kip-in.}} + \left(\dfrac{409\text{ kips}}{1{,}490\text{ kips}}\right)^2 + \left(\dfrac{442\text{ kips}}{990\text{ kips}}\right)^4 = 0.242 < 1.0$ **o.k.**	$\dfrac{1{,}020\text{ kip-in.}}{8{,}140\text{ kip-in.}} + \left(\dfrac{271\text{ kips}}{988\text{ kips}}\right)^2 + \left(\dfrac{292\text{ kips}}{660\text{ kips}}\right)^4 = 0.239 < 1.0$ **o.k.**

For rupture:

LRFD	ASD
$\dfrac{1{,}550\text{ kip-in.}}{7{,}510\text{ kip-in.}} + \left(\dfrac{409\text{ kips}}{907\text{ kips}}\right)^2 + \left(\dfrac{442\text{ kips}}{544\text{ kips}}\right)^4 = 0.845 < 1.0$ **o.k.**	$\dfrac{1{,}020\text{ kip-in.}}{5{,}010\text{ kip-in.}} + \left(\dfrac{271\text{ kips}}{605\text{ kips}}\right)^2 + \left(\dfrac{292\text{ kips}}{363\text{ kips}}\right)^4 = 0.823 < 1.0$ **o.k.**

Because the tab plate and the gusset plate are both 1 in. thick, the above checks on the tab plate suffice for the gusset plate.

Truss B-to-Truss D Connection (Secondary Truss Chord to Primary Truss Chord)

Figure 5-27 gives the forces, and Figure 5-28 gives greater definition as to the force distribution in the shaped tab plate denoted Tab Plate C in Figure 5-24.

Check bolt shear strength

The bolts must support the following required strengths:

LRFD	ASD
$P_u = 189\text{ kips}$ $V_u = 409\text{ kips} + 62.0\text{ kips}$ $= 471\text{ kips}$	$P_a = 125\text{ kips}$ $V_a = 271\text{ kips} + 41.0\text{ kips}$ $= 312\text{ kips}$

Note that P_u = 189 kips (LRFD) and P_a =125 kips (ASD) is not a transfer force, but rather the "H_c" required to maintain equilibrium.

The secondary chord reaction of 62 kips for LRFD and 41 kips for ASD must be delivered to the centerline of the primary truss member. Thus, a moment of (62.0 kips) (11⅜ in.) = 705 kip-in. for LRFD and (41.0 kips)(11⅜ in.) = 466 kip-in. for ASD must be carried by the bolts. Also, because the control point A is 3.5 in. from the centroid of the bolt group, an additional moment is required to deliver the V_b = 409 kips (LRFD) and V_b = 271 kips (ASD) force from the control point to the bolt group centroid; thus (409 kips)(3.5 in.) = 1,430 kip-in. (LRFD) and (271 kips)(3.5 in.) = 949 kip-in. (ASD). The total moment at the Truss B to Truss D interface is:

LRFD	ASD
M_u = 705 kip-in. + 1,430 kip-in. = 2,140 kip-in. The resultant load is: $R = \sqrt{(189\text{ kips})^2 + (471\text{ kips})^2}$ = 508 kips	M_a = 466 kip-in. + 949 kip-in. = 1,420 kip-in. The resultant load is: $R = \sqrt{(125\text{ kips})^2 + (312\text{ kips})^2}$ = 336 kips

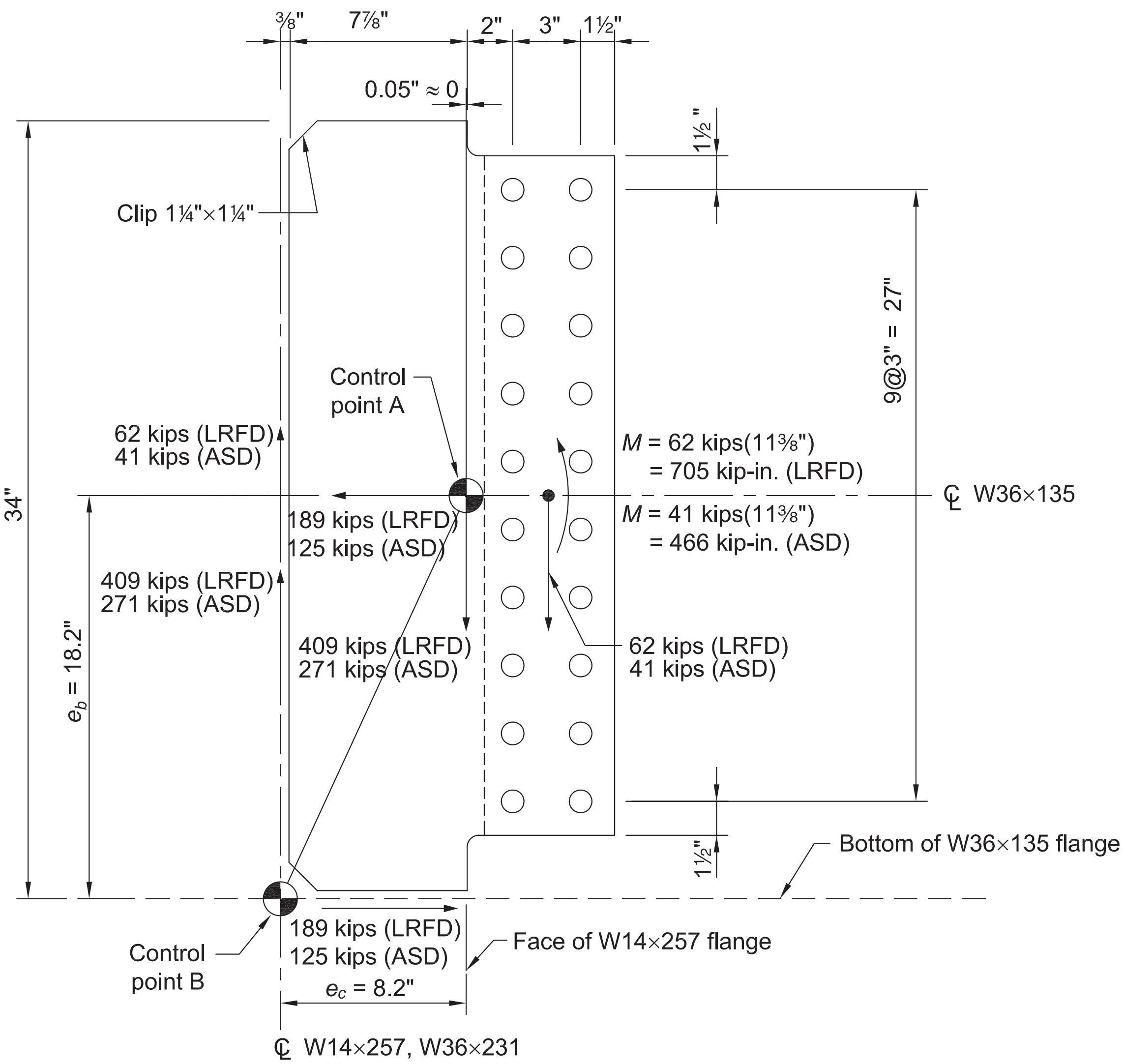


Fig. 5-28. Tab Plate C admissible force field—transfer of shear force only.

LRFD	ASD
The resultant load angle is: $\theta = \tan^{-1}\left(\frac{189 \text{ kips}}{471 \text{ kips}}\right)$ $= 21.9°$ The effective eccentricity is: $e_x = \frac{2{,}140 \text{ kip-in.}}{471 \text{ kips}}$ $= 4.54 \text{ in.}$ From AISC *Manual* Table 7-7 with Angle = 15°, $n = 10$, $e_x = 4.54$ in., and $s = 3$ in.: $C = 17.0$ $\phi R_n = C\phi r_n$ $= 17.0(30.7 \text{ kips/bolt})$ $= 522 \text{ kips} > 508 \text{ kips}$ **o.k.**	The resultant load angle is: $\theta = \tan^{-1}\left(\frac{125 \text{ kips}}{312 \text{ kips}}\right)$ $= 21.8°$ The effective eccentricity is: $e_x = \frac{1{,}420 \text{ kip-in.}}{312 \text{ kips}}$ $= 4.55 \text{ in.}$ From AISC *Manual* Table 7-7 with Angle = 15°, $n = 10$, $e_x = 4.55$ in., and $s = 3$ in.: $C = 17.0$ $\frac{R_{nv}}{\Omega} = \frac{Cr_{nv}}{\Omega}$ $= 17.0(20.5 \text{ kips/bolt})$ $= 349 \text{ kips} > 336 \text{ kips}$ **o.k.**

Check bolt bearing on the Truss B top chord web and Tab Plate C

A 1-in.-thick plate is used for the initial calculations. Figure 5-24 shows a 1¼-in.-thick plate that is required to satisfy the interaction checks.

The maximum force per bolt is:

LRFD	ASD
$r_u = \frac{508 \text{ kips}}{17.0}$ $= 29.9 \text{ kips/bolt} < 30.7 \text{ kips/bolt}$ **o.k.**	$r_a = \frac{336 \text{ kips}}{17.0}$ $= 19.8 \text{ kips/bolt} < 20.5 \text{ kips/bolt}$ **o.k.**

Note that the eccentricity degrades the strength of the bolts so that the 20 bolts can only support a force equivalent to that which could be supported by 17 directly loaded bolts.

Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(\textit{Spec}. Eq. J3-6a)}$$

For the Truss B top chord web, assume a 1.75-in. end distance and 1 1/16-in.-diameter holes. The clear distance perpendicular to the outer line of bolts to the beam end is:

$$\begin{aligned} l_c &= 1.75 \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.}) \\ &= 1.22 \text{ in.} \end{aligned}$$

The available tearout and bearing strengths of the Truss B top chord web are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.22\text{ in.})(0.600\text{ in.})(65\text{ ksi})$ $= 42.8\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(0.600\text{ in.})(65\text{ ksi})$ $= 70.2\text{ kips/bolt}$ Therefore, $\phi r_n = 42.8$ kips/bolt. The bolt shear strength of 30.7 kips governs over bearing or tearout strength.	$\dfrac{1.2 L_c t F_u}{\Omega} = \dfrac{1.2(1.22\text{ in.})(0.600\text{ in.})(65\text{ ksi})}{2.00}$ $= 28.6\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(0.600\text{ in.})(65\text{ ksi})}{2.00}$ $= 46.8\text{ kips/bolt}$ Therefore, $r_n/\Omega = 28.6$ kips/bolt. The bolt shear strength of 20.5 kips governs over bearing or tearout strength.

Note that because of eccentric loading, the directions of the bolt forces are unknown. The tearout strength calculations for both the top chord web and the tab plate are conservative and assume the worst case, where the bolt force is perpendicular to the edge. Note also that all bolts in the outer bolt line are assumed to have the tearout capacity of the worst bolt. If the direction and magnitude of the force on each bolt is determined, it is acceptable to use a less conservative approach.

Assuming a 1.5-in. end distance and 1¼-in.-diameter holes, the clear distance from the outer bolt line to the Tab Plate C edge is:

$$\begin{aligned} l_c &= 1.50\text{ in.} - 0.5(1\tfrac{1}{4}\text{ in.}) \\ &= 0.875\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 51.2\text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 117\text{ kips}$ Therefore, $\phi r_n = 51.2$ kips/bolt The bolt shear strength of 30.7 kips governs over bearing or tearout strength.	$\dfrac{1.2 L_c t F_u}{\Omega} = \dfrac{1.2(0.875\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 34.1\text{ kips}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 78.0\text{ kips}$ Therefore, $r_n/\Omega = 34.1$ kips/bolt The bolt shear strength of 20.5 kips governs over bearing or tearout strength.

Note that spacing should also be checked for tearout between bolts, but this seldom controls. For example:

$$\begin{aligned} l_c &= 3.00\text{ in.} - 1(1\tfrac{1}{4}\text{ in.}) \\ &= 1.75\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.75\text{ in.})(1.00\text{ in.})(65\text{ ksi})$ $= 102\text{ kips}$	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.75\text{ in.})(1.00\text{ in.})(65\text{ ksi})}{2.00}$ $= 68.3\text{ kips}$

Again, bolt shear strength of 30.7 kips for LRFD and 20.5 kips for ASD controls over tearout strength; therefore, the effective strength of the bolt group is controlled by bolt shear strength, which was previously checked.

Check gross and net sections of Tab Plate C at Section a-a

The forces at Section a-a are:

LRFD	ASD
$V_u\ (shear) = 471$ kips $N_u\ (normal) = 189$ kips	$V_a\ (shear) = 312$ kips $N_a\ (normal) = 125$ kips

The moment at the centroid of the bolt group is 2,130 kip-in. (LRFD) and 1,420 kip-in. (ASD). The portion of this moment due to the beam shear [62 kips (LRFD) and 41 kips (ASD)] at the edge of Tab Plate C attached to the web of the Truss D top chord (W36×231) is zero. At the first line of bolts, the moment, due to the offset control point A, is reduced from the moment at the bolt centroid because of its closer proximity to control point A. Therefore:

LRFD	ASD
$M_u = (62.0\text{ kips})(11.375\text{ in.} - 1.50\text{ in.}) + (409\text{ kips})(2.00\text{ in.})$ $= 1,430\text{ kip-in.}$	$M_a = (41.0\text{ kips})(11.375\text{ in.} - 1.50\text{ in.}) + (271\text{ kips})(2.00\text{ in.})$ $= 947\text{ kip-in.}$

An interaction equation that includes shear, normal and flexural stresses is the fourth order interaction equation proposed by Neal (1977) and derived based on plastic theory:

LRFD	ASD
$\dfrac{M_u}{\phi M_n} + \left(\dfrac{N_u}{\phi N_n}\right)^2 + \left(\dfrac{V_u}{\phi V_n}\right)^4 \le 1.0$	$\dfrac{\Omega M_a}{M_n} + \left(\dfrac{\Omega N_a}{N_n}\right)^2 + \left(\dfrac{\Omega V_a}{V_n}\right)^4 \le 1.0$

where M_n, N_n and V_n, as well as ϕ and Ω, vary depending on whether yield or rupture is being considered.

The gross and net areas are:

$$\begin{aligned} A_g &= (1.00\text{ in.})(30.0\text{ in.}) \\ &= 30.0\text{ in.}^2 \\ A_n &= (1.00\text{ in.})\left[30.0\text{ in.} - 10\left(1\tfrac{1}{4}\text{ in.} + 1\tfrac{1}{16}\text{ in.}\right)\right] \\ &= 16.9\text{ in.}^2 \end{aligned}$$

The plate gross plastic section modulus is:

$$\begin{aligned} Z_g &= \tfrac{1}{4}(1.00\text{ in.})(30.0\text{ in.})^2 \\ &= 225\text{ in.}^3 \end{aligned}$$

The plate net plastic section modulus is:

$$Z_n = \tfrac{1}{4}t\left(s - d_h'\right)\left(n^2 s + d_h'\right) \qquad (\textit{Manual}\text{ Eq. 15-4})$$

$$\begin{aligned} &= \tfrac{1}{4}(1.00\text{ in.})\left[3.00\text{ in.} - \left(1\tfrac{1}{4}\text{ in.} + 1\tfrac{1}{16}\text{ in.}\right)\right]\left[\left(10^2\right)(3.00\text{ in.}) + \left(1\tfrac{1}{4}\text{ in.} + 1\tfrac{1}{16}\text{ in.}\right)\right] \\ &= 127\text{ in.}^3 \end{aligned}$$

The available shear yielding strength of Tab Plate C is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1 as follows:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(30.0\text{ in.}^2)$ $= 900\text{ kips} > 468\text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(50\text{ ksi})(30.0\text{ in.}^2)$ $= 1{,}350\text{ kips} > 188\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50\text{ ksi})(30.0\text{ in.}^2)}{1.50}$ $= 600\text{ kips} > 312\text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50\text{ ksi})(30.0\text{ in.}^2)}{1.67}$ $= 898\text{ kips} > 125\text{ kips}$ **o.k.**

The available shear rupture strength of Tab Plate C is determined from AISC *Specification* Equation J4-4, and the available tensile rupture strength is determined from AISC *Specification* Equation J4-2.

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(65\text{ ksi})(16.9\text{ in.}^2)$ $= 494\text{ kips} > 468\text{ kips}$ **o.k.** $\phi R_n = \phi F_u A_e$ $= 0.75(65\text{ ksi})(16.9\text{ in.}^2)$ $= 824\text{ kips} > 188\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(65\text{ ksi})(16.9\text{ in.}^2)}{2.00}$ $= 330\text{ kips} > 312\text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(65\text{ ksi})(16.9\text{ in.}^2)}{2.00}$ $= 549\text{ kips} > 125\text{ kips}$ **o.k.**

The available flexural yield strength of Tab Plate C is determined from AISC *Manual* Equation 15-2, and the available flexural rupture strength is determined from AISC *Manual* Equation 15-3 as follows:

LRFD	ASD
$\phi M_n = 0.90(50\text{ ksi})(225\text{ in.}^3)$ $= 10{,}100\text{ kip-in.} > 1{,}430\text{ kip-in.}$ **o.k.** $\phi M_n = 0.75(65\text{ ksi})(127\text{ in.}^3)$ $= 6{,}190\text{ kip-in.} > 1{,}430\text{ kip-in.}$ **o.k.**	$\dfrac{M_n}{\Omega} = \dfrac{(50\text{ ksi})(225\text{ in.}^3)}{1.67}$ $= 6{,}740\text{ kip-in.} > 947\text{ kip-in.}$ **o.k.** $\dfrac{M_n}{\Omega} = \dfrac{(65\text{ ksi})(127\text{ in.}^3)}{2.00}$ $= 4{,}130\text{ kip-in.} > 947\text{ kip-in.}$ **o.k.**

For yielding:

LRFD	ASD
$\dfrac{1{,}430\text{ kip-in.}}{10{,}100\text{ kip-in.}}+\left(\dfrac{189\text{ kips}}{1{,}350\text{ kips}}\right)^2+\left(\dfrac{471\text{ kips}}{900\text{ kips}}\right)^4=0.236<1.0\quad\textbf{o.k.}$	$\dfrac{947\text{ kip-in.}}{6{,}740\text{ kip-in.}}+\left(\dfrac{125\text{ kips}}{898\text{ kips}}\right)^2+\left(\dfrac{312\text{ kips}}{600\text{ kips}}\right)^4=0.233<1.0\quad\textbf{o.k.}$

For rupture:

LRFD	ASD
$\dfrac{1{,}430\text{ kip-in.}}{6{,}190\text{ kip-in.}}+\left(\dfrac{189\text{ kips}}{824\text{ kips}}\right)^2+\left(\dfrac{471\text{ kips}}{494\text{ kips}}\right)^4=1.11>1.0\quad\textbf{n.g.}$	$\dfrac{947\text{ kip-in.}}{4{,}130\text{ kip-in.}}+\left(\dfrac{125\text{ kips}}{549\text{ kips}}\right)^2+\left(\dfrac{312\text{ kips}}{330\text{ kips}}\right)^4=1.08>1.0\quad\textbf{n.g.}$

Plate C must be increased to 1⅛ in., which will give an interaction value of 0.762 < 1.0. Most fabricators will use a 1¼-in.-thick plate. Figure 5-24 shows Plate C as a 1¼ in. plate.

Check Truss B top chord web shear

The W36×135 web has $h/t_w = 54.1$, which exceeds the AISC *Specification* Section G2.1(a) limit of:

$$\begin{aligned}\frac{h}{t_w} &= 2.24\sqrt{\frac{E}{F_y}}\\ &= 53.9\end{aligned}$$

Therefore, from AISC *Specification* Section G2.1(b), with $k_v = 5.0$:

$$\begin{aligned}\frac{h}{t_w} &= 54.1 < 1.10\sqrt{\frac{5(29{,}000\text{ ksi})}{50\text{ ksi}}}\\ &= 59.2\end{aligned}$$

Therefore, $C_v = 1.0$, $\phi = 0.90$ and $\Omega = 1.67$.

The available shear strength is determined from AISC *Specification* Equation G2-1:

LRFD	ASD
$\begin{aligned}\phi V_n &= \phi 0.6 F_y A_w C_v\\ &= 0.90(0.6)(50\text{ ksi})(35.6\text{ in.})(0.600\text{ in.})(1.0)\\ &= 577\text{ kips} > 471\text{ kips}\quad\textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{V_n}{\Omega} &= \frac{0.6F_yA_wC_v}{\Omega}\\ &= \frac{0.6(50\text{ ksi})(35.6\text{ in.})(0.600\text{ in.})(1.0)}{1.67}\\ &= 384\text{ kips} > 312\text{ kips}\quad\textbf{o.k.}\end{aligned}$

Design welds of Tab Plate C to Truss D top chord

Figure 5-28 shows that the resultant of the brace induced forces, 189 kips and 409 kips (LRFD) and 125 kips and 271 kips (ASD), acts through the control point B at the center of the W14×257 and W36×231 at the elevation of the bottom flange of the W36×135 secondary truss chord. This means that the vertical component of 409 kips (LRFD) and 271 kips (ASD) can be carried by the Tab Plate C weld to the W36×231 web, and the horizontal component of 189 kips (LRFD) and 125 kips (ASD) can be carried by the Tab Plate C weld to the inside of the W36×231 bottom flange. This would be a correct solution (an admissible force field) if the load were to remain in the W36×231 primary truss chord. In this case, however, the load needs to be delivered to the primary truss vertical W14×257 member. It is best in situations like this to consider where the load needs to be delivered,

and then determine a force field that will accomplish this. With this approach, consider Figure 5-29. The forces acting on Tab Plate C need to be delivered to the chord/truss vertical interface. In order to do this a Stiffener D is introduced. The forces on Tab Plate C and the Stiffener D are shown in Figure 5-30. Point P for each plate is sufficiently close to the control point B shown in Figure 5-29 to assume that they are the same. For the forces shown in Figure 5-30 to be admissible (i.e., satisfy equilibrium) a couple, *M*, must be introduced. Taking moments about point P for either plate:

LRFD	ASD
$M_u = (94.5\text{ kips})(17.0\text{ in.}) - (205\text{ kips} + 31.0\text{ kips})(4\tfrac{9}{16}\text{ in.})$ $= 530\text{ kip-in.}$	$M_a = (62.5\text{ kips})(17\text{ in.}) - (136\text{ kips} + 20.5\text{ kips})(4\tfrac{9}{16}\text{ in.})$ $= 348\text{ kip-in.}$

This moment can be applied as shown in Figure 5-30, or it can be converted into horizontal forces at the top and bottom of the plates as H = 530 kip-in./34 in. = 15.6 kips (LRFD) or H = 348 kip-in./34 in. = 10.2 kips (ASD). The force field shown in Figure 5-30 will be used here.

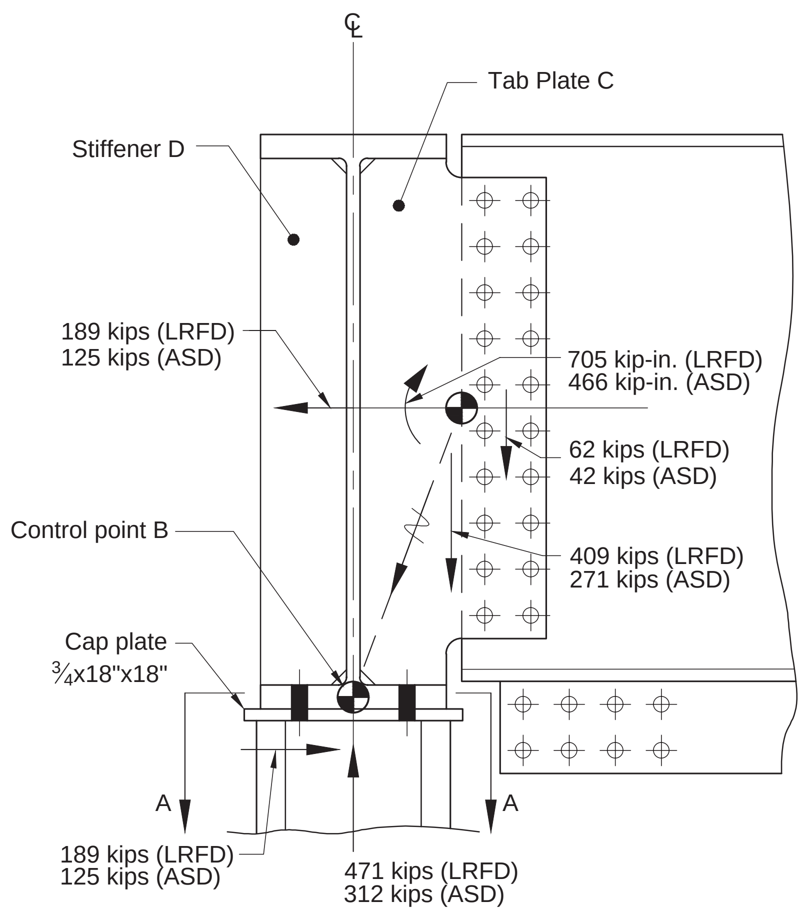


Fig. 5-29. Delivery point of Tab Plate C forces.

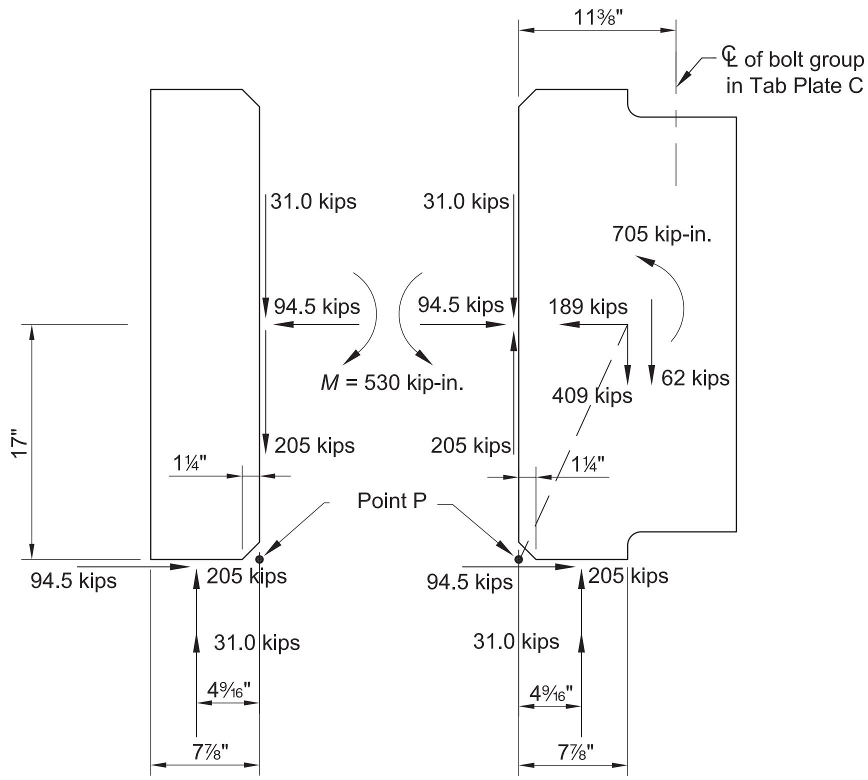


Fig. 5-30a. Admissible force fields for Tab Plate C and Stiffener D to deliver load to chord and vertical interface—LRFD

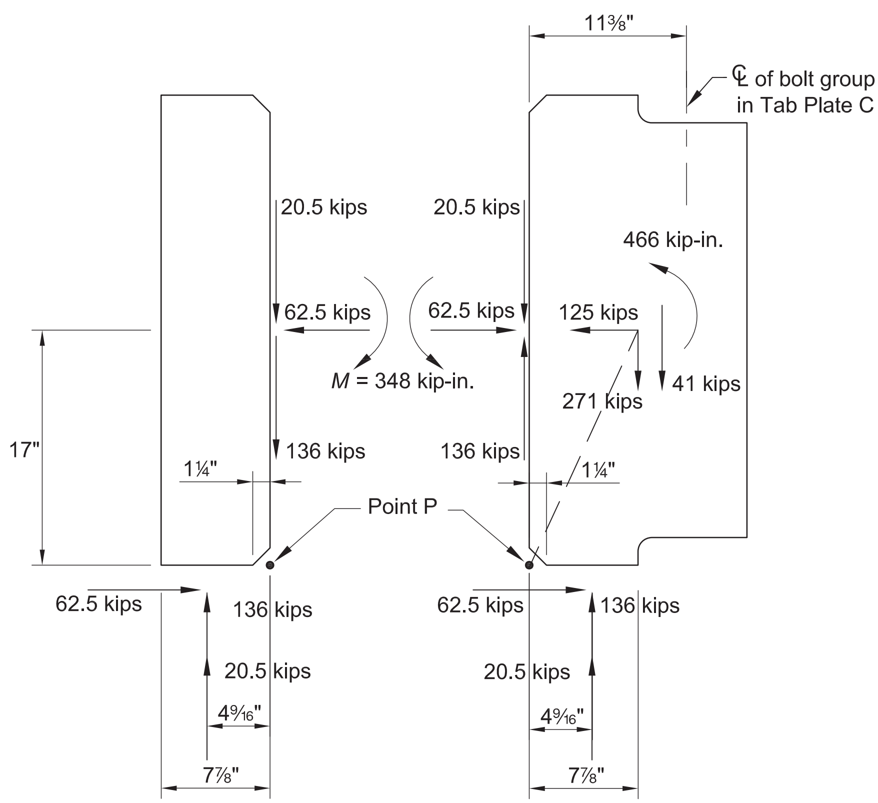


Fig. 5-30b. Admissible force fields for Tab Plate C and Stiffener D to deliver load to chord and vertical interface—ASD

Design the weld of Tab Plate C to the W36×231 (primary truss chord) web. Following Appendix B and the loading given in Figure 5-30:

LRFD	ASD
$N_u = 94.5$ kips $V_u = 205\text{ kips} + 31.0\text{ kips} = 236$ kips $M_u = 530$ kip-in. $l = 34.0\text{ in.} - 2(1\tfrac{1}{4}\text{ in.}) = 31.5$ in. From Appendix B, Equation B-1, the equivalent normal force is: $N_{ue} = N_u + \dfrac{4M}{l} = 94.5\text{ kips} + \dfrac{4(530\text{ kip-in.})}{31.5\text{ in.}} = 162$ kips $R_u = \sqrt{(235\text{ kips})^2 + (162\text{ kips})^2} = 285$ kips $\theta = \tan^{-1}\left(\dfrac{162\text{ kips}}{235\text{ kips}}\right) = 34.6°$ The required weld size is determined using the directional strength increase from AISC *Specification* Section J2.4, and the simplified equation given in AISC *Manual* Part 8 as Equation 8-2 : $D_{req} = \dfrac{R_u}{2l(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)} = \dfrac{284\text{ kips}}{2(31.5\text{ in.})(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5}34.6°)} = 2.67$ sixteenths	$N_a = 62.5$ kips $V_a = 136\text{ kips} + 20.5\text{ kips} = 157$ kips $M_a = 348$ kip-in. $l = 34.0\text{ in.} - 2(1\tfrac{1}{4}\text{ in.}) = 31.5$ in. From Appendix B, Equation B-1, the equivalent normal force is: $N_{ae} = N_a + \dfrac{4M}{l} = 62.5\text{ kips} + \dfrac{4(348\text{ kip-in.})}{31.5\text{ in.}} = 107$ kips $R_a = \sqrt{(157\text{ kips})^2 + (107\text{ kips})^2} = 190$ kips $\theta = \tan^{-1}\left(\dfrac{107\text{ kips}}{157\text{ kips}}\right) = 34.3°$ The required weld size is determined using the directional strength increase from AISC *Specification* Section J2.4, and the simplified equation given in AISC *Manual* Part 8 as Equation 8-2 : $D_{req} = \dfrac{R_a}{2l(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)} = \dfrac{190\text{ kips}}{2(31.5\text{ in.})(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5}34.3°)} = 2.68$ sixteenths

Although a 3⁄16-in. fillet weld is indicated, use the minimum 5⁄16-in. fillet weld required for this connection according to AISC *Specification* Table J2.4. The stresses at the weld in the plate could be checked as was done in previous examples, or by using the conservative formula, $t_{min} = 6.19D/F_u$.

Taking the latter approach, check the rupture strength of the tab plate at the weld using AISC *Manual* Equation 9-3:

LRFD	ASD
$t_{min} = \dfrac{6.19D}{F_u} = \dfrac{6.19(2.67\text{ sixteenths})}{65\text{ ksi}} = 0.254\text{ in.} < 1.00\text{ in.}$ **o.k.**	$t_{min} = \dfrac{6.19D}{F_u} = \dfrac{6.19(2.68\text{ sixteenths})}{65\text{ ksi}} = 0.255\text{ in.} < 1.00\text{ in.}$ **o.k.**

Design the weld of Tab Plate C to the W36×231 (primary truss chord) flange. The resultant load and load angle on the weld is:

LRFD	ASD
$R = \sqrt{(94.5\text{ kips})^2 + (205\text{ kips} + 31.0\text{ kips})^2}$ $= 254\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{236\text{ kips}}{94.5\text{ kips}}\right)$ $= 68.2°$	$R = \sqrt{(62.5\text{ kips})^2 + (136\text{ kips} + 20.5\text{ kips})^2}$ $= 169\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{157\text{ kips}}{62.5\text{ kips}}\right)$ $= 68.3°$

Length of weld, $l = 7\frac{7}{8}\text{ in.} - 1\frac{1}{4}\text{ in.} = 6.625\text{ in.}$

From AISC *Manual* Table 8-4 with Angle = 60°:

$$a = k = 0$$
$$C = 5.21$$
$$C_1 = 1.0$$

LRFD	ASD
$D_{min} = \dfrac{254\text{ kips}}{0.75(5.21)(1.0)(6.625\text{ in.})}$ $= 9.81\text{ sixteenths}$	$D_{min} = \dfrac{2.00(169\text{ kips})}{5.21(1.0)(6.625\text{ in.})}$ $= 9.79\text{ sixteenths}$

Use ⅝-in. fillet welds.

This weld could be reduced if the Tab Plate C were "finished to bear" on the inside flange of the W36, as the normal force can then be ignored. This is not the usual practice because of mill tolerances and shop operations. If the plate were finished to bear, the fillet weld size required could be calculated from AISC *Manual* Equation 8-2:

LRFD	ASD
$D = \dfrac{94.5\text{ kips}}{(1.392\text{ kip/in.})(6.625\text{ in.})(2)}$ $= 5.12\text{ sixteenths}$	$D = \dfrac{62.5\text{ kips}}{(0.928\text{ kip/in.})(6.625\text{ in.})(2)}$ $= 5.08\text{ sixteenths}$

Therefore, a ⅜-in. fillet weld would be required in this case.

Check the rupture strength of the plate at the weld using AISC *Manual* Equation 9-3, and assume the plate is not finished to bear:

LRFD	ASD
$t_{min} = \dfrac{6.19D_{min}}{F_u}$ $= \dfrac{6.19(9.81\text{ sixteenths})}{65\text{ ksi}}$ $= 0.934\text{ in.} < 1.00\text{ in.}$ **o.k.**	$t_{min} = \dfrac{6.19D_{min}}{F_u}$ $= \dfrac{6.19(9.79\text{ sixteenths})}{65\text{ ksi}}$ $= 0.932\text{ in.} < 1.00\text{ in.}$ **o.k.**

Therefore, by inspection, the plate is adequate for either size of weld.

Design weld of Stiffener D to the W36×231 web and flange

The loads are the same as for Tab Plate C, so likewise the welds will be the same. Also, use 1-in.-thick plate for this stiffener.

Figure 5-24 shows 5⁄16-in. fillet welds connecting Tab Plate C and Stiffener D to the inside of the W36×231 top flange. These welds are not required for strength, but are used to control distortion during fabrication, to provide stability to the plates, and for aesthetics.

Connection of Primary Truss D Top Chord (W36×231) to Primary Truss D Vertical Web Member (W14×257)

This connection is shown in Figure 5-29. The forces are 471 kips (LRFD) and 312 kips (ASD) axial compression and 189 kips (LRFD) and 125 kips (ASD) shear. The cap plate to W14 connection will be "finished to bear." The axial compressive force will be carried in bearing to the W14×257 web. All of the force goes to the web because Tab Plate C and Stiffener D are aligned with the W14 web.

Check the bearing strength of the W14×257 web

A_{pb} is the projected bearing area, which is the W14×257 web area in this case. Therefore:

$$\begin{aligned} A_{pb} &= (16.4 \text{ in.})(1.18 \text{ in.}) \\ &= 19.4 \text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section J7:

LRFD	ASD
$\begin{aligned} \phi R_n &= \phi 1.8 F_y A_{pb} \\ &= 0.75(1.8)(50 \text{ ksi})(19.4 \text{ in.}^2) \\ &= 1{,}310 \text{ kips} > 471 \text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{1.8 F_y A_{pb}}{\Omega} \\ &= \frac{1.8(50 \text{ ksi})(19.4 \text{ in.}^2)}{2.00} \\ &= 873 \text{ kips} > 312 \text{ kips} \quad \textbf{o.k.} \end{aligned}$

Design weld of cap plate to W14×257

The weld to the W14 web, plus weld to the inside and outside of the W14 flanges up to the k_1 distance, will be effective in resisting the shear load of 189 kips (LRFD) and 125 kips (ASD). Therefore, the effective length of weld is:

$$\begin{aligned} l_{eff} &= 2(d - 2t_f) + 4k_1 + 4\left(k_1 - \frac{t_w}{2}\right) \\ &= 2[16.4 \text{ in.} - 2(1.89 \text{ in.})] + 4(1^{13}\!/\!_{16} \text{ in.}) + 4\left(1^{13}\!/\!_{16} \text{ in.} - \frac{1.18 \text{ in.}}{2}\right) \\ &= 37.4 \text{ in.} \end{aligned}$$

LRFD	ASD
$\begin{aligned} D_{min} &= \frac{189 \text{ kips}}{(1.392 \text{ kip/in.})(37.4 \text{ in.})} \\ &= 3.63 \text{ sixteenths} \end{aligned}$	$\begin{aligned} D_{min} &= \frac{125 \text{ kips}}{(0.928 \text{ kip/in.})(37.4 \text{ in.})} \\ &= 3.60 \text{ sixteenths} \end{aligned}$

Use the minimum fillet weld required by AISC *Manual* Table J2.4 of 5⁄16-in. as shown in Figure 5-24.

Check bolt shear strength

The bolts carry the shear load into the cap plate from the W36×231 flange. Using the bolt shear value for ASTM A490 1-in.-diameter bolts in slip-critical connections, Class B, in oversized holes, single shear, the number of bolts required is:

LRFD	ASD
$N_b = \dfrac{189 \text{ kips}}{30.7 \text{ kips/bolt}}$ $= 6.16 \text{ bolts}$	$N_b = \dfrac{125 \text{ kips}}{20.5 \text{ kips/bolt}}$ $= 6.10 \text{ bolts}$

Use eight bolts.

Check bolt bearing on the cap plate

Try a ¾-in.-thick cap plate. Check the bearing strength using AISC *Specification* Equation J3-6a, where the right side of the equation controls by inspection:

LRFD	ASD
$\phi r_n = \phi 2.4dtF_u$ $= 0.75(2.4)(1.00 \text{ in.})(\tfrac{3}{4} \text{ in.})(65 \text{ ksi})$ $= 87.8 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = \dfrac{2.4dtF_u}{\Omega}$ $= \dfrac{2.4(1.00 \text{ in.})(\tfrac{3}{4} \text{ in.})(65 \text{ ksi})}{2.00}$ $= 58.5 \text{ kips/bolt}$

Bolt shear strength controls the effective strength of the bolt group, which was checked previously. Figure 5-31 shows the bolt pattern.

This completes the design of this truss-to-truss connection. At every step of the way, admissible force fields were determined. The final design is, by the lower bound theorem of limit analysis, a safe design.

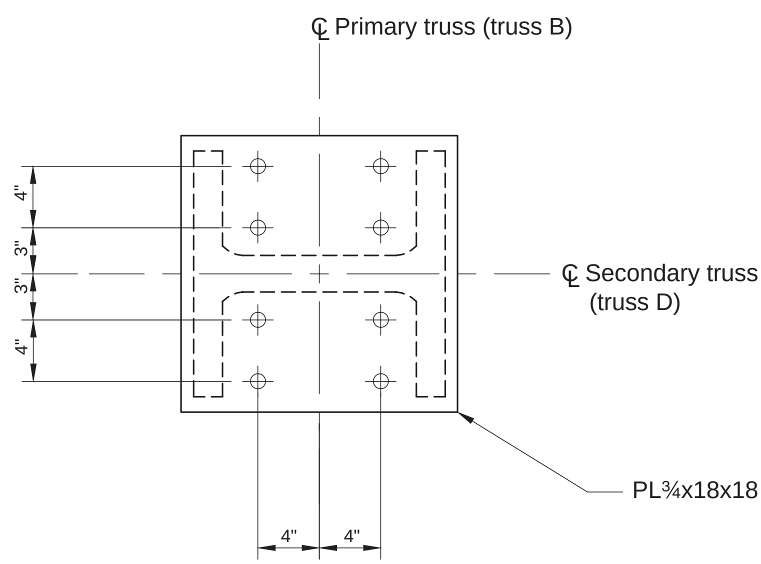


Fig 5-31. Cap plate bolt pattern.

Example 5.12—Brace-to-Column Base Plate Connection

Example 5.12.1—Strong-Axis Case

Given:

The theory for this case is given in Section 4.3 and Figure 4-22. Figure 5-32 shows a completed connection design, the calculations for which follow. The brace force given in Figure 5-32 is 180 kips (LRFD) and 120 kips (ASD). Use ⅞-in.-diameter ASTM A325-N bolts in standard holes and 70-ksi electrodes.

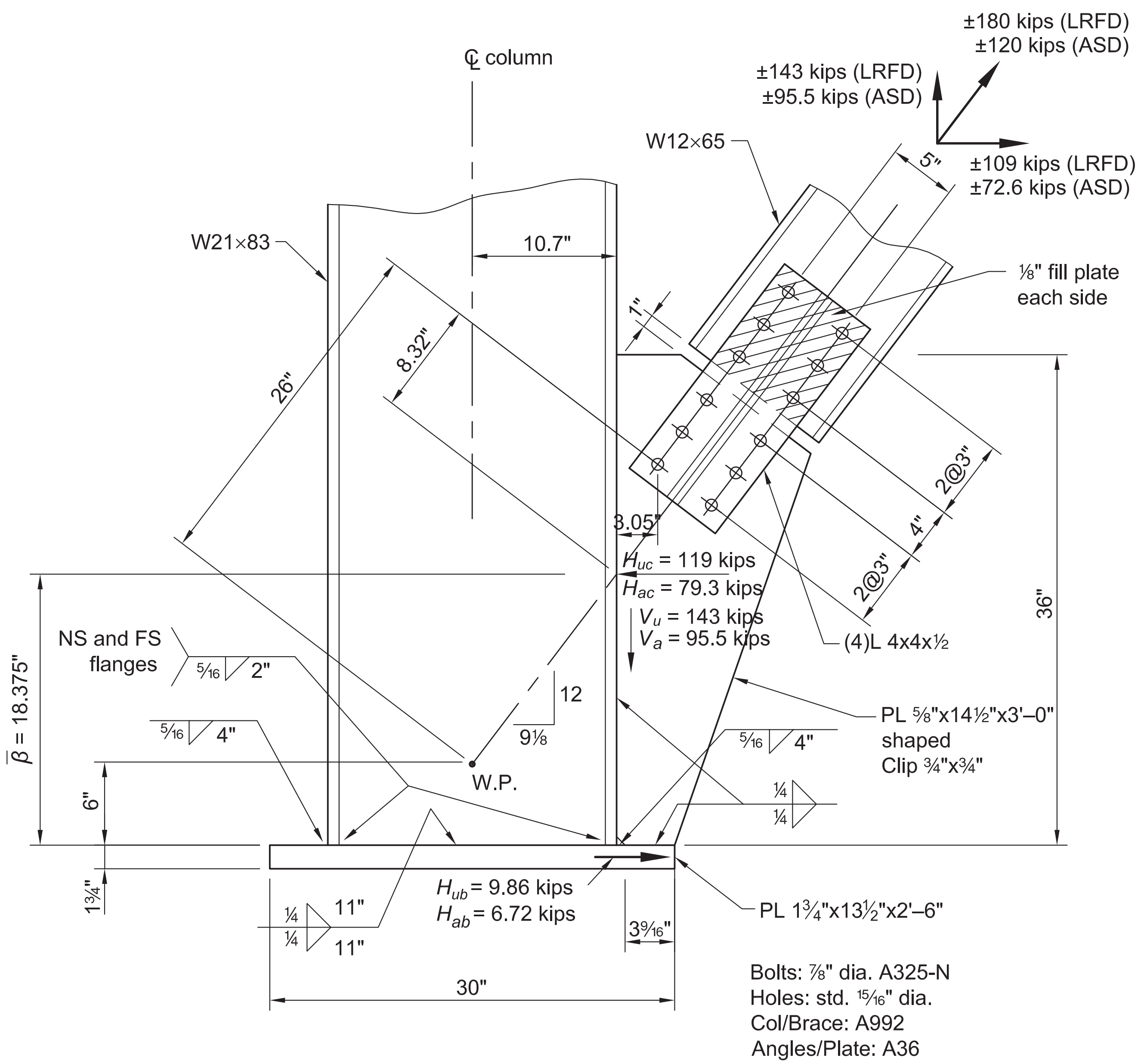


Fig. 5-32. Brace-to-column base plate, strong axis.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

From AISC *Manual* Table 1-1, the member properties are as follows:

Column
W21×83
$d = 21.4$ in. $\quad t_w = 0.515$ in. $\quad b_f = 8.36$ in. $\quad t_f = 0.835$ in. $\quad k = 1.34$ in.

Brace
W12×65
$A_g = 19.1$ in.^{2} $\quad d = 12.1$ in. $\quad t_w = 0.390$ in. $\quad b_f = 12.0$ in. $\quad t_f = 0.605$ in.

Angles
L4×4×½
$t = ½$ in. $\quad A_g = 3.75$ in.^{2} $\quad \bar{x} = 1.18$ in.

From AISC *Manual* Table 7-1 the available shear strength of the ASTM A325-N ⅞-in.-diameter bolts in standard holes are:

LRFD	ASD
$\phi r_{nv} = 24.3$ kips/bolt single shear $\phi r_{nv} = 48.7$ kips/bolt double shear	$\dfrac{r_{nv}}{\Omega} = 16.2$ kips/bolt single shear $\dfrac{r_{nv}}{\Omega} = 32.5$ kips/bolt double shear

Brace-to–Gusset Connection

As noted in the previous example, this part of the connection should be designed first in order to obtain a gusset plate size.

Check tension yielding on the brace

From AISC *Specification* Section J4.1, the available tension yielding strength is:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(50\text{ ksi})(19.1\text{ in.}^2)$ $= 860\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(50\text{ ksi})(19.1\text{ in.}^2)}{1.67}$ $= 572\text{ kips} > 120\text{ kips}$ **o.k.**

It may seem that the brace is oversized, but remember that the brace force can be compression, and this will control the brace size.

Check tensile rupture on the brace

From AISC *Specification* Section J4.1, the available tensile rupture strength is determined as follows. The effective net area is determined from AISC *Specification* Section D3, using the hole diameter as given in AISC *Specification* Table J3.3 of 15⁄16 in.:

$$A_n = 19.1\text{ in.}^2 - 2\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(0.390\text{ in.})$$
$$= 18.3\text{ in.}^2$$

$$A_e = A_n U \qquad \text{(}Spec.\text{ Eq. D3-1)}$$

The value of U is obtained from AISC *Specification* Table D3.1, Case 2, by considering that the W12×65 can be assumed to be two channels back-to-back with:

$$t_w = \frac{0.390 \text{ in.}}{2}$$
$$= 0.195 \text{ in.}$$
$$b_f = \frac{12.0 \text{ in.}}{2}$$
$$= 6.00 \text{ in.}$$
$$t_f = 0.605 \text{ in.}$$

Then determine the distance to the center of gravity for the channel section, $\bar{x}$:

$$\bar{x} = \frac{\sum A_i x_i}{A}$$

where

$$A = (6.00 \text{ in.})(0.605 \text{ in.})(2) + (0.195 \text{ in.})[12.1 \text{ in.} - 2(0.605 \text{ in.})]$$
$$= 7.26 \text{ in.}^2 + 2.12 \text{ in.}^2$$
$$= 9.38 \text{ in.}^2$$
$$A_1 = \text{area of flanges}$$
$$= 7.26 \text{ in.}^2$$
$$x_1 = 6.00 \text{ in.}/2$$
$$= 3.00 \text{ in.}$$
$$A_2 = \text{area of web}$$
$$= 2.12 \text{ in.}^2$$
$$x_2 = 0.195 \text{ in.}/2$$
$$= 0.0975 \text{ in.}$$
$$\bar{x} = \frac{(7.26 \text{ in.}^2)(3.00 \text{ in.}) + (2.12 \text{ in.}^2)(0.0975 \text{ in.})}{9.38 \text{ in.}^2}$$
$$= 2.34 \text{ in.}$$
$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{2.34 \text{ in.}}{6.00 \text{ in.}}$$
$$= 0.610$$

$$A_e = A_n U$$
$$= (18.3 \text{ in.}^2)(0.610)$$
$$= 11.2 \text{ in.}^2$$

The available tensile rupture strength is:

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75(65 \text{ ksi})(11.2 \text{ in.}^2)$ $= 546 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(65 \text{ ksi})(11.2 \text{ in.}^2)}{2.00}$ $= 364 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check bolt bearing on the brace

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad (Spec. \text{ Eq. J3-6a})$$

For the inner bolts, the clear distance is:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - 1.0 d_h \\ &= 3.00 \text{ in.} - 1.0(\tfrac{15}{16} \text{ in.}) \\ &= 2.06 \text{ in.} \end{aligned}$$

The available tearout and bearing strengths are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06 \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})$ $= 47.0 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8} \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})$ $= 39.9 \text{ kips/bolt}$ Therefore, $\phi r_n = 39.9$ kips/bolt.	$1.2 l_c t F_u / \Omega = 1.2(2.06 \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})/2.00$ $= 31.3 \text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8} \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 26.6 \text{ kips/bolt}$ Therefore, $r_n/\Omega = 26.6$ kips/bolt.

Because the available bolt shear strength for double shear determined previously (48.7 kips for LRFD and 32.5 kips for ASD) is greater than the bearing strength, the limit state of bearing controls the strength of the inner bolts.

For the outer bolts:

$$\begin{aligned} l_c &= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.}) \\ &= 1.03 \text{ in.} \end{aligned}$$

The available tearout and bearing strengths are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.03 \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})$ $= 23.5 \text{ kips/bolt}$	$1.2 l_c t F_u / \Omega = 1.2(1.03 \text{ in.})(0.390 \text{ in.})(65 \text{ ksi})/2.00$ $= 15.7 \text{ kips/bolt}$

LRFD	ASD
$\phi 2.4dtF_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(0.390\text{ in.})(65\text{ ksi})$ $= 39.9\text{ kips/bolt}$ Therefore, $\phi r_n = 23.5$ kips/bolt.	$\dfrac{2.4dtF_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8}\text{ in.})(0.390\text{ in.})(65\text{ ksi})}{2.00}$ $= 26.6\text{ kips/bolt}$ Therefore, $r_n/\Omega = 15.7$ kips/bolt.

Because the available bolt shear strength for double shear determined previously (48.7 kips for LRFD and 32.5 kips for ASD) is greater than the tearout strength, the limit state of tearout controls the strength of the outer bolts.

The effective strength of the bolt group based on bearing, tearout and shear strength is:

LRFD	ASD
$\phi R_n = 2(23.5\text{ kips/bolt}) + 4(39.9\text{ kips/bolt})$ $= 207\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2(15.7\text{ kips}) + 4(26.6\text{ kips})$ $= 138\text{ kips} > 120\text{ kips}$ **o.k.**

Bearing and tearout on the connection angles will be checked subsequently.

Check block shear rupture on the brace

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (\textit{Spec}.\ \text{Eq. J4-5})$$

Assume the controlling failure path forms a rectangle bounded by the two bolt lines on the brace web.

Shear yielding component:

$$A_{gv} = 2(7.50\text{ in.})(0.390\text{ in.}) = 5.85\text{ in.}^2$$

$$0.60F_yA_{gv} = 0.60(50\text{ ksi})(5.85\text{ in.}^2) = 176\text{ kips}$$

Shear rupture component:

$$A_{nv} = 5.85\text{ in.}^2 - 2.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.390\text{ in.})(2) = 3.90\text{ in.}^2$$

$$0.60F_uA_{nv} = 0.60(65\text{ ksi})(3.90\text{ in.}^2) = 152\text{ kips}$$

Tension rupture component:

$$A_{nt} = [5.00\text{ in.} - (0.5 + 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})](0.390\text{ in.}) = 1.56\text{ in.}^2$$

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$U_{bs}F_uA_{nt} = 1(65\text{ ksi})(1.56\text{ in.}^2) = 101\text{ kips}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 152 \text{ kips} + 101 \text{ kips}$$
$$= 253 \text{ kips}$$
$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 176 \text{ kips} + 101 \text{ kips}$$
$$= 277 \text{ kips}$$

Therefore, $R_n = 253$ kips.

LRFD	ASD
$\phi R_n = 0.75(253 \text{ kips})$ $= 190 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{253 \text{ kips}}{2.00}$ $= 127 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check tensile yielding on the connection angles

The gross area of each angle is 3.75 in.^{2} From AISC *Specification* Section J4.1, the available tensile yielding strength is:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(3.75 \text{ in.}^2)(4)$ $= 486 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36 \text{ ksi}(3.75 \text{ in.}^2)(4)}{1.67}$ $= 323 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check tensile rupture on connection angles

From AISC *Specification* Section J4.1, the available tensile rupture strength is determined as follows. The effective net area is determined from AISC *Specification* Section D3, using the hole diameter as given in AISC *Specification* Table J3.3 of 15⁄16 in. The net area of one angle is:

$$A_{nt} = 3.75 \text{ in.}^2 - (\tfrac{1}{2} \text{ in.})(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})$$
$$= 3.25 \text{ in.}^2$$

From AISC *Specification* Table D3.1, Case 2:

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{1.18 \text{ in.}}{6.00 \text{ in.}}$$
$$= 0.803$$

From AISC *Specification* Table D3.1, Case 8:

$$U = 0.600$$

The AISC *Specification* allows the larger value determined from Case 2 and Case 8 to be used for angles; therefore, in this example, use the larger value of $U = 0.803$:

$$
\begin{aligned}
A_e &= UA_{nt} \\
&= 0.803\left(3.25\text{ in.}^2\right) \\
&= 2.61\text{ in.}^2
\end{aligned}
$$

The available tensile rupture strength is:

LRFD	ASD
$\phi R_{nt} = \phi F_u A_e$ $= 0.75(58\text{ ksi})\left(2.61\text{ in.}^2\right)(4)$ $= 454\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_{nt}}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(58\text{ ksi})\left(2.61\text{ in.}^2\right)}{2.00}(4)$ $= 303\text{ kips} > 120\text{ kips}$ **o.k.**

Check bolt bearing on the connection angles

For the inner bolts, using AISC *Specification* Equation J3-6a, and $l_c = 2.06$ as determined previously for the brace, the available tearout and bearing strengths are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 53.8\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 45.7\text{ kips/bolt}$ Therefore, $\phi r_n = 45.7$ kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = 1.2(2.06\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})/2.00$ $= 35.8\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00}$ $= 30.5\text{ kips/bolt}$ Therefore, $r_n/\Omega = 30.5$ kips/bolt.

Because the available bolt shear strength for single shear determined previously (24.3 kips for LRFD and 16.2 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the inner bolts.

For the outer bolts, using $l_c = 1.03$ in. determined previously:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.03\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 26.9\text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 45.7\text{ kips/bolt}$ Therefore, $\phi r_n = 26.9$ kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.03\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00}$ $= 17.9\text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00}$ $= 30.5\text{ kips/bolt}$ Therefore, $r_n/\Omega = 17.9$ kips/bolt.

Because the available bolt shear strength for single shear determined previously (24.3 kips for LRFD and 16.2 kips for ASD) is less than the bearing strength, the limit state of bolt shear also controls the strength of the outer bolts. The determination of the effective strength of the bolt group based on bolt shear follows.

Check bolt shear strength

The available bolt shear strength at the brace-to-gusset connection is:

LRFD	ASD
$\phi R_n = (48.7 \text{ kips/bolt})(6 \text{ bolts})$ $= 292 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (32.5 \text{ kips/bolt})(6 \text{ bolts})$ $= 195 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check block shear rupture on the connection angles

Determine the available strength for the limit state of block shear rupture from AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \quad \text{(Spec. Eq. J4-5)}$$

Assume the controlling failure path follows each bolt line and then runs perpendicular to the edge of each angle.

Shear yielding component:

$$\begin{aligned} A_{gv} &= 4(7.50 \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 15.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(36 \text{ ksi})(15.0 \text{ in.}^2) \\ &= 324 \text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= 4\left[3.75 \text{ in.}^2 - 2.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})\right] \\ &= 10.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(58 \text{ ksi})(10.0 \text{ in.}^2) \\ &= 348 \text{ kips} \end{aligned}$$

Tension rupture component:

$$\begin{aligned} A_{nt} &= 4\left\{\left[(8.00 \text{ in.} - 5.00 \text{ in.})/2\right](\tfrac{1}{2} \text{ in.}) - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})\right\} \\ &= 2.00 \text{ in.}^2 \end{aligned}$$

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(58 \text{ ksi})(2.00 \text{ in.}^2) \\ &= 116 \text{ kips} \end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 348 \text{ kips} + 116 \text{ kips} \\ &= 464 \text{ kips} \\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 324 \text{ kips} + 116 \text{ kips} \\ &= 440 \text{ kips} \end{aligned}$$

Therefore, $R_n = 440$ kips.

LRFD	ASD
$\phi R_n = 0.75(440 \text{ kips})$ $= 330 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{440 \text{ kips}}{2.00}$ $= 220 \text{ kips} > 120 \text{ kips}$ **o.k.**

Check bolt bearing on gusset plate

Try a ⅝-in.-thick gusset plate.

Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

For the inner bolts, the clear distance between bolts is:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - \tfrac{15}{16} \text{ in.} \\ &= 2.06 \text{ in.} \end{aligned}$$

The available tearout and bearing strengths are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.06 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$ $= 67.2 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$ $= 57.1 \text{ kips/bolt}$ Therefore, $\phi r_n = 57.1$ kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(2.06 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 44.8 \text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 38.1 \text{ kips/bolt}$ Therefore, $r_n/\Omega = 38.1$ kips/bolt.

The available bolt shear strength of 48.7 kips (LRFD) and 32.5 kips (ASD) controls over bearing strength.

For the outer bolts, the clear end distance is:

$$\begin{aligned} l_c &= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.}) \\ &= 1.03 \text{ in.} \end{aligned}$$

The available tearout and bearing strengths are:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.03 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$ $= 33.6 \text{ kips/bolt}$	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.03 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})}{2.00}$ $= 22.4 \text{ kips}$

LRFD	ASD
$\phi 2.4dtF_u = 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 57.1\text{ kips/bolt}$ Therefore, $\phi r_n = 33.6$ kips/bolt.	$\dfrac{2.4dtF_u}{\Omega} = \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 38.1\text{ kips/bolt}$ Therefore, $r_n/\Omega = 22.4$ kips/bolt.

Note that the tearout strength controls over bolt shear strength for the outer bolts.

The effective strength of the bolt group in the gusset plate is:

LRFD	ASD
$\phi R_n = (33.6\text{ kips/bolt})(2\text{ bolts}) + (48.7\text{ kips/bolt})(4\text{ bolts})$ $= 262\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (22.4\text{ kips/bolt})(2\text{ bolts}) + (32.5\text{ kips/bolt})(4\text{ bolts})$ $= 175\text{ kips} > 120\text{ kips}$ **o.k.**

Check the gusset plate for tension yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore section is:

$$
\begin{aligned}
l_w &= 5.00\text{ in.} + 2(6.00\text{ in.})\tan 30° \\
&= 11.9\text{ in.}
\end{aligned}
$$

The Whitmore section remains in the gusset plate.

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= \phi F_y l_w t_g$ $= 0.90(36\text{ ksi})(11.9\text{ in.})(\tfrac{5}{8}\text{ in.})$ $= 241\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{F_y l_w t_g}{\Omega}$ $= \dfrac{(36\text{ ksi})(11.9\text{ in.})(\tfrac{5}{8}\text{ in.})}{1.67}$ $= 160\text{ kips} > 120\text{ kips}$ **o.k.**

Check the gusset plate for compression buckling on the Whitmore section

From Dowswell (2006), this is an extended gusset, for which $K = 0.60$ is recommended. With a laterally unbraced length of 8.32 in., as given in Figure 5-32, the effective slenderness ratio is:

$$
\begin{aligned}
\frac{KL}{r} &= \frac{0.60(8.32\text{ in.})}{\tfrac{5}{8}\text{ in.}/\sqrt{12}} \\
&= 27.7
\end{aligned}
$$

Because $\dfrac{KL}{r} > 25$, according to AISC *Specification* Section J4.4, Chapter E applies. From AISC *Manual* Table 4-22, $\phi_c F_{cr} = 31.1$ ksi and $F_{cr}/\Omega_c = 20.7$ ksi.

From AISC *Specification* Section E3:

LRFD	ASD
$$\begin{aligned}\phi_c R_n &= \phi_c F_{cr} A_w \\ &= \phi_c F_{cr} l_w t_g \\ &= (31.1\text{ ksi})(11.9\text{ in.})(5/8\text{ in.}) \\ &= 231\text{ kips} > 180\text{ kips} \quad \textbf{o.k.}\end{aligned}$$	$$\begin{aligned}\frac{R_n}{\Omega_c} &= \frac{F_{cr} A_w}{\Omega_c} \\ &= \frac{F_{cr} l_w t_g}{\Omega_c} \\ &= (20.7\text{ ksi})(11.9\text{ in.})(5/8\text{ in.}) \\ &= 154\text{ kips} > 120\text{ kips} \quad \textbf{o.k.}\end{aligned}$$

Check block shear rupture on the gusset plate

Determine the available strength for the limit state of block shear rupture from AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}\text{. Eq. J4-5})$$

Assume the controlling failure path forms a rectangle bounded by the two bolt lines on the gusset plate.

Shear yielding component:

$$\begin{aligned}A_{gv} &= (7.50\text{ in.})(5/8\text{ in.})(2) \\ &= 9.38\text{ in.}^2\end{aligned}$$

$$\begin{aligned}0.60F_y A_{gv} &= 0.60(36\text{ ksi})\left(9.38\text{ in.}^2\right) \\ &= 203\text{ kips}\end{aligned}$$

Shear rupture component:

$$\begin{aligned}A_{nv} &= 9.38\text{ in.}^2 - 2.5(15/16\text{ in.} + 1/16\text{ in.})(5/8\text{ in.})(2) \\ &= 6.26\text{ in.}^2\end{aligned}$$

$$\begin{aligned}0.60F_u A_{nv} &= 0.60(58\text{ ksi})\left(6.26\text{ in.}^2\right) \\ &= 218\text{ kips}\end{aligned}$$

Tension rupture component:

$$\begin{aligned}A_{nt} &= (5.00\text{ in.})(5/8\text{ in.}) - 1.0(15/16\text{ in.} + 1/16\text{ in.})(5/8\text{ in.}) \\ &= 2.50\text{ in.}^2\end{aligned}$$

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned}U_{bs} F_u A_{nt} &= 1(58\text{ ksi})\left(2.50\text{ in.}^2\right) \\ &= 145\text{ kips}\end{aligned}$$

The available strength for the limit state of block shear rupture is determined as follows:

$$\begin{aligned}0.60F_uA_{nv}+U_{bs}F_uA_{nt} &= 218\text{ kips}+145\text{ kips}\\ &= 363\text{ kips}\\ 0.60F_yA_{gv}+U_{bs}F_uA_{nt} &= 203\text{ kips}+145\text{ kips}\\ &= 348\text{ kips}\end{aligned}$$

Therefore, $R_n = 348$ kips.

LRFD	ASD
$\phi R_n = 0.75(348\text{ kips})$ $= 261\text{ kips} > 180\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{348\text{ kips}}{2.00}$ $= 174\text{ kips} > 120\text{ kips}$ **o.k.**

Distribution of Forces

The force distribution is obtained from the formulas of Figure 4-22 and shown in Figure 5-32:

$$\begin{aligned}\bar{\beta} &= \frac{36.0\text{ in.}-3/4\text{ in.}}{2}+3/4\text{ in.}\\ &= 18.375\text{ in.}\\ e &= 6\text{ in.}\\ e_c &= \frac{21.4\text{ in.}}{2}\\ &= 10.7\text{ in.}\end{aligned}$$

LRFD	ASD
$H_u = 109$ kips $V_u = 143$ kips $H_{ub} = \dfrac{H_u(\bar{\beta}-e)-V_ue_c}{\bar{\beta}}$ $= \dfrac{(109\text{ kips})(18.375\text{ in.}-6.00\text{ in.})-(143\text{ kips})(10.7\text{ in.})}{18.375\text{ in.}}$ $= -9.86$ kips $H_{uc} = \dfrac{H_ue+V_ue_c}{\bar{\beta}}$ $= \dfrac{(109\text{ kips})(6.00\text{ in.})+(143\text{ kips})(10.7\text{ in.})}{18.375\text{ in.}}$ $= 119$ kips	$H_a = 72.6$ kips $V_a = 95.5$ kips $H_{ab} = \dfrac{H_a(\bar{\beta}-e)-V_ae_c}{\bar{\beta}}$ $= \dfrac{(72.6\text{ kips})(18.375\text{ in.}-6.00\text{ in.})-(95.5\text{ kips})(10.7\text{ in.})}{18.375\text{ in.}}$ $= -6.72$ kips $H_{ac} = \dfrac{H_ae+V_ae_c}{\bar{\beta}}$ $= \dfrac{(72.6\text{ kips})(6.00\text{ in.})+(95.5\text{ kips})(10.7\text{ in.})}{18.375\text{ in.}}$ $= 79.3$ kips

Gusset-to-Column Connection

Check shear and tensile yielding on the gusset plate along the column flange

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1:

The weld length is:

$$l = 36.0 \text{ in.} - \tfrac{3}{4} \text{ in.}$$
$$= 35.3 \text{ in.}$$

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(\tfrac{5}{8} \text{ in.})(35.3 \text{ in.})$ $= 477 \text{ kips} > 143 \text{ kips}$ **o.k.** $\phi R_n = F_y A_g$ $= 0.90(36 \text{ ksi})(\tfrac{5}{8} \text{ in.})(35.3 \text{ in.})$ $= 715 \text{ kips} > 119 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(\tfrac{5}{8} \text{ in.})(35.3 \text{ in.})}{1.50}$ $= 318 \text{ kips} > 95.5 \text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(36 \text{ ksi})(\tfrac{5}{8} \text{ in.})(35.3 \text{ in.})}{1.67}$ $= 476 \text{ kips} > 79.3 \text{ kips}$ **o.k.**

Design weld for combined forces at gusset-to-column flange connection

As discussed previously in Example 5.1, and in AISC *Manual* Part 13, to allow for redistribution of force in welded gusset connections, the weld is designed for an increased strength of 1.25R. The strength of the fillet weld can be determined from AISC *Manual* Equation 8-2, incorporating the increased strength permitted from AISC *Specification* Equation J2-5, as follows:

LRFD	ASD
The resultant load is: $R_u = \sqrt{(119 \text{ kips})^2 + (143 \text{ kips})^2}$ $= 186 \text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{119 \text{ kips}}{143 \text{ kips}}\right)$ $= 39.8°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{1.25 R_u}{2(l)(1.392 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5}\theta)}$ $= \dfrac{1.25(186 \text{ kips})}{2(35.3 \text{ in.})(1.392 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5} 39.8°)}$ $= 1.88 \text{ sixteenths}$	The resultant load is: $R_a = \sqrt{(79.3 \text{ kips})^2 + (95.5 \text{ kips})^2}$ $= 124 \text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{79.3 \text{ kips}}{95.5 \text{ kips}}\right)$ $= 39.7°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{1.25 R_a}{2(l)(0.928 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5}\theta)}$ $= \dfrac{1.25(124 \text{ kips})}{2(35.3 \text{ in.})(0.928 \text{ kip/in.})(1.0 + 0.50 \sin^{1.5} 39.7°)}$ $= 1.88 \text{ sixteenths}$

Use the AISC *Specification* Table J2.4 minimum fillet weld size of ¼ in. Note that the 1.25 ductility factor is conservatively applied to the resultant force, which is based on both the peak stress and the average stress in this case. See Hewitt and Thornton (2004).

Check local yielding on the column web

The normal force is applied at a distance, $\bar{\beta}$ = 18.375 in., from the base plate edge, which is less than the column depth. Therefore, use AISC *Specification* Equation J10-3:

LRFD	ASD
$\phi R_n = \phi F_y t_w (2.5k + l_b)$ $= 1.00\left\{\begin{array}{l}50\text{ ksi}(0.515\text{ in.})\\ \times\left[2.5(1.34\text{ in.}) + (36.0\text{ in.} - \tfrac{3}{4}\text{ in.})\right]\end{array}\right\}$ = 994 kips > 119 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w (2.5k + l_b)}{\Omega}$ $= \dfrac{(50\text{ ksi})(0.515\text{ in.})\left[2.5(1.34\text{ in.}) + (36.0\text{ in.} - \tfrac{3}{4}\text{ in.})\right]}{1.50}$ = 663 kips > 79.3 kips **o.k.**

Check local crippling on the column web

The normal force is applied at $\bar{\beta}$ = 18.375 in. from the base plate edge, which is greater than a distance $d/2$, where d is the column depth. Therefore, AISC *Specification* Equation J10-4 is applicable:

LRFD	ASD
$\phi R_n = \phi\left\{0.80t_w^2\left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_y t_f}{t_w}}\right\}$ $= 0.75\left\{\begin{array}{l}0.80(0.515\text{ in.})^2\\ \times\left[1+3\left(\dfrac{(36.0\text{ in.} - \tfrac{3}{4}\text{ in.})}{21.4\text{ in.}}\right)\left(\dfrac{0.515\text{ in.}}{0.835\text{ in.}}\right)^{1.5}\right]\\ \times\sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(0.835\text{ in.})}{0.515\text{ in.}}}\end{array}\right\}$ = 829 kips > 119 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1}{\Omega}\left\{0.80t_w^2\left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_y t_f}{t_w}}\right\}$ $= \dfrac{1}{2.00}\left\{\begin{array}{l}0.80(0.515\text{ in.})^2\\ \times\left[1+3\left(\dfrac{(36.0\text{ in.} - \tfrac{3}{4}\text{ in.})}{21.4\text{ in.}}\right)\left(\dfrac{0.515\text{ in.}}{0.835\text{ in.}}\right)^{1.5}\right]\\ \times\sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(0.835\text{ in.})}{0.515\text{ in.}}}\end{array}\right\}$ = 553 kips > 79.3 kips **o.k.**

Gusset-to-Base Plate Connection

Check shear yielding on gusset plate

From AISC *Specification* Section J4.2, the available shear yielding strength is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(3\tfrac{9}{16}\text{ in.})$ = 48.1 kips > 9.86 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(3\tfrac{9}{16}\text{ in.})}{1.50}$ = 32.1 kips > 6.72 kips **o.k.**

Design weld at gusset-to-base plate connection

The strength of the fillet weld can be determined from AISC *Manual* Equation 8-2:

LRFD	ASD
$$\begin{aligned} D_{req'd} &= \frac{R_u}{2(l)(1.392\text{ kip/in.})} \\ &= \frac{9.86\text{ kips}}{2(3\tfrac{9}{16}\text{ in.})(1.392\text{ kip/in.})} \\ &= 0.994\text{ sixteenths} \end{aligned}$$	$$\begin{aligned} D_{req'd} &= \frac{R_a}{2(l)(0.928\text{ kip/in.})} \\ &= \frac{6.72\text{ kips}}{2(3\tfrac{9}{16}\text{ in.})(0.928\text{ kip/in.})} \\ &= 1.02\text{ sixteenths} \end{aligned}$$

Use the AISC *Specification* Table J2.4 minimum fillet weld of ¼ in.

Note that an extension plate similar to that which is shown in Figure 4-22 could be used if the gusset stress and gusset weld to the base plate were excessive. Alternatively, the base plate could be extended to accommodate the gusset.

Column-to-Base Plate Connection

AISC *Manual* Figure 14-2 shows a typical column-to-base plate weld for axial compression. The welds of AISC *Manual* Figure 14-2 are for handling, shipping and erection. The column compressive axial load is carried by bearing for which no weld is required. Figure 5-33 shows a more commonly used weld arrangement. The purpose of both these arrangements is to provide enough weld for handling, shipping and erection without turning the column in the shop.

When axial tension and shear are present, as in this example, welds specific for these loads are required. For gussets to the column strong axis at the base plate, the most effective force distribution is to take the horizontal shear to the web-to-base plate weld, and the axial tension to the flange-to-base plate welds.

This force distribution assumes that the anchor bolts are placed close to the flanges to avoid excessive base plate bending due to the axial tension (which is also called uplift).

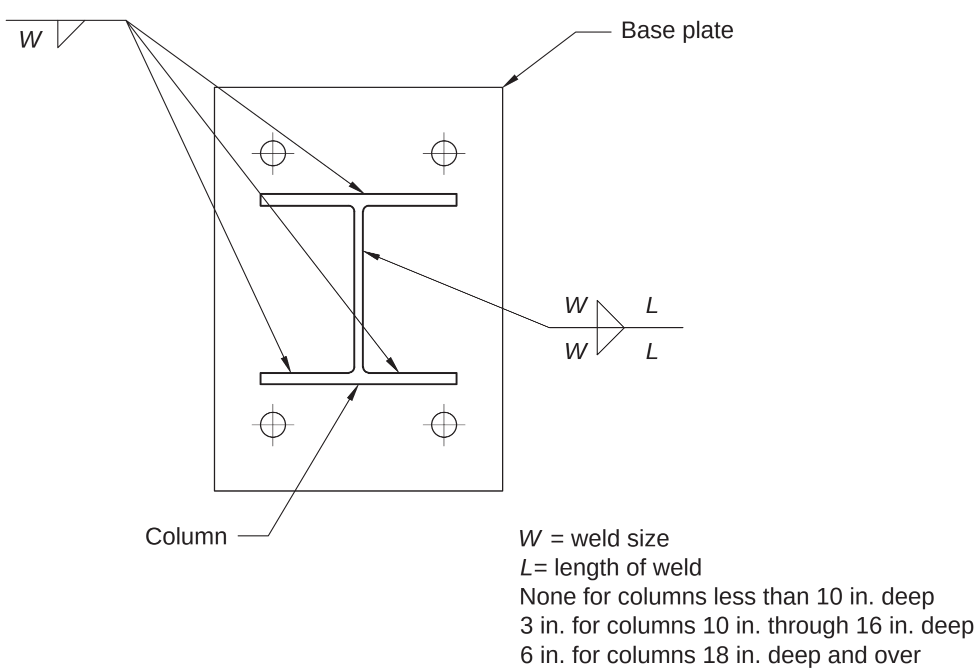


Fig. 5-33. Weld pattern for column-to-base plate for axial column compression.

Base plate-to-column web connection

Check shear yielding on the column web

From AISC *Specification* Section J4.2, the available shear yielding strength is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50 \text{ ksi})(0.515 \text{ in.})(21.4 \text{ in.})$ $= 331 \text{ kips} > 119 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(0.515 \text{ in.})(21.4 \text{ in.})}{1.50}$ $= 220 \text{ kips} > 79.3 \text{ kips}$ **o.k.**

Design column web-to-base plate welds

Use the minimum ¼-in. fillet weld required by AISC *Specification* Table J2.3 and determine the required length from AISC *Manual* Equation 8-2:

LRFD	ASD
$H_{uc} \le 2(1.392 \text{ kip/in.})Dl$ $l \ge \dfrac{119 \text{ kips}}{2(1.392 \text{ kip/in.})(4 \text{ sixteenths})} = 10.7 \text{ in.}$	$H_{ac} \le 2(0.928 \text{ kip/in.})Dl$ $l \ge \dfrac{79.3 \text{ kips}}{2(0.928 \text{ kip/in.})(4 \text{ sixteenths})} = 10.7 \text{ in.}$

Use 11-in.-long ¼-in. fillet welds on each side of the web. Check that the column web shear rupture strength along the weld is adequate:

$$t_{min} = \frac{6.19D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-3})$$

$$= \frac{6.19(4 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.381 \text{ in.} < 0.515 \text{ in.} \quad \textbf{o.k.}$$

Base plate-to-column flange connection

Each flange carries $\dfrac{143 \text{ kips}}{2} = 71.5$ kips (LRFD) and $\dfrac{95.5 \text{ kips}}{2} = 47.8$ kips (ASD) tension. Using the minimum 5⁄16-in. fillet weld required by AISC *Specification* Table J2.4, the length required can be determined from AISC *Manual* Equation 8-2. The directional strength increase factor from AISC *Specification* Section J2.4 is also incorporated due to the 90° angle of loading:

LRFD	ASD
$l = \dfrac{V_u/2}{2(1.392 \text{ kip/in.})D\left(1.0 + 0.50\sin^{1.5}\theta\right)}$ $= \dfrac{71.5 \text{ kips}}{2(1.392 \text{ kip/in.})(5 \text{ sixteenths})\left(1.0 + 0.50\sin^{1.5} 90^\circ\right)}$ $= 3.42 \text{ in.}$	$l = \dfrac{V_a/2}{2(0.928 \text{ kip/in.})D\left(1.0 + 0.50\sin^{1.5}\theta\right)}$ $= \dfrac{47.8 \text{ kips}}{2(0.928 \text{ kip/in.})(5 \text{ sixteenths})\left(1.0 + 0.50\sin^{1.5} 90^\circ\right)}$ $= 3.43 \text{ in.}$

Use a 4-in.-long 5⁄16-in. fillet weld on the outside of each flange, and (2) 2-in. long welds on the inside of each flange. Check that the column-flange shear rupture strength along the weld is adequate:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-3)}$$

$$= \frac{6.19(5 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.476 \text{ in.} < 0.835 \text{ in.} \quad \textbf{o.k.}$$

Example 5.12.2—Weak-Axis Case

Given:

The theory for this case is given in Section 4.3 and Figure 4-23. Figure 5-34 shows a completed sketch of the connection designed in this example. The brace is the same as that shown in Figure 5-32, as is the brace load. Therefore, with the exception of gusset plate buckling, the brace, connection angle and gusset plate limit state checks shown in Example 5.12.1 apply here also and will not be repeated. The given forces and section properties are also the same as those given in the previous example, but the bevel changes to 10⅛ on 12.

Solution:

Check compression buckling of gusset plate on Whitmore section

As given in Figure 5-34, the buckling length is 14 in. The K factor is 0.60 from Dowswell (2006). Therefore, the effective slenderness ratio is:

$$\frac{KL}{r} = \frac{0.60(14.0 \text{ in.})}{\text{⅝ in.}/\sqrt{12}}$$

$$= 46.6$$

As determined in Example 5.12.1, the Whitmore section width, l_w, is 11.9 in. Because $\frac{KL}{r} > 25$, AISC *Specification* Chapter E applies, as indicated in AISC *Specification* Section J4.4. From AISC *Manual* Table 4-22:

$\phi_c F_{cr} = 28.9$ ksi and $F_{cr}/\Omega_c = 19.2$ ksi

From AISC *Specification* Section E3:

LRFD	ASD
$\phi_c R_n = \phi_c F_{cr} A_w$ $= \phi_c F_{cr} l_w t_g$ $= (28.9 \text{ ksi})(11.9 \text{ in.})(\text{⅝ in.})$ $= 215 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega_c} = \frac{F_{cr} A_w}{\Omega_c}$ $= \frac{F_{cr} l_w t_g}{\Omega_c}$ $= (19.2 \text{ ksi})(11.9 \text{ in.})(\text{⅝ in.})$ $= 143 \text{ kips} > 120 \text{ kips}$ **o.k.**

Distribution of Forces

From Figure 4-23, and the specific case shown in Figure 5-34:

$$\bar{\beta} = \frac{25 \text{ in.} - \frac{3}{4} \text{ in.}}{2} + \frac{3}{4} \text{ in.}$$
$$= 12.875 \text{ in.}$$
$$e = 6 \text{ in.}$$
$$e_c = 0 \text{ in.}$$

LRFD	ASD
$H_u = 116 \text{ kips}$ $V_u = 138 \text{ kips}$ $H_{ub} = \dfrac{H_u\left(\bar{\beta} - e\right)}{\bar{\beta}}$ $= \dfrac{(116 \text{ kips})(12.875 \text{ in.} - 6.00 \text{ in.})}{12.875 \text{ in.}}$ $= 61.9 \text{ kips}$ $H_{uc} = \dfrac{H_u e}{\bar{\beta}}$ $= \dfrac{(116 \text{ kips})(6.00 \text{ in.})}{12.875 \text{ in.}}$ $= 54.1 \text{ kips}$	$H_a = 77.4 \text{ kips}$ $V_a = 91.7 \text{ kips}$ $H_{ab} = \dfrac{H_a\left(\bar{\beta} - e\right)}{\bar{\beta}}$ $= \dfrac{(77.4 \text{ kips})(12.875 \text{ in.} - 6.00 \text{ in.})}{12.875 \text{ in.}}$ $= 41.3 \text{ kips}$ $H_{ac} = \dfrac{H_a e}{\bar{\beta}}$ $= \dfrac{(77.4 \text{ kips})(6.00 \text{ in.})}{12.875 \text{ in.}}$ $= 36.1 \text{ kips}$

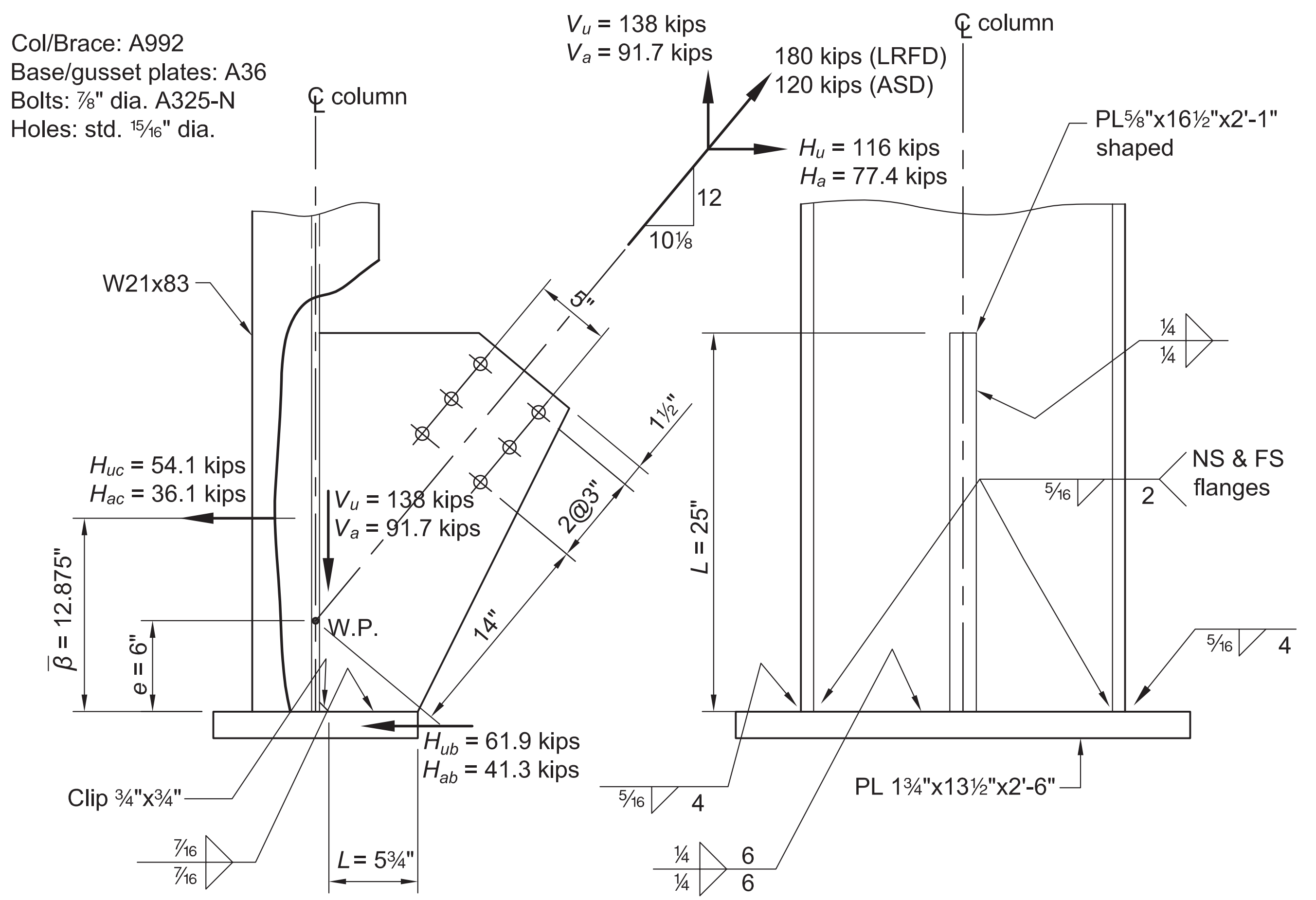


Fig. 5-34. Brace-to-column base plate, weak axis.

Gusset-to-Column Connection

Check shear and tension yielding on the gusset plate

The available shear yielding strength of the gusset plate is determined from AISC *Specification* Equation J4-3, and the available tensile yielding strength is determined from AISC *Specification* Equation J4-1:

The weld length is,

$$\begin{aligned} l &= 25.0\text{ in.} - \tfrac{3}{4}\text{ in.} \\ &= 24.3\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(24.3\text{ in.})$ $= 328\text{ kips} > 138\text{ kips}$ **o.k.** $\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(24.3\text{ in.})$ $= 492\text{ kips} > 54.1\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(24.3\text{ in.})}{1.50}$ $= 219\text{ kips} > 91.7\text{ kips}$ **o.k.** $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(24.3\text{ in.})}{1.67}$ $= 327\text{ kips} > 36\ 1\text{ kips}$ **o.k.**

Design weld for combined forces at gusset-to-column web connection

The strength of the fillet weld can be determined from AISC *Manual* Equation 8-2, incorporating the directional strength increase permitted from AISC *Specification* Equation J2-5, as follows:

LRFD	ASD
The resultant load is: $R_u = \sqrt{(54.1\text{ kips})^2 + (138\text{ kips})^2}$ $= 148\text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{54.1\text{ kips}}{138\text{ kips}}\right)$ $= 21.4°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{R_u}{2(l)(1.392\text{ kip/in.})\left(1.0 + 0.50\sin^{1.5}\theta\right)}$ $= \dfrac{148\text{ kips}}{2(24.3\text{ in.})(1.392\text{ kip/in.})\left(1.0 + 0.50\sin^{1.5} 21.4°\right)}$ $= 1.97\text{ sixteenths}$	The resultant load is: $R_a = \sqrt{(36.1\text{ kips})^2 + (91.7\text{ kips})^2}$ $= 98.5\text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{36.1\text{ kips}}{91.7\text{ kips}}\right)$ $= 21.5°$ The number of sixteenths of fillet weld required is: $D_{req'd} = \dfrac{R_a}{2(l)(0.928\text{ kip/in.})\left(1.0 + 0.50\sin^{1.5}\theta\right)}$ $= \dfrac{98.5\text{ kips}}{2(24.3\text{ in.})(0.928\text{ kip/in.})\left(1.0 + 0.50\sin^{1.5} 21.5°\right)}$ $= 1.97\text{ sixteenths}$

Use the minimum fillet weld size of ¼ in. stipulated in AISC *Specification* Table J2.4. Note that the ductility factor of 1.25 is omitted here. The flexibility of the web will allow redistribution to occur and maintain the assumed uniform force distribution.

Check column web strength to carry H$_c$

The theory for this limit state is discussed in Section 4.3 and Figure 4-24.

From Section 4.3:

LRFD	ASD
$H_{uc} \le \dfrac{4\phi F_y t_w^2 L}{\bar{\beta}}$ $= \dfrac{4(0.90)(50\text{ ksi})(0.515\text{ in.})^2(25.0\text{ in.})}{12\tfrac{7}{8}\text{ in.}}$ $= 92.7\text{ kips}$	$H_{ac} \le \dfrac{4F_y t_w^2 L}{\bar{\beta}\Omega}$ $= \dfrac{4(50\text{ ksi})(0.515\text{ in.})^2(25.0\text{ in.})}{(12\tfrac{7}{8}\text{ in.})(1.67)}$ $= 61.7\text{ kips}$

Because $H_{uc} = 54.1$ kips < 92.7 kips for LRFD and $H_{ac} = 36.1$ kips < 61.7 kips for ASD, the optional stiffener of Figure 4-23 is not required.

Gusset-to-Base Plate Connection

Check shear yielding on the gusset plate

From AISC *Specification* Section J4.2, the available shear yielding strength is:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(5\tfrac{3}{4}\text{ in.})$ $= 77.6\text{ kips} > 61.9\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(\tfrac{5}{8}\text{ in.})(5\tfrac{3}{4}\text{ in.})}{1.50}$ $= 51.8\text{ kips} > 41.3\text{ kips}$ **o.k.**

Design weld

Design weld at gusset-to-base plate connection

The strength of the fillet weld can be determined from AISC *Manual* Equation 8-2:

LRFD	ASD
$D_{req'd} = \dfrac{R_u}{2(l)(1.392\text{ kip/in.})}$ $= \dfrac{61.9\text{ kips}}{2(5\tfrac{3}{4}\text{ in.})(1.392\text{ kip/in.})}$ $= 3.87\text{ sixteenths}$ Check weld for ductility using the recommendations in Section 4.3: $D_{min} = \tfrac{5}{8}(\tfrac{5}{8}\text{ in.})(16)$ $= 6.25\text{ sixteenths}$	$D_{req'd} = \dfrac{R_a}{2(l)(0.928\text{ kip/in.})}$ $= \dfrac{41.3\text{ kips}}{2(5\tfrac{3}{4}\text{ in.})(0.928\text{ kip/in.})}$ $= 3.87\text{ sixteenths}$ Check weld for ductility using the recommendation in Section 4.3: $D_{min} = \tfrac{5}{8}(\tfrac{5}{8}\text{ in.})(16)$ $= 6.25\text{ sixteenths}$

Use a 7⁄16-in. fillet weld. As noted in Section 4.3, this larger weld can be avoided by the use of a top stiffener as shown in Figure 4-23.

Column-to-Base Plate Connection

The force $H_{ub} = 61.9$ kips (LRFD) and $H_{ab} = 41.3$ kips (ASD) is already in the base plate and needs no further consideration. The gusset-to-column shear force $V_u = 138$ kips (LRFD) and $V_a = 91.7$ kips (ASD) is an axial tension force between the column and base plate and needs to exit the column near the anchor rods. Because the anchor rods are usually near the flanges, one half of this shear force will be distributed to each flange. The force H_{uc} (LRFD) and H_{ac} (ASD) also needs to exit the column through the column flanges.

Thus, the forces per flange are:

LRFD	ASD
$\text{Axial} = \dfrac{V_u}{2} = \dfrac{138\text{ kips}}{2} = 69.0\text{ kips}$ $\text{Shear} = \dfrac{H_{uc}}{2} = \dfrac{54.1\text{ kips}}{2} = 27.1\text{ kips}$ The resultant force per flange is: $R_u = \sqrt{(69.0\text{ kips})^2 + (27.1\text{ kips})^2} = 74.1\text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{69.0}{27.1}\right) = 68.6°$	$\text{Axial} = \dfrac{V_a}{2} = \dfrac{91.7\text{ kips}}{2} = 45.9\text{ kips}$ $\text{Shear} = \dfrac{H_{ac}}{2} = \dfrac{36.1\text{ kips}}{2} = 18.1\text{ kips}$ The resultant force per flange is: $R_a = \sqrt{(45.9\text{ kips})^2 + (18.1\text{ kips})^2} = 49.3\text{ kips}$ The resultant load angle is: $\theta = \tan^{-1}\left(\dfrac{45.9}{18.1}\right) = 68.5°$

If a 5⁄16-in. fillet weld is assumed on each of the flanges, the length required is determined from AISC *Manual* Equation 8-2,

LRFD	ASD
$l = \dfrac{R_u}{2(1.392\text{ kip/in.})D(1.0+0.50\sin^{1.5}\theta)}$ $= \dfrac{74.1\text{ kips}}{2(1.392\text{ kip/in.})(5\text{ sixteenths})(1.0+0.50\sin^{1.5}68.6°)}$ $= 3.67\text{ in.}$	$l = \dfrac{R_a}{2(0.928\text{ kip/in.})D(1.0+0.50\sin^{1.5}\theta)}$ $= \dfrac{49.3\text{ kips}}{2(0.928\text{ kip/in.})(5\text{ sixteenths})(1.0+0.50\sin^{1.5}68.5°)}$ $= 3.67\text{ in.}$

Use a 4-in.-long 5⁄16-in. fillet weld on the outside of the flanges and two 2-in.-long welds each on the inside of the flanges.

Check that the column flange shear rupture strength along the weld is adequate:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-3)}$$

$$= \frac{6.19(5 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.476 \text{ in.} < t_f = 0.835 \text{ in.} \quad \textbf{o.k.}$$

Check that the web can deliver ½V to each flange by checking the available shear yielding strength of the column web along the gusset plate:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y t_w L \geq \dfrac{V}{2}$ $= 1.00(0.60)(50 \text{ ksi})(0.515 \text{ in.})(25.0 \text{ in.}) \geq \dfrac{138 \text{ kips}}{2}$ $= 386 \text{ kips} > 69.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y t_w L}{\Omega} \geq \dfrac{V}{2}$ $= \dfrac{0.60(50 \text{ ksi})(0.515 \text{ in.})(25.0 \text{ in.})}{1.50} \geq \dfrac{91.7 \text{ kips}}{2}$ $= 258 \text{ kips} > 45.9 \text{ kips}$ **o.k.**

Design weld of column web-to-base plate

The column web delivers no load to the base plate in this case. A nominal AISC minimum weld of the length given in Figure 5-33 can be used to facilitate shipping, handling and erection.

Concluding Remarks:

The designs shown in Figures 5-32 and 5-34 could occur at the same point. In this case, a question arises as to whether or not the column shears and tensions (uplifts) need to be considered simultaneously. In most cases this is not necessary because the lateral loads do not act simultaneously in two orthogonal directions. Some building codes, however, require that the full lateral load in one direction be combined with 70% of the lateral load in the other (orthogonal) direction.

Chapter 6
Design of Bracing Connections for Seismic Resistance

As discussed in Section 4.2.6, Effect of Frame Distortion, for high-seismic applications where story drifts of 2 to 2.5% must be accommodated, frame distortion cannot be ignored. These story drifts of 2 to 2.5% are on the order of ten times the drifts that are expected for wind and low seismic ($R \leq 3$) design. These large deformations occur because the actual maximum considered earthquake (MCE) forces are reduced to about one ninth of the forces the MCE could produce. This is done by first using 2/3 of the MCE forces and then dividing the forces resulting from the reduced earthquake by an R factor on the order of 6, so the MCE load reduction factor is 6(3)/2 = 9 (ASCE, 2010).

The rationale for this reduction factor is twofold: (1) the forces are of short duration and reversing so the response to them does not necessarily achieve the maximum values; and (2) when the structure deforms into the plastic response range, its fundamental natural frequency is reduced, and its period of vibration is increased. With an increased period of vibration, the maximum effect of the seismic base input is reduced. The softening of the structure tends to isolate it from the ground motion. The price paid for this MCE force reduction is the high drift ratios, and the requirement for ductile response that allows large distortions without fracture and resulting building collapse. If $R = 1$ is used, or even 2/3, the drift would be no greater (and probably less because of the short duration) than traditional wind design. Some designers of hospitals and nuclear power plants use these lower R factors.

The AISC *Seismic Provisions for Structural Steel Buildings* (AISC, 2010b), hereafter referred to as the AISC *Seismic Provisions*, provide three ways to design special concentrically braced frame (SCBF) connections involving a beam, a column and a brace. The requirements of these SCBF connections are given by AISC *Seismic Provisions* Section F2.6b.

The first option, given in AISC *Seismic Provisions* Section F2.6b(a), is to provide a simple connection. This is one that satisfies AISC *Specification* Section B3.6a, except using a required rotation of 0.025 rad. The simple shear connections of AISC *Manual* Parts 9 and 10 are deemed to satisfy this requirement. Basically, the moment that can exist in the beam-column-brace connection, because of continuity of the column and the rigid connection that results between the beam and the column because of the gusset plate connection for the brace, needs to be released. This can be done by means of a beam moment release as shown in Figure 4-20 with a beam splice or Figure 4-19 with a pin. An option given by Thornton and Muir (2009) is to move the workpoint and use long slots to accommodate the 0.025 rad rotation.

The second and third options, given in AISC *Seismic Provisions* Section F2.6b(b), are to provide a connection that can resist a moment equal to the lesser of 1.1 times the expected beam flexural strength $(1.1R_yM_p)_{beam}$ for LRFD, or the expected column flexural strength $\Sigma(1.1R_yM_p)_{column}$ for LRFD. There are two ways to accomplish this (providing options two and three):

1. Provide a prescriptive fully restrained beam-to-column moment connection designed for the lesser of the beam or column expected flexural strength. One way to achieve this connection is to satisfy the prescriptive option for beam-to-column connections in ordinary moment frames (OMF) given in AISC *Seismic Provisions* Section E1.6b(c).

2. Provide a connection in which the beam, column and gusset plate participate in carrying the required flexural strength prescribed. This approach requires distributing the rotational or distortional forces that result from the required flexural strength, $M_D = \min[(1.1R_yM_p)_{beam}, \Sigma(1.1R_yM_p)_{column}]$. M_D will be referred to as the distortional moment. Figure 4-18 shows an admissible distortional force distribution that can be added algebraically to the uniform force method (UFM) distribution.

The AISC *Seismic Design Manual* (AISC, 2012) gives examples of the use of these three methods.

6.1 COMPARISON BETWEEN HIGH-SEISMIC DUCTILE DESIGN AND ORDINARY LOW-SEISMIC DESIGN

In the following examples, two connections are designed. The connections are at the same plan location, in the same building, and at the same geographic location. This location allows for a structure not specifically detailed for seismic resistance with $R = 3$, but the designer has the option to use a lower seismic force level when the seismic detailing requirements of a special concentrically braced frame (SCBF), with $R = 6$, are satisfied. In Example 6.1a, the SCBF option is designed. In Example 6.1b, the $R = 3$ version is designed and a comparison between the two is discussed.

Example 6.1a—High-Seismic Design in Accordance with the AISC *Specification* and the AISC *Seismic Provisions*

Given:

The frame is designed as a special concentrically braced frame (SCBF) with R = 6. Design the HSS heavy bracing connection, shown in its final state in Figure 6-1, between the W16×100 beam and the W14×283 column. Use 1-in.-diameter ASTM A490-X bolts in standard holes and 70-ksi electrodes. The shape materials are given in Figure 6-1. The brace length is 24 ft (work point-to-work point). The moment release option will be used.

The required axial strengths (tension or compression) of the brace from seismic and wind loads are as follows:

LRFD	ASD
$P_u = 300$ kips (wind) $P_u = 145$ kips (seismic)	$P_a = 200$ kips (wind) $P_a = 96.7$ kips (seismic)

The shear reaction on the beam due to factored gravity loads is:

LRFD	ASD
$V_u = 70.0$ kips	$V_a = 46.7$ kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $F_u = 65$ ksi

ASTM A500, Grade B
$F_y = 46$ ksi $F_u = 58$ ksi

ASTM A572, Grade 50
$F_y = 50$ ksi $F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Table 1-12 the geometric properties are as follows:

Beam
W16×100
$A_g = 29.4$ in.^{2} $d = 17.0$ in. $t_w = 0.585$ in. $b_f = 10.4$ in. $t_f = 0.985$ in. $k_{des} = 1.39$ in.

Column
W14×283
$d = 16.7$ in. $t_w = 1.29$ in. $t_f = 2.07$ in. $k_{des} = 2.67$ in.

Brace
HSS8×8×⅝
$A_g = 16.4$ in.^{2} $t = 0.581$ in. $b/t = 10.8$ in. $r = 2.99$ in.

Design HSS Brace

The AISC *Seismic Provisions* are applicable for the design of a special concentrically braced frame (SCBF). Because the required strength of the bracing connection is based on the expected strength of the brace, it is important not to oversize the brace. Try an HSS8×8×½. At a length of 24 ft, AISC *Manual* Table 4-4 gives an available compressive strength of:

LRFD	ASD
$\phi_c P_n = 306$ kips (compression) > 300 kips **o.k.**	$\dfrac{P_n}{\Omega_c} = 203$ kips (compression) > 200 kips **o.k.**

According to AISC *Seismic Provisions* Section F2.5a, braces shall satisfy the requirements of Section D1.1 for highly ductile members. From AISC *Seismic Design Manual* Table 1-5b, it can be seen that an HSS8×8×½ does not satisfy the width-to-thickness requirements for an SCBF brace. From AISC *Seismic Provisions* Table D1.1, for highly ductile HSS:

$$\frac{b}{t} \le 0.55\sqrt{\frac{E}{F_y}}$$

$$\le 0.55\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}} = 13.8$$

Try an HSS8×8×⅝. For this shape, $b/t = 10.8 < 13.8$, therefore the highly ductile width-to-thickness requirement for an SCBF brace is satisfied.

According to AISC *Seismic Provisions* Section F2.5, columns in SCBF must satisfy the requirements for highly ductile members and beams must satisfy the requirements for moderately ductile members. To meet the width-to-thickness limits, a similar check to the above could be performed. Alternatively, AISC *Seismic Design Manual* Table 1-3 demonstrates that the W14×283 column and the W16×100 beam satisfy the width-to-thickness requirements for SCBF columns and beams, respectively.

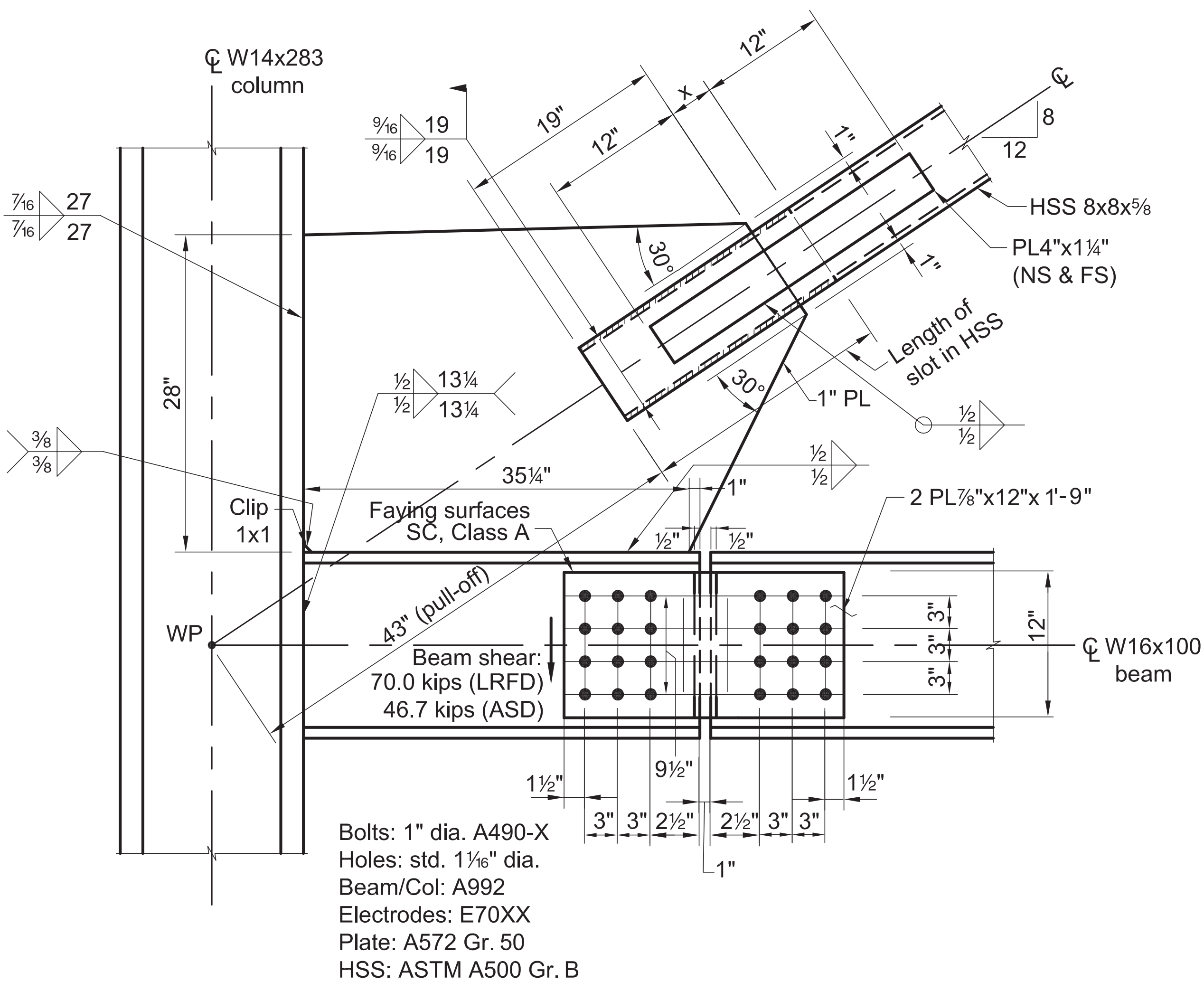


Fig. 6-1. High-seismic design with hinge (moment release).

Brace-to-Gusset Connection

Determine the required tensile strength of the connection

Assuming that AISC *Seismic Provisions* Section F2.6c(1)(a) controls, with R_y given in Table A3.1, the required tensile strength of the bracing connection is:

LRFD	ASD
$P_u = R_y F_y A_g$ $= 1.4(46\text{ ksi})(16.4\text{ in.}^2)$ $= 1{,}060\text{ kips}$	$P_a = \dfrac{R_y F_y A_g}{1.5}$ $= \dfrac{1.4(46\text{ ksi})(16.4\text{ in.}^2)}{1.5}$ $= 704\text{ kips}$

Note that the required strength of the brace and the required strength of the bracing connection are significantly different in an SCBF, whereas in an R = 3 design they are the same. Furthermore, the SCBF highly ductile width-to-thickness requirement requires a heavier section (HSS8×8×⅝) than is required for strength (HSS8×8×½), thereby driving up the required strength of the connection even more.

Determine the required compressive strength of the connection

AISC *Seismic Provisions* Section F2.5b(1) requires that the slenderness ratio of SCBF braces be less than 200. The calculation uses the work point-to-work point length of the brace, with K = 1.0. The actual buckling length will be less.

$$\begin{aligned}\frac{KL}{r} &= \frac{1.0(24.0\text{ ft})(12\text{ in./ft})}{2.99\text{ in.}}\\ &= 96.3 < 200 \quad \textbf{o.k.}\end{aligned}$$

From AISC *Seismic Provisions* Section F2.3, the expected brace buckling strength must be based on a length not more than the brace end-to-end length. With the pull-off dimension given in Figure 6-1 assumed to exist at each end of the brace, the length of the brace, l_{brace} = (24 ft)(12 in./ft) − 2(43 in.) = 202 in. = 16 ft 10 in. The geometry of Figure 6-2 can be used to determine the pull-off dimension. The "pull-off" dimension is the terminology used by fabricators and detailers to determine the actual cut length of the brace. It is based on the point where the brace ends.

With a K value of 1.0:

$$\begin{aligned}\frac{KL}{r} &= \frac{1.0(202\text{ in.})}{2.99\text{ in.}}\\ &= 67.6\end{aligned}$$

According to AISC *Seismic Provisions* Section F2.3, the expected brace strength in compression is permitted to be taken as the lesser of $R_yF_y A_g$ or $1.14F_{cre}A_g$, where F_{cre} is determined from AISC *Specification* Chapter E using R_yF_y in lieu of F_y. From AISC *Specification* Section E3:

$$\begin{aligned}F_e &= \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}\\ &= \frac{\pi^2(29{,}000\text{ ksi})}{(67.6)^2}\\ &= 62.6\text{ ksi}\end{aligned}$$

$$4.71\sqrt{\frac{E}{R_y F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{1.4(46 \text{ ksi})}}$$
$$= 99.9$$

Because $\dfrac{KL}{r} = 67.6 < 99.9$:

$$F_{cre} = R_y F_y \left[0.658^{\frac{R_y F_y}{F_e}} \right]$$
$$= 1.4(46 \text{ ksi}) \left[0.658^{\frac{1.4(46 \text{ ksi})}{62.6 \text{ ksi}}} \right]$$
$$= 41.9 \text{ ksi}$$

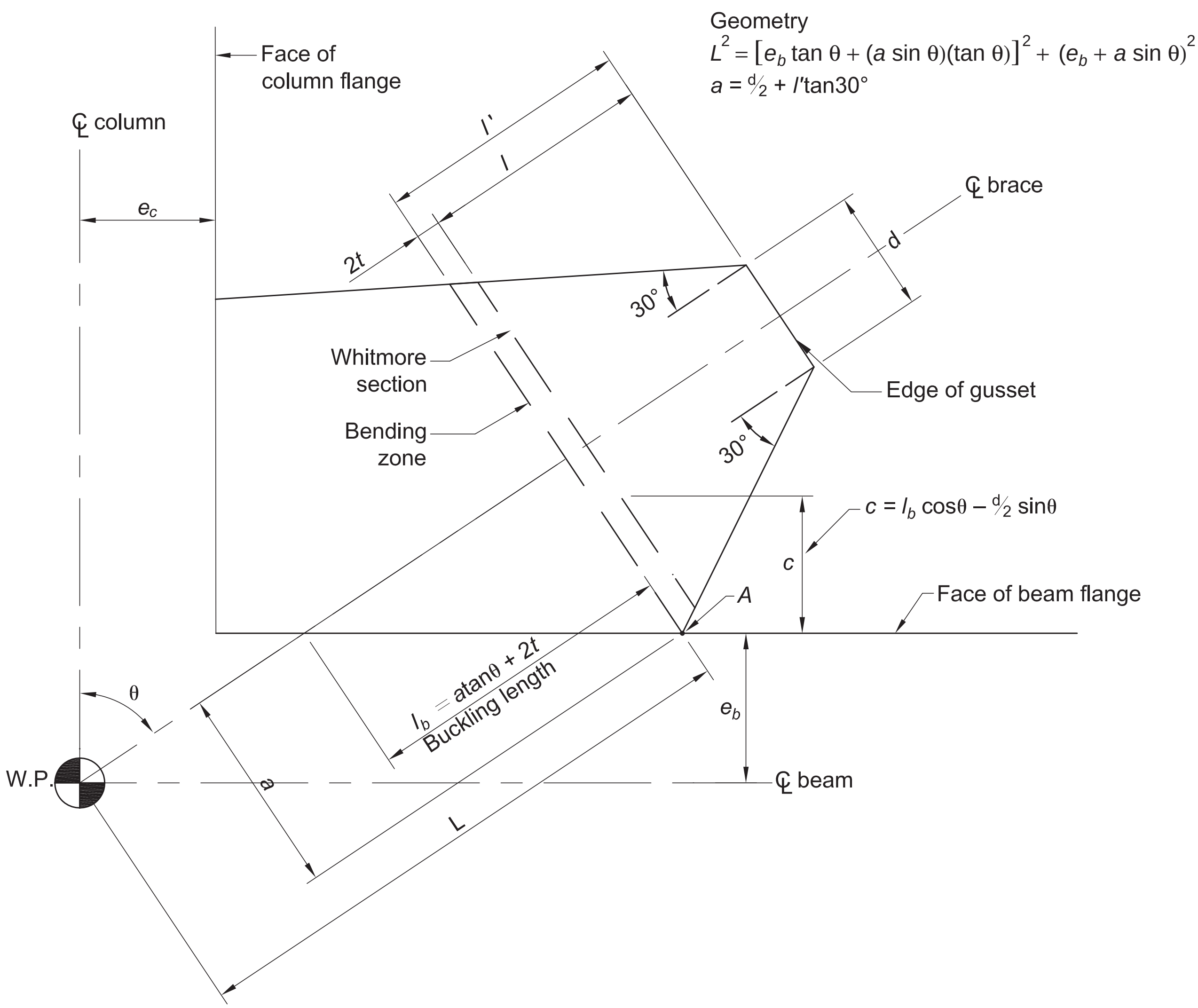


Fig. 6-2. Geometry to locate critical Point A of the bending zone.

The expected brace strength in compression is, according to AISC *Seismic Provisions* Section F2.3, the lesser of $R_yF_yA_g$ and $1.14F_{cre}A_g$ for LRFD or $R_yF_yA_g/1.15$ and $1.14F_{cre}A_g/1.5$ for ASD:

LRFD	ASD
$P_u = 1.14F_{cre}A_g$ $= 1.14(41.9\text{ ksi})(16.4\text{ in.}^2)$ $= 783\text{ kips} < 1{,}060\text{ kips determined previously}$ Therefore, use $P_u = 783$ kips.	$P_a = \dfrac{1.14F_{cre}A_g}{1.5}$ $= \dfrac{1.14(41.9\text{ ksi})(16.4\text{ in.}^2)}{1.5}$ $= 522\text{ kips} < 704\text{ kips determined previously}$ Therefore, use $P_a = 522$ kips.

The expected post-buckling strength does not govern in this example, which is equal to 0.3 times the expected brace strength in compression.

Summary of required strengths of the connection for tension and compression in the brace

LRFD	ASD
$P_u = 1{,}060$ kips (tension) $P_u = 783$ kips (compression)	$P_a = 704$ kips (tension) $P_a = 522$ kips (compression)

Check shear yielding strength in the brace wall

Choose a lap length between the brace and the gusset based on the limit state of shear yielding in the brace wall. Shear yielding is given by AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60F_ytl \le P_u$ $= 1.00(0.60)(46\text{ ksi})(0.581\text{ in.})l(4\text{ walls}) \le 1{,}060\text{ kips}$ $l \ge 16.5$ in. Try $l = 19.0$ in.	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_ytl}{\Omega} \le P_u$ $= \dfrac{(0.60)(46\text{ ksi})(0.581\text{ in.})l(4\text{ walls})}{1.50} \le 704\text{ kips}$ $l \ge 16.5$ in. Try $l = 19.0$ in.

Design brace-to-gusset weld

There are four lines of fillet weld between the brace and the gusset plate. From AISC *Manual* Part 8, Equation 8-2, where l = weld length:

LRFD	ASD
$\phi R_n = 1.392Dl$ $D = \dfrac{1{,}060\text{ kips}}{4(1.392\text{ kip/in.})(19.0\text{ in.})}$ $= 10.0$ sixteenths	$\dfrac{R_n}{\Omega} = 0.928Dl$ $D = \dfrac{704\text{ kips}}{4(0.928\text{ kip/in.})(19.0\text{ in.})}$ $= 9.98$ sixteenths

Try a ⅝-in. fillet weld.

AISC *Specification* Section J2.2b states that, for end-loaded fillet welds, a reduction factor must be applied to the length when the length exceeds 100 times the weld size:

$$\left(\tfrac{5}{8}\text{ in.}\right)100 = 62.5\text{ in.}$$
$$> 19\text{ in.} \quad \textbf{o.k.}$$

Therefore, no reduction on the length is required.

Figure 6-2 shows a "bending zone" of width equal to two times the gusset thickness. This is discussed in the AISC *Seismic Provisions* Commentary Section F2.6c(3). Figure 6-2 gives the geometry required to locate the critical Point A, which locates the bending zone. For steep braces, this point will be at the column and a similar geometry derivation can be developed. From Figures 6-1 and 6-2 and the member geometry, where d is the dimension of the gusset perpendicular to the brace at the beginning of the brace-to-gusset lap, the following parameters can be determined:

$e_c = 8.35$ in. $\quad e_b = 8.50$ in. $\quad \theta = 56.3°$ $\quad \tan\theta = 1.50$ $\quad l = 19.0$ in.

$$\begin{aligned} d &= 8.00\text{ in.} + 1.00\text{ in.} + 1.00\text{ in.} \\ &= 10.0\text{ in.} \end{aligned}$$

Check block shear rupture on the gusset plate

The block shear rupture failure path is assumed to follow the overlap of the brace on the gusset plate. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Assume a 1-in.-thick gusset plate.

Shear yielding component:

$$\begin{aligned} A_{gv} &= (19.0\text{ in.})(2)(1.00\text{ in.}) \\ &= 38.0\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_yA_{gv} &= 0.60(50\text{ ksi})\left(38.0\text{ in.}^2\right) \\ &= 1{,}140\text{ kips} \end{aligned}$$

Shear rupture component:

$$\begin{aligned} A_{nv} &= A_{gv} \\ &= 38.0\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_uA_{nv} &= 0.60(65\text{ ksi})\left(38.0\text{ in.}^2\right) \\ &= 1{,}480\text{ kips} \end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specification* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned} A_{nt} &= A_{gt} \\ &= (8.00\text{ in.})(1.00\text{ in.}) \\ &= 8.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs}F_uA_{nt} &= 1(65\text{ ksi})\left(8.00\text{ in.}^2\right) \\ &= 520\text{ kips} \end{aligned}$$

The available block shear rupture strength of the gusset plate is:

$$\begin{aligned} 0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 1{,}480 \text{ kips} + 520 \text{ kips} \\ &= 2{,}000 \text{ kips} \\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 1{,}140 \text{ kips} + 520 \text{ kips} \\ &= 1{,}660 \text{ kips} \end{aligned}$$

Therefore, R_n = 1,660 kips.

LRFD	ASD
$\begin{aligned} \phi R_n &= 0.75(1{,}660 \text{ kips}) \\ &= 1{,}250 \text{ kips} > 1{,}060 \text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{1{,}660 \text{ kips}}{2.00} \\ &= 830 \text{ kips} > 704 \text{ kips} \quad \textbf{o.k.} \end{aligned}$

Check tensile yielding on the Whitmore section

$$\begin{aligned} l_w &= 8.00 \text{ in.} + 2(19.0 \text{ in.})\tan 30° \\ &= 29.9 \text{ in.} \end{aligned}$$

LRFD	ASD
$\begin{aligned} \phi R_n &= 0.90F_yl_wt \\ &= 0.90(50 \text{ ksi})(29.9 \text{ in.})(1.00 \text{ in.}) \\ &= 1{,}350 \text{ kips} > 1{,}060 \text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{F_yl_wt}{\Omega} \\ &= \frac{(50 \text{ ksi})(29.9 \text{ in.})(1.00 \text{ in.})}{1.67} \\ &= 895 \text{ kips} > 704 \text{ kips} \quad \textbf{o.k.} \end{aligned}$

Check flexural buckling in compression on the Whitmore section

From Dowswell (2006), the critical gusset thickness is t_β. If the actual gusset thickness is greater than t_β, the gusset will yield before it buckles and the limit state of flexural buckling does not apply. From Figure 6-2:

$$\begin{aligned} l' &= l + 2t \\ &= 19.0 \text{ in.} + 2(1.00 \text{ in.}) \\ &= 21.0 \text{ in.} \end{aligned}$$

$$\begin{aligned} a &= \frac{d}{2} + l' \tan 30° \\ &= \frac{10.0 \text{ in.}}{2} + (21.0 \text{ in.})\tan 30° \\ &= 17.1 \text{ in.} \end{aligned}$$

$$\begin{aligned} l_b &= a\tan\theta + 2t \\ &= (17.1 \text{ in.})(1.50) + 2(1.00 \text{ in.}) \\ &= 27.7 \text{ in.} \end{aligned}$$

$$c = l_b \cos\theta - \frac{d}{2}\sin\theta$$

Use $d = 8.00$ in. here (the actual width of the brace) to locate the corner of the HSS connection at the Whitmore section:

$$c = (27.7\text{ in.})\cos 56.3° - \left(\frac{8.00\text{ in.}}{2}\right)\sin 56.3°$$
$$= 12.0\text{ in.}$$

From Dowswell (2006):

$$t_\beta = 1.5\sqrt{\frac{F_y c^3}{E_s l_b}}$$
$$= 1.5\sqrt{\frac{(50\text{ ksi})(12.0\text{ in.})^3}{(29{,}000\text{ ksi})(27.7\text{ in.})}}$$
$$= 0.492\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

Because $t = 1.00$ in. > 0.492 in., the gusset is "compact" and will yield before it buckles according to Dowswell (2006); however, the gusset flexural buckling will be checked here using AISC *Specification* Section J4.4.

The gusset satisfies Dowswell's compact criterion. Dowswell refers to this type of gusset as an "extended" gusset and recommends $K = 0.60$.

$$\frac{KL}{r} = \frac{0.60(27.7\text{ in.})}{(1.00\text{ in.})/\sqrt{12}}$$
$$= 57.6$$

Because $\dfrac{KL}{r} > 25$, AISC *Specification* Chapter E applies. Interpolating for $KL/r = 57.6$ in AISC *Manual* Table 4-22, for $F_y =$ 50 ksi:

LRFD	ASD
$\phi_c F_{cr} = 35.3\text{ ksi}$ From AISC *Specification* Section E3: $\phi_c R_n = \phi_c F_{cr} A_w$ $= \phi_c F_{cr} l_w t_g$ $= (35.3\text{ ksi})(29.9\text{ in.})(1.00\text{ in.})$ $= 1{,}060\text{ kips} > 783\text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 23.5\text{ ksi}$ From AISC *Specification* Section E3: $\dfrac{R_n}{\Omega_c} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= \dfrac{F_{cr} l_w t_g}{\Omega_c}$ $= (23.5\text{ ksi})(29.9\text{ in.})(1.00\text{ in.})$ $= 703\text{ kips} > 522\text{ kips}$ **o.k.**

Use a 1-in.-thick gusset plate.

Check tensile rupture on the HSS brace

The available tensile rupture strength of the HSS effective net area must be larger than the required tensile strength of the bracing connection. Also, in accordance with AISC *Seismic Provisions* Section F2.5b(3), the brace effective net area cannot be less than the brace gross area. Assume that the slot in the brace is 1⁄16 in. wider than the gusset plate on each side of the gusset plate, and determine the effective net area of the brace from AISC *Specification* Section D3:

$$A_g = 16.4 \text{ in.}^2$$

$$A_n = A_g - 2td_{slot} \qquad (d_{slot} = \text{width of slot})$$

$$= 16.4 \text{ in.}^2 - 2(0.581 \text{ in.})(1.00 \text{ in.} + \tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})$$

$$= 15.1 \text{ in.}^2$$

From AISC *Specification* Table D3.1, Case 6:

$$\bar{x} = \frac{B^2 + 2BH}{4(B+H)} \qquad (B = H = 8.00 \text{ in.})$$

$$= \frac{(8.00 \text{ in.})^2 + 2(8.00 \text{ in.})(8.00 \text{ in.})}{4(8.00 \text{ in.} + 8.00 \text{ in.})}$$

$$= 3.00 \text{ in.}$$

$$U = 1 - \left(\frac{\bar{x}}{l}\right)$$

$$= 1 - \left(\frac{3.00 \text{ in.}}{19.0 \text{ in.}}\right)$$

$$= 0.842$$

$$A_e = A_n U \qquad \text{(\textit{Spec}. Eq. D3-1)}$$

$$= (15.1 \text{ in.}^2)(0.842)$$

$$= 12.7 \text{ in.}^2$$

Because 12.7 in.^{2} < 16.4 in.^{2}, reinforcement is required. AISC *Seismic Provisions* Section F2.5b(3)(i) requires the specified minimum yield strength of the reinforcement to be at least that of the brace. Therefore, ASTM A572 Grade 50 must be used for the reinforcing plates.

Design reinforcing plates

Try 4 in. × 1¼ in. ASTM A572 Grade 50 plates, as shown in Figures 6-3 and 6-4:

$$A_r = (4.00 \text{ in.})(1\tfrac{1}{4} \text{ in.})$$
$$= 5.00 \text{ in.}^2$$

Determine the available strength of the HSS effective area with the reinforcing plates, which must be greater than the gross area. Using AISC *Specification* Table D3.1, Case 6, and expanding it to include the reinforcing plates, the shear lag factor is determined as follows:

$$\bar{x} = \frac{2BHt + B^2t + 2A_rB}{4(Bt + Ht + A_r)}$$

$$= \frac{2(8.00 \text{ in.})(8.00 \text{ in.})(0.581 \text{ in.}) + (8.00 \text{ in.})^2(0.581 \text{ in.}) + 2(5.00 \text{ in.}^2)(8.00 \text{ in.})}{4\left[(8.00 \text{ in.})(0.581 \text{ in.}) + (8.00 \text{ in.})(0.581 \text{ in.}) + 5.00 \text{ in.}^2\right]}$$

$$= 3.35 \text{ in.}$$

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{3.35 \text{ in.}}{19.0 \text{ in.}}$$

$$= 0.824$$

$$A_n = 15.1 \text{ in.}^2 + 2\left[(4.00 \text{ in.})(1\tfrac{1}{4} \text{ in.})\right]$$
$$= 25.1 \text{ in.}^2$$

$$A_e = A_n U \qquad \text{(}Spec\text{. Eq. D3-1)}$$
$$= \left(25.1 \text{ in.}^2\right)(0.824)$$
$$= 20.7 \text{ in.}^2 > 16.4 \text{ in.}^2 \quad \textbf{o.k.}$$

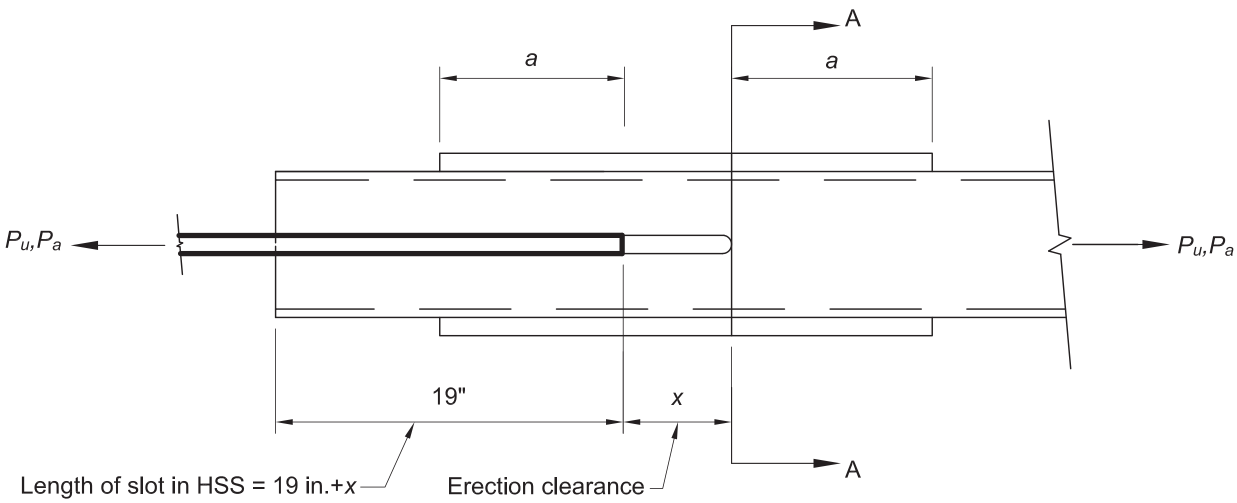


Fig. 6-3. Reinforcing plates positioning and dimensioning to transfer loads into and out of plates.

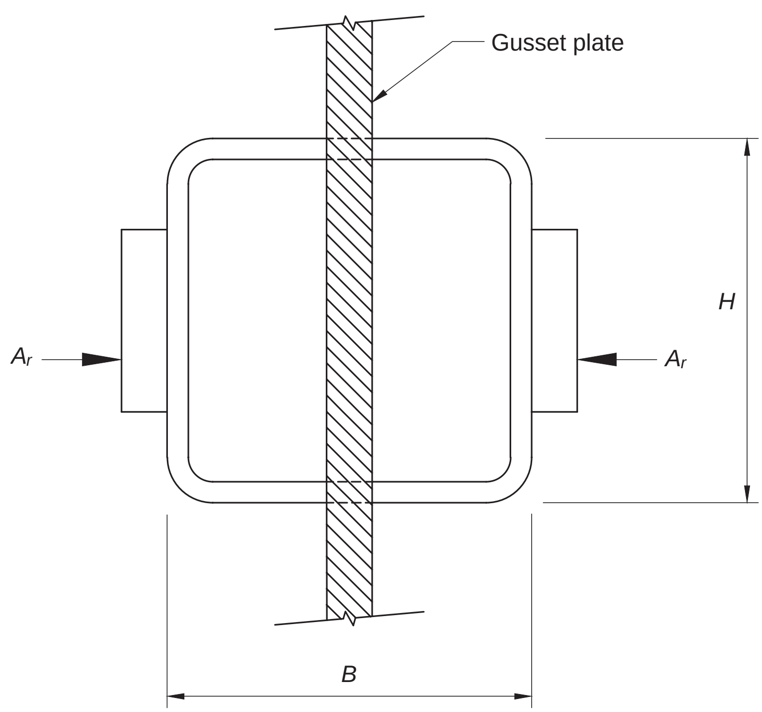


Fig. 6-4. Section A-A: HSS section with reinforcing plates of area A_r.

4 in. × 1¼ in. ASTM A572 Grade 50 reinforcing plates will be sufficient.

Attachment of reinforcing plates to HSS brace

The AISC *Seismic Provisions* Section F2.5b(3)(ii) requires that the attachment of the reinforcing plates to the HSS brace develop the expected reinforcing tensile strength on each side of the reduced section. The expected tensile strength of each 4 in. × 1¼ in. reinforcing plate is determined as follows, with R_y taken from AISC *Seismic Provisions* Table A3.1:

LRFD	ASD
$P_u = 1.1(50\text{ ksi})(5.00\text{ in.}^2)$ $= 275\text{ kips}$	$P_a = \dfrac{1.1}{1.5}(50\text{ ksi})(5.00\text{ in.}^2)$ $= 183\text{ kips}$

Determine what "*a*" length is required to transfer the force into the plates (refer to Figure 6-3). Applying AISC *Specification* Equation J2-10b to account for the fillet welds oriented longitudinally and transversely to the direction of the applied load, and AISC *Manual* Equation 8-2, for each reinforcing plate:

LRFD	ASD
$P_u = \left[2a(0.85)(1.392\text{ kip/in.})D\right]$ $+\left[(4.00\text{ in.})(1.5)(1.392\text{ kip/in.})D\right]$	$P_a = \left[2a(0.85)(0.928\text{ kip/in.})D\right]$ $+\left[(4.00\text{ in.})(1.5)(0.928\text{ kip/in.})D\right]$

The 0.85 factor in the equation above is the strength factor for longitudinally loaded fillet welds when the strength factor of 1.50 is used for the transversely loaded fillets, as given in AISC *Specification* Equation J2-10b.

Try a weld length, *a*, of 12 in. and solve for *D*:

LRFD	ASD
$D = \dfrac{275\text{ kips}}{2(12)(0.85)(1.392\text{ kip/in.})+4(1.5)(1.392\text{ kip/in.})}$ $= 7.48\text{ sixteenths}$ Use a ½-in. fillet weld.	$D = \dfrac{183\text{ kips}}{2(12)(0.85)(0.928\text{ kip/in.})+4(1.5)(0.928\text{ kip/in.})}$ $= 7.47\text{ sixteenths}$ Use a ½-in. fillet weld.

The final design for the reinforcing plates is shown in Figure 6-1. The *x* dimension is the erection clearance.

Force Distribution

The uniform force method (UFM) is used to determine the forces at the gusset-to-beam and gusset-to-column interfaces. As determined previously, the required parameters are:

$e_c = 8.35$ in. $\quad e_b = 8.50$ in. $\quad \theta = 56.3°$ $\quad \tan\theta = 1.50$ $\quad l = 19.0$ in.

$$\sin 56.3° = 0.832$$

$$\alpha - \beta\tan\theta = e_b\tan\theta - e_c \qquad (\textit{Manual}\text{ Eq. 13-1})$$

$$= (8.50\text{ in.})(1.50) - 8.35\text{ in.}$$

$$= 4.40\text{ in.}$$

The "pull-off" dimension is $L + 2t$. From the geometry shown in Figure 6-2:

$$L = \left[\left(e_b \tan\theta + a \sin\theta \tan\theta\right)^2 + \left(e_b + a \sin\theta\right)^2\right]^{0.5}$$

$$\begin{aligned}
a &= \frac{d}{2} + l' \tan 30^\circ \\
&= \frac{10.0 \text{ in.}}{2} + \left[19.0 \text{ in.} + 2\left(1.00 \text{ in.}\right)\right] \tan 30^\circ \\
&= 17.1 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
L &= \left\{\left[\left(8.50 \text{ in.}\right)\left(1.50\right) + \left(17.1 \text{ in.}\right)\left(0.832\right)\left(1.50\right)\right]^2 + \left[8.50 \text{ in.} + \left(17.1 \text{ in.}\right)\left(0.832\right)\right]^2\right\}^{0.5} \\
&= 41.0 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
\text{Pull-off} &= 41.0 \text{ in.} + 2.00 \text{ in.} \\
&= 43.0 \text{ in.}
\end{aligned}$$

Figure 6-1 shows the geometry required to guarantee that the hinge ($2t$) position clears the beam flange. For a steep brace, i.e., θ on the order of 30°, a similar formulation can be derived to guarantee that the hinge ($2t$) position clears the column.

It should be noted that the 30° spread angle shown is used to accommodate the Whitmore section. The 30° Whitmore section defines the maximum effective width of the load spread from the beginning to the end of the brace-to-gusset connection. When the entire Whitmore section is accommodated, the entire section is effective in resisting tensile and compressive forces in the gusset; thus, accommodating the entire Whitmore section will result in the minimum gusset thickness. If a smaller gusset is required, the spread angle can be taken as any angle between 0° and 30°. Such an assumption will result in smaller, thicker gussets. While smaller gussets may be more aesthetically appealing, or may cause fewer interferences with the building services and façade, they may result in difficult-to-accomplish and expensive gusset-to-beam and gusset-to-column connections. Complete-joint-penetration welds of the gusset-to-beam and gusset-to-column connections may be required, one of which will usually be a field weld. Also, column and beam web doublers may be required. Figure 6-1 shows the typical case for sizing of the gusset, which has been widely used, but it should be recognized that variations, where $0° \le \phi \le 30°$, are possible in Figures 6-1 and 6-2.

From the geometry shown in Figure 6-2, the distance from the face of the column flange to Point A, A_{dist}, is used to find the horizontal length of the gusset plate.

The horizontal length of the gusset plate (measured at the gusset-to-beam interface) is:

$$\begin{aligned}
A_{dist} &= e_b \tan\theta - e_c + \frac{a}{\cos\theta} \\
&= \left(8.50 \text{ in.}\right) \tan 56.3^\circ - 8.35 \text{ in.} + \frac{17.1 \text{ in.}}{\cos 56.3^\circ} \\
&= 35.2 \text{ in.}
\end{aligned}$$

Use 35¼ in.

Since both the gusset-to-column and gusset-to-beam connections are welded, choose the shorter connection length to be free of moment because it will be the more flexible of the two. A more formal way to distribute the moment between the two gusset plate edges is given in the AISC *Manual* Part 13 and Section 4.2.1 of this Design Guide. In this example, the gusset-to-column is the shorter length. Therefore, choose $\beta = \bar{\beta}$ and apply any moment that results from the UFM to the more rigid gusset-to-beam interface.

Determine forces on connection interfaces

Using the notation and equations in AISC *Manual* Part 13:

$$\beta = \bar{\beta} = 14.5 \text{ in.}$$

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \qquad \text{(\textit{Manual} Eq. 13-1)}$$

$$\begin{aligned}
\alpha &= e_b \tan\theta - e_c + \beta \tan\theta \\
&= (8.50 \text{ in.})\tan 56.3° - 8.35 \text{ in.} + (14.5 \text{ in.})\tan 56.3° \\
&= 26.1 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
\bar{\alpha} &= \frac{35\tfrac{1}{4} \text{ in.} - 1.00 \text{ in.}}{2} + 1.00 \text{ in.} \\
&= 18.1 \text{ in.}
\end{aligned}$$

Since $\alpha \neq \bar{\alpha}$, there will be a moment on the gusset-to-beam interface.

From AISC *Manual* Part 13, determine r:

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad \text{(\textit{Manual} Eq. 13-6)}$$

$$\begin{aligned}
&= \sqrt{(26.1 \text{ in.} + 8.35 \text{ in.})^2 + (14.5 \text{ in.} + 8.50 \text{ in.})^2} \\
&= 41.4 \text{ in.}
\end{aligned}$$

LRFD	ASD
$\dfrac{P_u}{r} = \dfrac{1{,}060 \text{ kips}}{41.4 \text{ in.}} = 25.6 \text{ kip/in.}$	$\dfrac{P_a}{r} = \dfrac{704 \text{ kips}}{41.4 \text{ in.}} = 17.0 \text{ kip/in.}$
From AISC *Manual* Equation 13-2:	From AISC *Manual* Equation 13-2:
$V_{uc} = \beta \dfrac{P_u}{r} = (14.5 \text{ in.})(25.6 \text{ kip/in.}) = 371 \text{ kips}$	$V_{ac} = \beta \dfrac{P_a}{r} = (14.5 \text{ in.})(17.0 \text{ kip/in.}) = 247 \text{ kips}$
From AISC *Manual* Equation 13-4:	From AISC *Manual* Equation 13-4:
$V_{ub} = e_b \dfrac{P_u}{r} = (8.50 \text{ in.})(25.6 \text{ kip/in.}) = 218 \text{ kips}$	$V_{ab} = e_b \dfrac{P_a}{r} = (8.50 \text{ in.})(17.0 \text{ kip/in.}) = 145 \text{ kips}$
$\Sigma(V_{uc} + V_{ub}) = 371 \text{ kips} + 218 \text{ kips} = 589 \text{ kips} \approx (1{,}060 \text{ kips})\cos 56.3° = 588 \text{ kips}$ **o.k.**	$\Sigma(V_{ac} + V_{ab}) = 247 \text{ kips} + 145 \text{ kips} = 392 \text{ kips} \approx (704 \text{ kips})\cos 56.3° = 391 \text{ kips}$ **o.k.**

LRFD	ASD
From AISC *Manual* Equation 13-3: $$H_{uc} = e_c \frac{P_u}{r}$$ $$= (8.35 \text{ in.})(25.6 \text{ kip/in.})$$ $$= 214 \text{ kips}$$ From AISC *Manual* Equation 13-5: $$H_{ub} = \alpha \frac{P_u}{r}$$ $$= (26.1 \text{ in.})(25.6 \text{ kip/in.})$$ $$= 668 \text{ kips}$$ $$\Sigma(H_{uc} + H_{ub}) = 214 \text{ kips} + 668 \text{ kips}$$ $$= 882 \text{ kips}$$ $$\approx (1{,}060 \text{ kips})\sin 56.3°$$ $$= 882 \text{ kips} \quad \textbf{o.k.}$$ $$M_{ub} = V_{ub}(\alpha - \bar{\alpha})$$ $$= (218 \text{ kips})(26.1 \text{ in.} - 18.1 \text{ in.})$$ $$= 1{,}740 \text{ kip-in.}$$	From AISC *Manual* Equation 13-3: $$H_{ac} = e_c \frac{P_a}{r}$$ $$= (8.35 \text{ in.})(17.0 \text{ kip/in.})$$ $$= 142 \text{ kips}$$ From AISC *Manual* Equation 13-5: $$H_{ab} = \alpha \frac{P_a}{r}$$ $$= (26.1 \text{ in.})(17.0 \text{ kip/in.})$$ $$= 444 \text{ kips}$$ $$\Sigma(H_{ac} + H_{ab}) = 142 \text{ kips} + 444 \text{ kips}$$ $$= 586 \text{ kips}$$ $$\approx (704 \text{ kips})\sin 56.3°$$ $$= 586 \text{ kips} \quad \textbf{o.k.}$$ $$M_{ab} = V_{ab}(\alpha - \bar{\alpha})$$ $$= (145 \text{ kips})(26.1 \text{ in.} - 18.1 \text{ in.})$$ $$= 1{,}160 \text{ kip-in.}$$

The admissible force field of Figure 6-5 assumes the bracing configuration of Figure 6-6, called Configuration A. For this configuration there is no significant transfer force or drag force entering the braced bay. Figure 6-7 shows a possible Configuration B. In this case, there is a large transfer or drag force at the joint. The admissible force field for Configuration B is shown in Figure 6-8. As noted in Figure 6-8, the beam-to-column axial force increases from Configuration A to Configuration B. This example will proceed with Configuration B because the beam splice functions as a hinge, which should be designed as a "simple" connection in accordance with AISC *Manual* Parts 9 and 10.

Beam-to-Column Connection

Use a directly-welded connection between the beam and column.

LRFD	ASD
Shear: $V_u = 218 \text{ kips} + 70 \text{ kips}$ $= 288 \text{ kips}$ Normal: $N_u = H_{ub} = 668 \text{ kips}$	Shear: $V_a = 145 \text{ kips} + 46.7 \text{ kips}$ $= 192 \text{ kips}$ Normal: $N_a = H_{ab} = 444 \text{ kips}$

Check beam shear

From AISC *Manual* Table 3-2, the available shear strength of the W16×100 beam is:

LRFD	ASD
$\phi_v V_{nx} = 298 \text{ kips} > 288 \text{ kips}$ **o.k.**	$\dfrac{V_{nx}}{\Omega_v} = 199 \text{ kips} > 192 \text{ kips}$ **o.k.**

The axial force will be distributed to the column by the beam flange and beam web in proportion to their areas, as follows:

$$
\begin{aligned}
A_f &= t_f b_f \\
&= (0.985\text{ in.})(10.4\text{ in.}) \\
&= 10.2\text{ in.}^2 \\
A_w &= A_g - 2A_f \\
&= 29.4\text{ in.}^2 - 2\left(10.2\text{ in.}^2\right) \\
&= 9.00\text{ in.}^2
\end{aligned}
$$

LRFD	ASD
$N_{uf} = \dfrac{10.2\text{ in.}^2}{29.4\text{ in.}^2}(668\text{ kips})$ $= 232\text{ kips}$ $N_{uw} = \dfrac{9.00\text{ in.}^2}{29.4\text{ in.}^2}(668\text{ kips})$ $= 204\text{ kips}$	$N_{af} = \dfrac{10.2\text{ in.}^2}{29.4\text{ in.}^2}(444\text{ kips})$ $= 154\text{ kips}$ $N_{aw} = \dfrac{9.00\text{ in.}^2}{29.4\text{ in.}^2}(444\text{ kips})$ $= 136\text{ kips}$

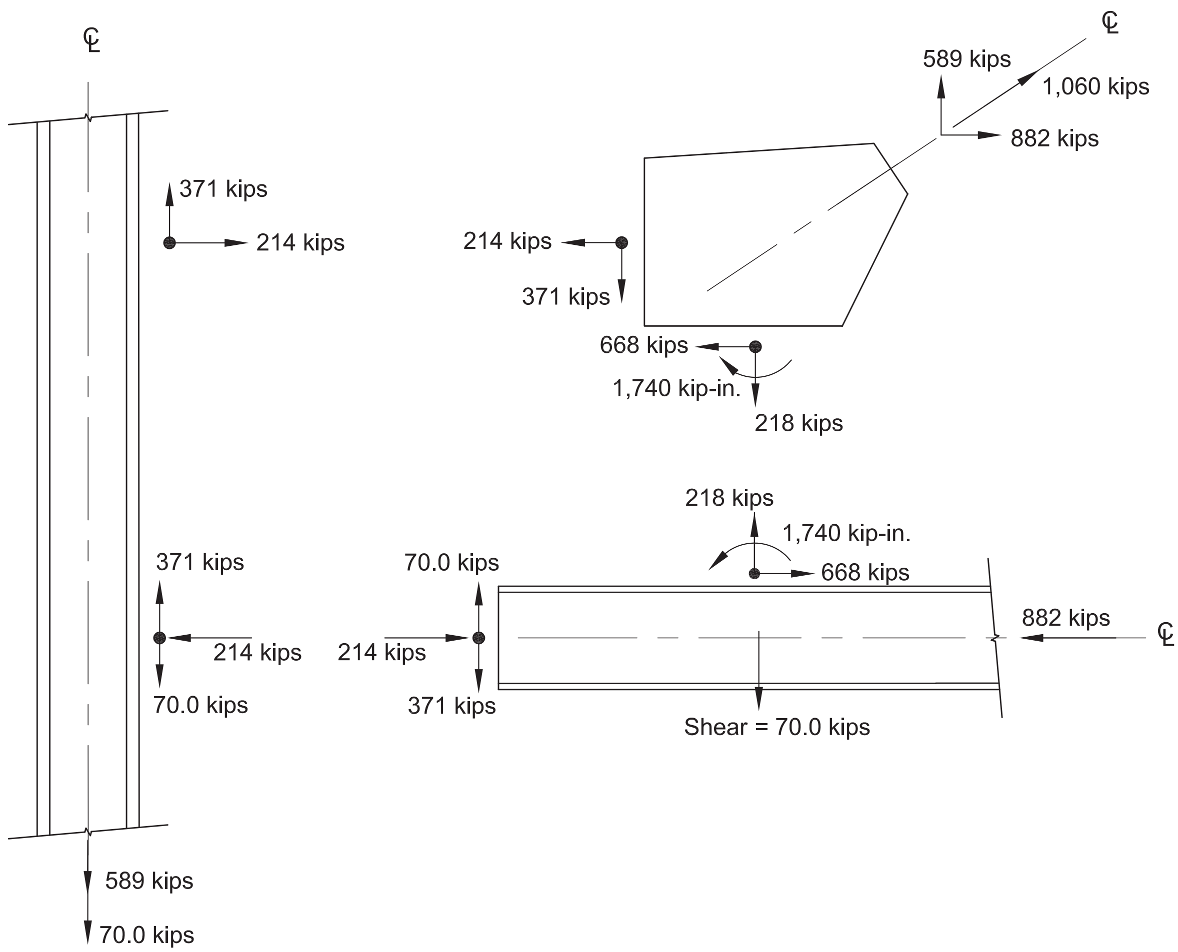


Fig. 6-5a. Admissible LRFD force field for connection shown in Figure 6-1—Configuration A of Figure 6-6.

Design beam web-to-column flange weld

Assume all shear is carried by the web weld.

Size the weld joining the beam web to the column flange. The length of weld is taken as 13¼ in. From AISC *Manual* Equation 8-2 and the directional strength increase permitted in AISC *Specification* Section J2.4 for fillet welds:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{204\text{ kips}}{288\text{ kips}}\right)$ $= 35.3°$ $D = \dfrac{\phi R_n}{1.392l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{\sqrt{(204\text{ kips})^2+(288\text{ kips})^2}}{(1.392\text{ kip/in.})(2)(13\tfrac{1}{4}\text{ in.})\left(1.0+0.50\sin^{1.5}35.3°\right)}$ $= 7.84$ sixteenths Use a ½-in. fillet weld.	$\theta = \tan^{-1}\left(\dfrac{136\text{ kips}}{192\text{ kips}}\right)$ $= 35.3°$ $D = \dfrac{R_n/\Omega}{0.928l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{\sqrt{(136\text{ kips})^2+(192\text{ kips})^2}}{(0.928\text{ kip/in.})(2)(13\tfrac{1}{4}\text{ in.})\left(1.0+0.50\sin^{1.5}35.3°\right)}$ $= 7.84$ sixteenths Use a ½-in. fillet weld.

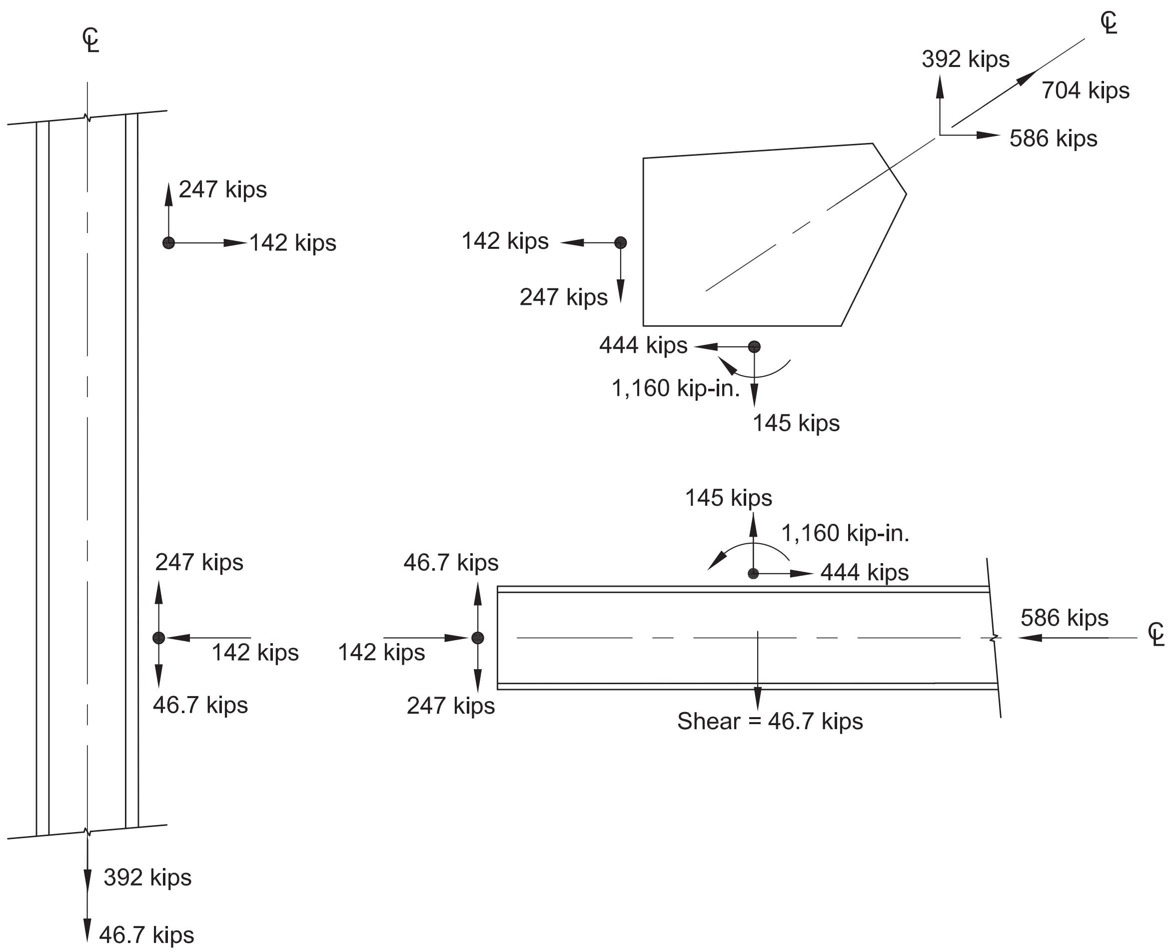


Fig. 6-5b. Admissible ASD force field for connection shown in Figure 6-1—Configuration A of Figure 6-6.

Design beam flange-to-column flange weld

The required weld length is:

$$\text{flange weld length} = 10.4\text{ in.} + 10.4\text{ in.} - 0.585\text{ in.}$$
$$= 20.2\text{ in.}$$

From AISC *Manual* Equation 8-2 and the directional strength increase permitted in AISC *Specification* Section J2.4 for fillet welds:

LRFD	ASD
$D = \dfrac{\phi R_n}{1.392l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{232\text{ kips}}{(1.392\text{ kip/in.})(20.2\text{ in.})\left(1.0+0.50\sin^{1.5}90°\right)}$ $= 5.50$ sixteenths Use a ⅜-in. fillet weld.	$D = \dfrac{R_n/\Omega}{0.928l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{154\text{ kips}}{(0.928\text{ kip/in.})(20.2\text{ in.})\left(1.0+0.50\sin^{1.5}90°\right)}$ $= 5.48$ sixteenths Use a ⅜-in. fillet weld.

A 1 in. × 1 in. clip is provided in the gusset plate to accommodate the top flange weld.

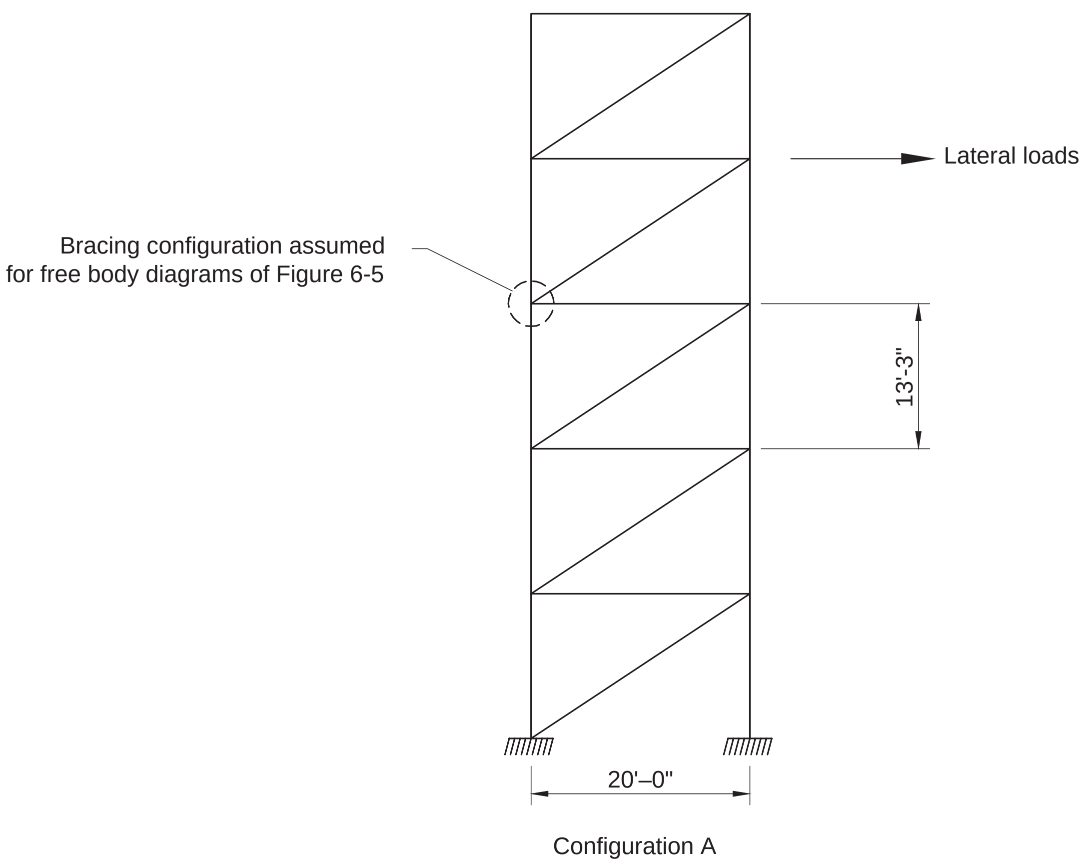


Fig. 6-6. Bracing arrangement with little to no transfer force.

Check for column stiffeners (continuity plates)

From AISC *Manual* Equation 4-2, Equation 4-4 and Table 4-1 for the W14×283 column, check web local yielding and flange local bending of the column:

LRFD	ASD
Web local yielding: $\phi R_n = P_{wo} + P_{wi} t_f$ $= 861 \text{ kips} + (64.5 \text{ kips})(0.985 \text{ in.})$ $= 925 \text{ kips} > 232 \text{ kips}$ **o.k.** Web compression buckling does not apply because only one flange is loaded.	Web local yielding: $R_n/\Omega = P_{wo} + t_f P_{wi}$ $= 574 \text{ kips} + (43.0 \text{ kips})(0.985 \text{ in.})$ $= 616 \text{ kips} > 154 \text{ kips}$ **o.k.** Web buckling does not apply because only one flange is loaded.

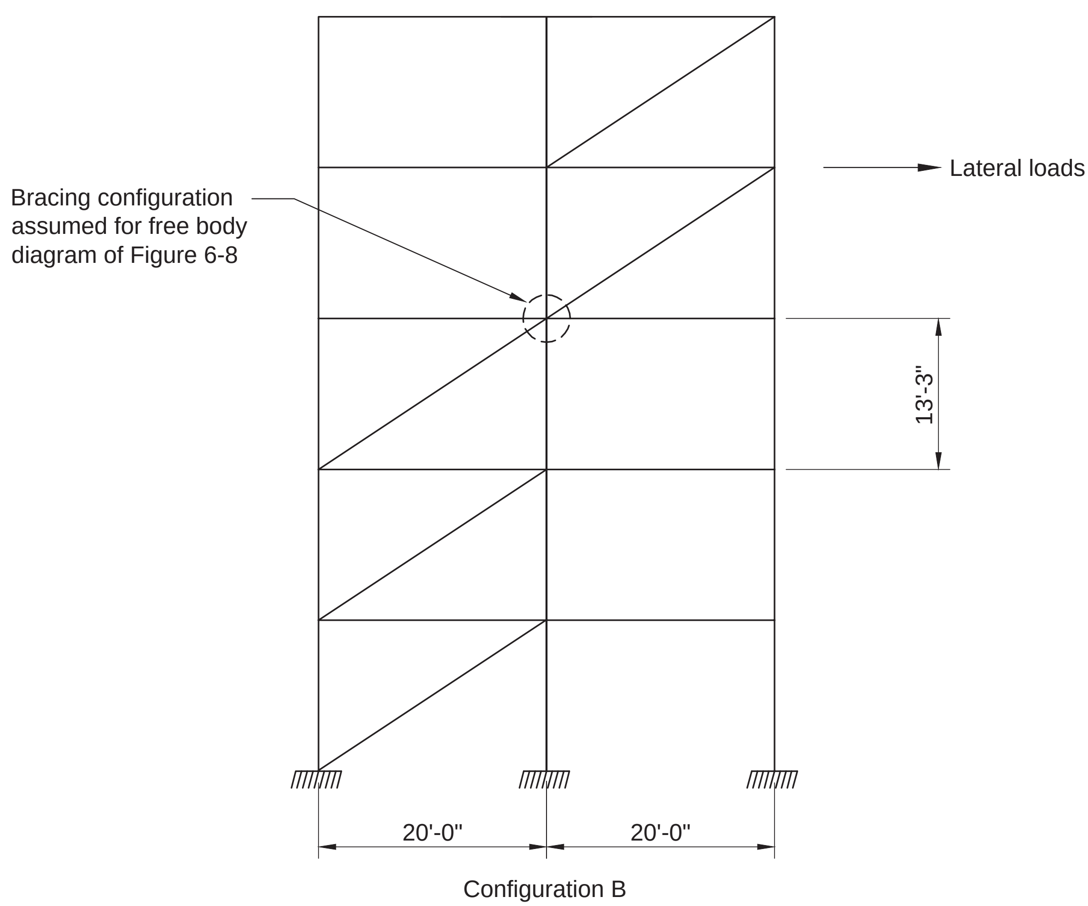


Fig. 6-7. Bracing arrangement with large transfer force.

LRFD	ASD
Flange local bending: $$\phi R_n = P_{fb}$$ $$= 1{,}210 \text{ kips} > 232 \text{ kips} \quad \textbf{o.k.}$$	Flange local bending: $$\frac{R_n}{\Omega} = P_{fb}$$ $$= 802 \text{ kips} > 154 \text{ kips} \quad \textbf{o.k.}$$

Therefore, no column stiffeners (continuity plates) are required.

Check column web local yielding and web local crippling adjacent to the beam web

AISC *Specification* Equation J10-2 is applicable for web local yielding in this case (the concentrated load is applied at a distance from the column end that is greater than the column depth, d); however, note that the $5k$ term is not used here because that portion of the web is already used by the flange connection. Refer to AISC *Manual* Figure 4-1.

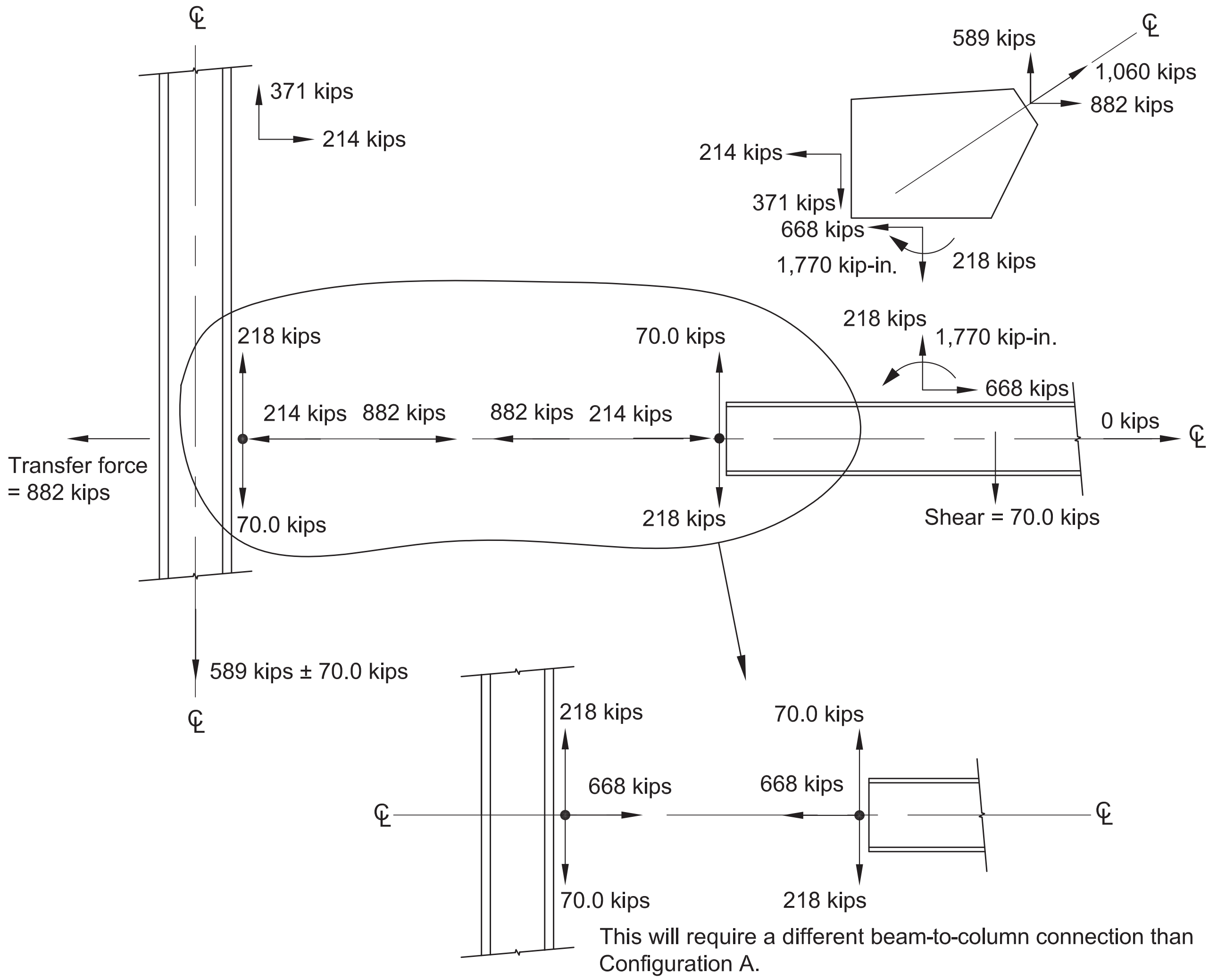


Fig. 6-8a. Admissible LRFD force field including transfer force—Configuration B.

LRFD	ASD
Web local yielding: $$\begin{aligned}\phi R_n &= \phi F_{yw} t_w l_b \\ &= 1.00(50\text{ ksi})(1.29\text{ in.})(13\tfrac{1}{4}\text{ in.}) \\ &= 855\text{ kips} > 204\text{ kips} \quad \textbf{o.k.}\end{aligned}$$	Web local yielding: $$\begin{aligned}\frac{R_n}{\Omega} &= \frac{F_{yw} t_w l_b}{\Omega} \\ &= \frac{(50\text{ ksi})(1.29\text{ in.})(13\tfrac{1}{4}\text{ in.})}{1.50} \\ &= 570\text{ kips} > 136\text{ kips} \quad \textbf{o.k.}\end{aligned}$$

Because $l_b/d = 13\tfrac{1}{4}\text{ in.}/16.7\text{ in.} = 0.793 > 0.2$, use AISC *Manual* Equation 9-48 with Table 9-4 to check the web local crippling strength of the W14×283 column:

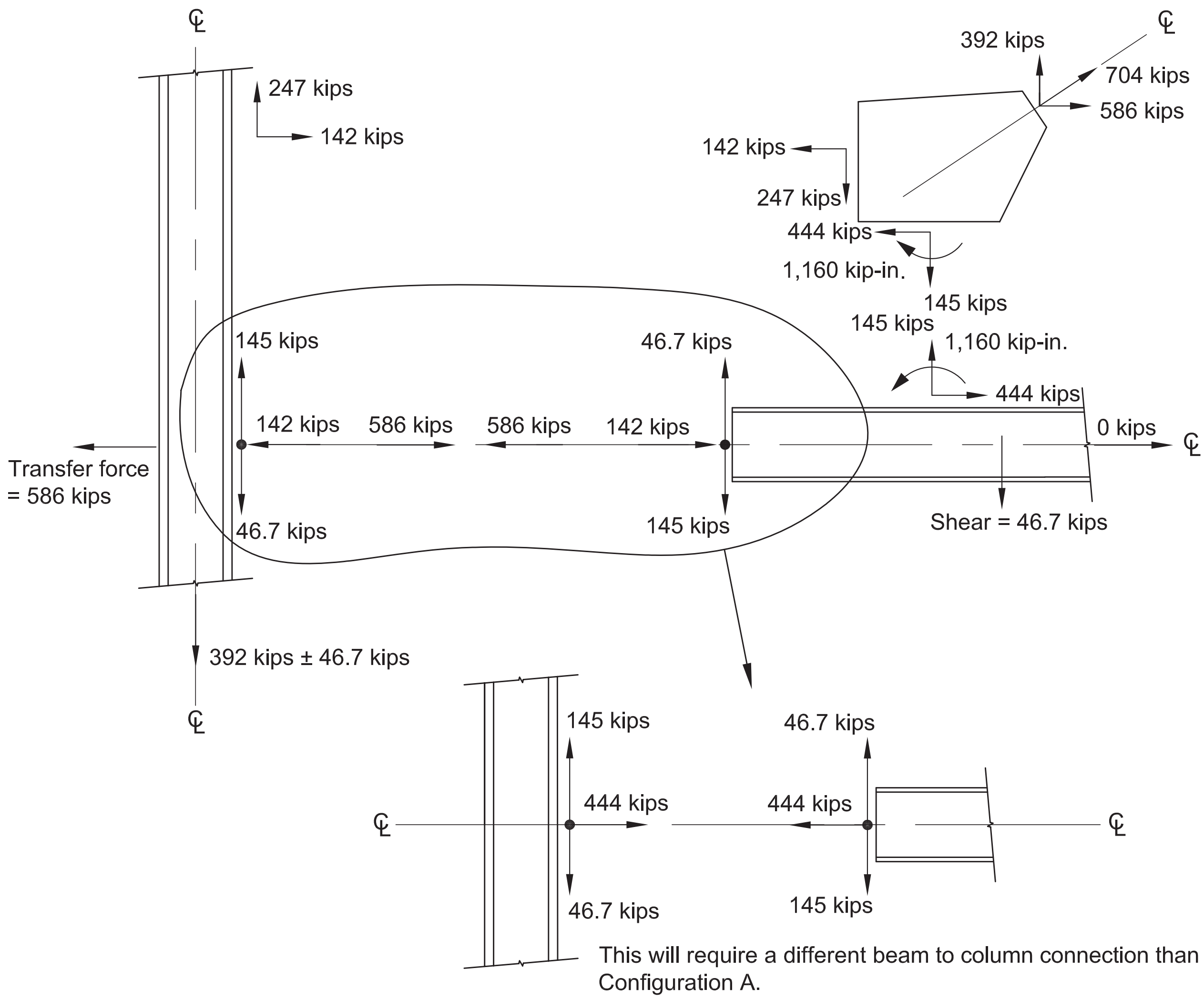


Fig. 6-8b. Admissible ASD force field including transfer force—Configuration B.

LRFD	ASD
Web local crippling: $$\phi R_n = \phi R_5 + l_b(\phi R_6)$$ $$= 2(687\text{ kips}) + 2(13\tfrac{1}{4}\text{ in.})(89.7\text{ kips})$$ $$= 3{,}750\text{ kips} > 204\text{ kips} \quad \textbf{o.k.}$$	Web local crippling: $$\frac{R_5}{\Omega} + l_b\left(\frac{R_6}{\Omega}\right) = 2(458\text{ kips}) + 2(13\tfrac{1}{4}\text{ in.})(59.8\text{ kips})$$ $$= 2{,}500\text{ kips} > 136\text{ kips} \quad \textbf{o.k.}$$

Gusset-to-Beam Connection

Using the same symbols used for the uniform force method, the required strengths at the gusset-to-beam connection interface as determined previously and shown in Figures 6-8a and 6-8b are:

LRFD	ASD
Required shear strength, H_{ub} = 668 kips	Required shear strength, H_{ab} = 444 kips
Required normal strength, V_{ub} = 218 kips	Required normal strength, V_{ab} = 145 kips
Required flexural strength, M_{ub} = 1,740 kip-in.	Required flexural strength, M_{ab} = 1,160 kip-in.

Gusset-to-beam contact length = 35¼ in. − 1-in. clip = 34.25 in.

Check gusset plate for shear yielding and tensile yielding along the beam flange

As determined, the gusset-to-beam interface length is the length of the gusset less the clip, or 34.25 in. In this example, to facilitate combining the required normal strength and flexural strength, stresses will be used to check the tensile and shear yielding limit states. Tensile yielding is checked using AISC *Specification* Section J4.1 and shear yielding is checked using Section J4.2:

LRFD	ASD
$$f_{ua} = \frac{V_{ub}}{A_g} = \frac{218\text{ kips}}{(1.00\text{ in.})(34.25\text{ in.})} = 6.36\text{ ksi}$$ $$f_{ub} = \frac{M_{ub}}{Z} = \frac{1{,}740\text{ kip-in.}}{(1.00\text{ in.})(34.25\text{ in.})^2/4} = 5.93\text{ ksi}$$ Total normal stress = f_{un} Tensile yielding strength: $$f_{un} = f_{ua} + f_{ub} = 6.36\text{ ksi} + 5.93\text{ ksi} = 12.3\text{ ksi} < 0.90(50\text{ ksi}) = 45\text{ ksi} \quad \textbf{o.k.}$$	$$f_{aa} = \frac{V_{ab}}{A_g} = \frac{145\text{ kips}}{(1.00\text{ in.})(34.25\text{ in.})} = 4.23\text{ ksi}$$ $$f_{ab} = \frac{M_{ab}}{Z} = \frac{1{,}160\text{ kip-in.}}{(1.00\text{ in.})(34.25\text{ in.})^2/4} = 3.96\text{ ksi}$$ Total normal stress = f_{an} Tensile yielding strength $$f_{an} = f_{aa} + f_{ab} = 4.23\text{ ksi} + 3.96\text{ ksi} = 8.19\text{ ksi} < \frac{50\text{ ksi}}{1.67} = 29.9\text{ ksi} \quad \textbf{o.k.}$$

LRFD	ASD
Shear yielding strength: $f_v = \dfrac{668\text{ kips}}{(1.00\text{ in.})(34.25\text{ in.})}$ $= 19.5\text{ ksi} < 1.00(0.60)(50\text{ ksi}) = 30\text{ ksi}$ **o.k.**	Shear yielding strength $f_v = \dfrac{444\text{ kips}}{(1.00\text{ in.})(34.25\text{ in.})}$ $= 13.0\text{ ksi} < \dfrac{0.60(50\text{ ksi})}{1.50} = 20.0\text{ ksi}$ **o.k.**

Consider the interaction equation derived from plasticity theory from Astaneh-Asl (1998) and Neal (1977) for gusset plates. A fourth order interaction equation is recommended. Neal states that this approximate interaction equation produces results within 5% of the theoretical lower-bound failure load for all possible combinations of shear, normal and flexural strengths.

The interaction equation is:

LRFD	ASD
$\left(\dfrac{M_{ub}}{\phi M_n}\right) + \left(\dfrac{V_{ub}}{\phi N_n}\right)^2 + \left(\dfrac{H_{ub}}{\phi V_n}\right)^4 \le 1$	$\left(\dfrac{M_{ab}}{M_n/\Omega}\right) + \left(\dfrac{V_{ab}}{N_n/\Omega}\right)^2 + \left(\dfrac{H_{ab}}{V_n/\Omega}\right)^4 \le 1$

where

$$
\begin{aligned}
M_n &= M_p \\
&= F_y Z_x && (\textit{Spec}.\text{ Eq. F2-1}) \\
&= F_y\left(\frac{t_g l^2}{4}\right) \\
&= (50\text{ ksi})\left[\frac{(1.00\text{ in.})(34.25\text{ in.})^2}{4}\right] \\
&= 14{,}700\text{ kip-in.}
\end{aligned}
$$

$$
\begin{aligned}
N_n &= F_y A_g && (\textit{Spec}.\text{ Eq. J4-1}) \\
&= F_y t_g l \\
&= (50\text{ ksi})(1.00\text{ in.})(34.25\text{ in.}) \\
&= 1{,}710\text{ kips}
\end{aligned}
$$

$$
\begin{aligned}
V_n &= 0.60 F_y A_g && (\textit{Spec}.\text{ Eq. J4-3}) \\
&= 0.60 F_y t l \\
&= 0.60(50\text{ ksi})(1.00\text{ in.})(34.25\text{ in.}) \\
&= 1{,}030\text{ kips}
\end{aligned}
$$

LRFD	ASD
$\left[\dfrac{1{,}740\text{ kip-in.}}{0.90(14{,}700\text{ kip-in.})}\right] + \left[\dfrac{218\text{ kips}}{0.90(1{,}710\text{ kips})}\right]^2$ $+ \left[\dfrac{668\text{ kips}}{1.00(1{,}030\text{ kips})}\right]^4 = 0.328 < 1$ **o.k.**	$\left[\dfrac{1{,}160\text{ kip-in.}}{(14{,}700\text{ kip-in.}/1.67)}\right] + \left[\dfrac{145\text{ kips}}{(1{,}710\text{ kips}/1.67)}\right]^2$ $+ \left[\dfrac{444\text{ kips}}{(1{,}030\text{ kips}/1.50)}\right]^4 = 0.327 < 1$ **o.k.**

It can be seen from this interaction calculation that interaction is not critical. It in fact very seldom controls. The authors have checked many connections of this type and have never had a case where interaction controlled. It is checked here for the sake of illustration.

Design gusset-to-beam weld

The AISC *Manual* Part 13 recommends an increase in design force for a welded connection between a gusset and a member flange to account for uneven load distribution across the weld's surface. The average stress should thus be multiplied by a factor of 1.25 and then compared to the peak stress, and the greater of the two should be selected as the design stress for the weld.

From Hewitt and Thornton (2004):

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{2l}$	$f_{aa} = \dfrac{H_{ab}}{2l}$
$= \dfrac{218 \text{ kips}}{2(34.25 \text{ in.})}$	$= \dfrac{145 \text{ kips}}{2(34.25 \text{ in.})}$
$= 3.18 \text{ kip/in.}$	$= 2.12 \text{ kip/in.}$
$f_{ub} = \dfrac{2M_{ub}}{l^2}$	$f_{ab} = \dfrac{2M_a}{l^2}$
$= \dfrac{2(1{,}740 \text{ kip-in.})}{(34.25 \text{ in.})^2}$	$= \dfrac{2(1{,}160 \text{ kip-in.})}{(34.25 \text{ in.})^2}$
$= 2.97 \text{ kip/in.}$	$= 1.98 \text{ kip/in.}$
$f_{un} = f_{ua} + f_{ub}$	$f_{an} = f_{aa} + f_{ab}$
$= 3.18 \text{ kip/in.} + 2.97 \text{ kip/in.}$	$= 2.12 \text{ kip/in.} + 1.98 \text{ kip/in.}$
$= 6.15 \text{ kip/in.}$	$= 4.10 \text{ kip/in.}$
$f_{uv} = \dfrac{H_{ub}}{2l}$	$f_{av} = \dfrac{V_a}{2l}$
$= \dfrac{668 \text{ kips}}{2(34.25 \text{ in.})}$	$= \dfrac{444 \text{ kips}}{2(34.25 \text{ in.})}$
$= 9.75 \text{ kip/in.}$	$= 6.48 \text{ kip/in.}$
$f_{peak} = \sqrt{(f_{ua} + f_{ub})^2 + {f_{uv}}^2}$	$f_{peak} = \sqrt{{f_{an}}^2 + {f_{av}}^2}$
$= \sqrt{(6.15 \text{ kip/in.})^2 + (9.75 \text{ kip/in.})^2}$	$= \sqrt{(4.10 \text{ kip/in.})^2 + (6.48 \text{ kip/in.})^2}$
$= 11.5 \text{ kip/in.}$	$= 7.67 \text{ kip/in.}$
$f_{avg} = \dfrac{1}{2}\left[\sqrt{(f_{ua} - f_{ub})^2 + {f_{uv}}^2} + \sqrt{(f_{ua} + f_{ub})^2 + {f_{uv}}^2}\right]$	$f_{avg} = \dfrac{1}{2}\left[\sqrt{(f_{aa} - f_{ab})^2 + {f_{av}}^2} + \sqrt{(f_{aa} + f_{ab})^2 + {f_{av}}^2}\right]$
$= \dfrac{1}{2}\left[\begin{array}{l}\sqrt{(3.18 \text{ kip/in.} - 2.97 \text{ kip/in.})^2 + (9.75 \text{ kip/in.})^2} \\ +11.5 \text{ kip/in.}\end{array}\right]$	$= \dfrac{1}{2}\left[\begin{array}{l}\sqrt{(2.12 \text{ kip/in.} - 1.98 \text{ kip/in.})^2 + (6.48 \text{ kip/in.})^2} \\ +7.67 \text{ kip/in.}\end{array}\right]$
$= 10.6 \text{ kip/in.}$	$= 7.08 \text{ kip/in.}$

LRFD	ASD
$1.25f_{avg} = 1.25(10.6\text{ kip/in.})$ $= 13.3\text{ kip/in.} > 11.5\text{ kip/in.}$ Use $1.25 f_{avg}$. $\theta = \tan^{-1}\left(\dfrac{f_{un}}{f_{uv}}\right)$ $= \tan^{-1}\left(\dfrac{6.15\text{ kip/in.}}{9.75\text{ kip/in.}}\right)$ $= 32.2°$ From AISC *Manual* Equation 8-2: $D_{req'd} = \dfrac{13.3\text{ kip/in.}}{(1.392\text{ kip/in.})(1.0+0.50\sin^{1.5} 32.2°)}$ $= 8.00$ sixteenths	$1.25f_{avg} = 1.25(7.08\text{ kip/in.})$ $= 8.85\text{ kip/in.} > 7.67\text{ kip/in.}$ Use $1.25 f_{avg}$. $\theta = \tan^{-1}\left(\dfrac{f_{an}}{f_{av}}\right)$ $= \tan^{-1}\left(\dfrac{4.10\text{ kip/in.}}{6.48\text{ kip/in.}}\right)$ $= 32.3°$ From AISC *Manual* Equation 8-2: $D_{req'd} = \dfrac{8.85\text{ kip/in.}}{(0.928\text{ kip/in.})(1.0+0.50\sin^{1.5} 32.3°)}$ $= 7.98$ sixteenths

Use a 9⁄16-in. fillet weld on both sides of the gusset plate.

Determine equivalent normal force, N_e

The concentrated force for which the beam must be designed includes both the normal force shown in Figure 6-8 and an additional normal force due to the moment. For convenience, convert the moment into an equivalent normal force and add it to the normal force:

LRFD	ASD
$N_e = V_{ub} + \dfrac{2M_{ub}}{\left(\dfrac{l}{2}\right)}$ $= 218\text{ kips} + \dfrac{2(1{,}740\text{ kip-in.})}{\left(\dfrac{34.25\text{ in.}}{2}\right)}$ $= 421\text{ kips}$	$N_e = V_{ab} + \dfrac{2M_{ab}}{\left(\dfrac{l}{2}\right)}$ $= 145\text{ kips} + \dfrac{2(1{,}160\text{ kip-in.})}{\left(\dfrac{34.25\text{ in.}}{2}\right)}$ $= 280\text{ kips}$

Check beam web local yielding

The concentrated force, which is the resultant force from the gusset plate, acts at a distance of 34.25 in./2 = 17.1 in. from the end of the beam. From AISC *Specification* Section J10.2, use Equation J10-2 because this distance is greater than the beam depth:

LRFD	ASD
$\phi R_n = \phi F_{yw} t_w (5k_{des} + l_b)$ $= 1.00(50\text{ ksi})(0.585\text{ in.})[5(1.39\text{ in.}) + 34.25\text{ in}]$ $= 1{,}210\text{ kips} > 421\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_{yw} t_w (5k_{des} + l_b)}{\Omega}$ $= \dfrac{(50\text{ ksi})(0.585\text{ in.})[5(1.39\text{ in.}) + 34.25\text{ in.}]}{1.50}$ $= 803\text{ kips} > 280\text{ kips}$ **o.k.**

Check beam web local crippling

As determined for web local yielding, the concentrated force acts at a distance of 17.1 in. from the beam end which is greater than $d/2$. Therefore, from AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= 0.75(0.80)(0.585\text{ in.})^2\left[1+3\left(\dfrac{34.25\text{ in.}}{17.0\text{ in.}}\right)\left(\dfrac{0.585\text{ in.}}{0.985\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(0.985\text{ in.})}{0.585\text{ in.}}}$ $= 1{,}210\text{ kips} > 421\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80t_w^2}{\Omega}\left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= \dfrac{(0.80)(0.585\text{ in.})^2}{2.00}\left[1+3\left(\dfrac{34.25\text{ in.}}{17.0\text{ in.}}\right)\left(\dfrac{0.585\text{ in.}}{0.985\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{29{,}000\text{ ksi}(50\text{ ksi})(0.985\text{ in.})}{0.585\text{ in.}}}$ $= 806\text{ kips} > 280\text{ kips}$ **o.k.**

Column-to-Gusset Connection

The required strengths are:

LRFD	ASD
Required normal strength: $H_{uc} = 214$ kips Required shear strength: $V_{uc} = 371$ kips	Required normal strength: $H_{ac} = 142$ kips Required shear strength: $V_{ac} = 247$ kips

The length of the weld is 27 in. as shown in Figure 6-1, with the 1 in. × 1 in. clip. Because there is no moment on this connection, the average and peak stresses are the same; therefore, the weld ductility factor discussed in AISC *Manual* Part 13 of 1.25 will be included. From AISC *Manual* Equation 8-2 and the directional strength increase permitted in AISC *Specification* Section J2.4 for fillet welds, the fillet weld size is determined as follows:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{214\text{ kips}}{371\text{ kips}}\right)$ $= 30.0°$ $D = 1.25\left[\dfrac{R_u}{1.392l\left(1.0+0.50\sin^{1.5}\theta\right)}\right]$ $= 1.25\left[\dfrac{\sqrt{(214\text{ kips})^2+(371\text{ kips})^2}}{(1.392\text{ kip/in.})(2)(27\text{ in.})\left(1.0+0.50\sin^{1.5}30°\right)}\right]$ $= 6.05$ sixteenths Use a 7⁄16-in. fillet weld.	$\theta = \tan^{-1}\left(\dfrac{142\text{ kips}}{247\text{ kips}}\right)$ $= 29.9°$ $D = 1.25\left[\dfrac{R_a}{0.928l\left(1.0+0.50\sin^{1.5}\theta\right)}\right]$ $= 1.25\left[\dfrac{\sqrt{(142\text{ kips})^2+(247\text{ kips})^2}}{(0.928\text{ kip/in.})(2)(27\text{ in.})\left(1.0+0.50\sin^{1.5}29.9°\right)}\right]$ $= 6.04$ sixteenths Use a 7⁄16-in. fillet weld.

Column web local yielding and web local crippling

Based on the beam web check, these are assumed to be o.k. because the column is a heavier shape and the required normal strength is nearly the same as that applied to the beam.

Check column shear strength

The column required shear strength is equal to the required normal force of 214 kips (LRFD) or 142 kips (ASD). From AISC *Manual* Table 3-6:

LRFD	ASD
$\phi_v V_n = 646 \text{ kips} > 214 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 431 \text{ kips} > 142 \text{ kips}$ **o.k.**

Moment Release Shear Splice

There are two design possibilities.

1. The splice must be capable of transporting the 70 kips (LRFD) or 46.7 kips (ASD) beam gravity shear force to the face of the column. This is necessary because this is the boundary condition usually assumed in the design of the beam. Also, this splice must be checked for rotational ductility in accordance with AISC *Manual* Part 10 to qualify as a simple connection in accordance with AISC *Specification* Section B3.6a. It might be possible to satisfy AISC *Specification* Section B3.6a without satisfying the checks of Part 10 of the AISC *Manual* if the rotational demand at the end of the beam is small. However, a good deal of judgment may be involved and in most instances satisfying the rotational ductility checks in the AISC *Manual* provides a cleaner solution.

2. If the structural analysis is performed with an actual moment release at the splice location, then the splice could be designed for the gravity shear without consideration of the moment required to transport the shear to the column face. This splice, being a simple shear splice, will satisfy AISC *Specification* Section B3.6a.

The first option will be used here. Use two ½-in.-thick ASTM A572 Grade 50 splice plates.

Check bolt shear strength

With three vertical rows of four bolts, the distance, e_x, from the face of the column to the centroid of the farthest bolt group is, from Figure 6-1:

$$\begin{aligned} e_x &= 35\tfrac{1}{4} \text{ in.} + 1 \text{ in.} + 1 \text{ in.} + 2.5 \text{ in.} + 3 \text{ in.} \\ &= 42.75 \text{ in.} \end{aligned}$$

This value exceeds the maximum eccentricity given in AISC *Manual* Table 7-10 of 36 in. A computer program based on the AISC *Manual* inelastic theory used in Table 7-10 yields $C = 1.04$. The 1.04 value will be used.

The bolts are 1-in.-diameter ASTM A490-X in standard 1$\tfrac{1}{16}$-in.-diameter holes. From AISC *Manual* Table 7-1, the available shear strength of the bolts in double shear is:

LRFD	ASD
$\phi r_n = 98.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 65.9 \text{ kips/bolt}$

The available strength of the eccentrically loaded bolt group is:

LRFD	ASD
$\begin{aligned} \phi R_n &= 1.04(98.9 \text{ kips/bolt}) \\ &= 103 \text{ kips} > 70.0 \text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= 1.04(65.9 \text{ kips/bolt}) \\ &= 68.5 \text{ kips} > 46.7 \text{ kips} \quad \textbf{o.k.} \end{aligned}$

Check bolt bearing on the beam web and splice plates

Assuming that deformation at the bolt hole is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(Spec. Eq. J3-6a)}$$

With a 2.5-in. end distance in the beam (to the top of the beam) and the bolt hole diameter of 1 1⁄16 in., the bolt spacing controls l_c:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - 1\tfrac{1}{16} \text{ in.} \\ &= 1.94 \text{ in.} \end{aligned}$$

The available strength of one bolt on the beam web is determined as follows:

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.94 \text{ in.})(0.585 \text{ in.})(65 \text{ ksi})$ $= 66.4 \text{ kips/bolt}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(0.585 \text{ in.})(65 \text{ ksi})$ $= 68.4 \text{ kips/bolt}$ Therefore, $\phi r_n = 66.4$ kips/bolt < 98.9 kips/bolt.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.94 \text{ in.})(0.585 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 44.3 \text{ kips/bolt}$ $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(1.00 \text{ in.})(0.585 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 45.6 \text{ kips/bolt}$ Therefore, $r_n/\Omega = 44.3$ kips/bolt < 65.9 kips/bolt.

The bearing strength controls over the bolt shear strength. The available strength of the eccentrically loaded bolt group is:

LRFD	ASD
$\phi R_n = 1.04(66.4 \text{ kips/bolt})$ $= 69.1 \text{ kips} < 70.0 \text{ kips}$ **n.g.**	$\dfrac{R_v}{\Omega} = 1.04(44.3 \text{ kips/bolt})$ $= 46.1 \text{ kips} < 46.7 \text{ kips}$ **n.g.**

Note that the calculations assume the worst case for tearout, the resultant force per bolt acts in the most critical direction, and all bolts are assumed to be in the same critical condition. In reality, the force resultant per bolt will have a different direction for each bolt, and some will have a much larger edge distance than 2½ in. or 3 in. If the true distribution is obtained from a computer program based on the AISC *Manual* inelastic bolt strength, and using the actual edge distances when determining l_c for each bolt, the available strength is:

LRFD	ASD
$\phi R_n = 71.1 \text{ kips} > 70.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 47.4 \text{ kips} > 46.7 \text{ kips}$ **o.k.**

Therefore, a web doubler is not required based on the bearing strength of the beam web.

The available strength per bolt on the splice plates (two ½-in.-thick splice plates) based on bearing and tearout strength is determined as follows:

$$\begin{aligned} l_c &= 1.50 \text{ in.} - 1\tfrac{1}{16} \text{ in.}/2 \\ &= 0.969 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(0.969\text{ in.})(2)(\tfrac{1}{2}\text{ in.})(65\text{ ksi})$ $= 56.7\text{ kips/bolt}$ $\phi r_n = \phi 2.4 d t F_u$ $= 0.75(2.4)(1.00\text{ in.})(2)(\tfrac{1}{2}\text{ in.})(65\text{ ksi})$ $= 117\text{ kips/bolt}$ Therefore, $\phi r_n = 56.7$ kips/bolt < 98.9 kips/bolt (bolt shear).	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(0.969\text{ in.})(2)(\tfrac{1}{2}\text{ in.})(65\text{ ksi})}{2.00}$ $= 37.8\text{ kips/bolt}$ $\dfrac{r_n}{\Omega} = \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{2.4(1.00\text{ in.})(2)(\tfrac{1}{2}\text{ in.})(65\text{ ksi})}{2.00}$ $= 78.0\text{ kips/bolt}$ Therefore, $r_n/\Omega = 37.8$ kips/bolt < 65.9 kips/bolt (bolt shear).

The available strength of the eccentrically loaded bolt group is:

LRFD	ASD
$\phi R_n = 1.04(56.7\text{ kips/bolt})$ $= 59.0\text{ kips} < 70.0\text{ kips}$ **n.g.**	$\dfrac{R_n}{\Omega} = 1.04(37.8\text{ kips/bolt})$ $= 39.3\text{ kips} < 46.7\text{ kips}$ **n.g.**

Thicker splice plates are required. Try two ⅝-in.-thick plates:

LRFD	ASD
$\phi r_n = (56.7\text{ kips/bolt})\left[\dfrac{2(\tfrac{5}{8}\text{ in.})}{2(\tfrac{1}{2}\text{ in.})}\right]$ $= 70.9\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (37.8\text{ kips/bolt})\left[\dfrac{2(\tfrac{5}{8}\text{ in.})}{2(\tfrac{1}{2}\text{ in.})}\right]$ $= 47.3\text{ kips/bolt}$

Tearout on the splice plates controls over bolt shear (98.9 kips for LRFD and 65.9 kips for ASD). The available strength of the eccentrically loaded bolt group is:

LRFD	ASD
$\phi R_n = 1.04(70.9\text{ kips/bolt})$ $= 73.7\text{ kips} > 70.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 1.04(47.3\text{ kips/bolt})$ $= 49.2\text{ kips} > 46.7\text{ kips}$ **o.k.**

Check splice plates for global effects

The shear on the splice is 70.0 kips (LRFD) or 46.7 kips (ASD). The moment on the splice, at the critical location (the first vertical line of bolts to the right of the splice) is:

LRFD	ASD
$M_u = (70.0\text{ kips})(39.75\text{ in.})$ $= 2{,}780\text{ kip-in.}$	$M_a = (46.7\text{ kips})(39.75\text{ in.})$ $= 1{,}860\text{ kip-in.}$

The available shear rupture and flexural strengths of the splice plates are:

LRFD	ASD
From AISC *Specification* Section J4.2: $$\phi V_n = \phi 0.60 F_y A_g$$ $$= 1.00(0.60)(50\text{ ksi})(2)(\tfrac{5}{8}\text{ in.})(12\text{ in.})$$ $$= 450\text{ kips} > 70\text{ kips} \quad \textbf{o.k.}$$ $$\phi V_n = \phi 0.60 F_u A_{nv}$$ $$= 0.75(0.60)(65\text{ ksi})(2)(\tfrac{5}{8}\text{ in.})$$ $$\times\left[12.0\text{ in.} - 4\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$$ $$= 274\text{ kips} > 70\text{ kips} \quad \textbf{o.k.}$$ From AISC *Specification* Section J4.5: $$\phi M_n = \phi F_y Z_x$$ $$= 0.90(50\text{ ksi})\left[2(\tfrac{5}{8}\text{ in.})(12\text{ in.})^2/4\right]$$ $$= 2{,}030\text{ kip-in.} < 2{,}780\text{ kip-in.} \quad \textbf{n.g.}$$	From AISC *Specification* Section J4.2: $$\frac{V_n}{\Omega} = \frac{0.60 F_y A_g}{\Omega}$$ $$= \frac{0.60(50\text{ ksi})(2)(\tfrac{5}{8}\text{ in.})(12\text{ in.})}{1.50}$$ $$= 300\text{ kips} > 46.7\text{ kips} \quad \textbf{o.k.}$$ $$\frac{V_n}{\Omega} = \frac{0.60 F_u A_{nv}}{\Omega}$$ $$= 0.60(65\text{ ksi})(2)(\tfrac{5}{8}\text{ in.})$$ $$\times\left[12.0\text{ in.} - 4\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]/2.00$$ $$= 183\text{ kips} > 46.7\text{ kips} \quad \textbf{o.k.}$$ From AISC *Specification* Section J4.5: $$\frac{M_n}{\Omega_b} = \frac{F_y Z_x}{\Omega_b}$$ $$= \frac{(50\text{ ksi})\left[2(\tfrac{5}{8}\text{ in.})(12\text{ in.})^2/4\right]}{1.67}$$ $$= 1{,}350\text{ kip-in.} < 1{,}860\text{ kip-in.} \quad \textbf{n.g.}$$

Try two ⅞-in.-thick splice plates:

LRFD	ASD
$$\phi M_n = 2{,}030\text{ kip-in.}\left[\frac{2(\tfrac{7}{8}\text{ in.})}{2(\tfrac{5}{8}\text{ in.})}\right]$$ $$= 2{,}840\text{ kip-in.} > 2{,}780\text{ kip-in.} \quad \textbf{o.k.}$$	$$\frac{M_n}{\Omega} = 1{,}350\text{ kip-in.}\left[\frac{2(\tfrac{7}{8}\text{ in.})}{2(\tfrac{5}{8}\text{ in.})}\right]$$ $$= 1{,}890\text{ kip-in.} > 1{,}860\text{ kip-in.} \quad \textbf{o.k.}$$

Use two ⅞-in.-thick splice plates.

Ductility checks on the splice

The splice can be considered to be a double extended shear plate connection as presented in the AISC *Manual* Part 10. Equation 10-4 of the AISC *Manual* is a ductility check. When the shear plate or splice plates satisfy this equation, the plate or plates are guaranteed to yield before the bolts fracture. For a shear plate connection, the weld must satisfy a ductility requirement also; that is, the fillet weld size on each side of the plate must be at least ⅝ times the plate thickness. This weld requirement is satisfied by the two bolt groups that form the splice. Following the AISC *Manual*:

$$M_{max} = \frac{F_v}{0.90}(A_b C') \qquad \text{(\textit{Manual} Eq. 10-4)}$$

where

F_v = 84 ksi (from AISC *Specification* Table J3.2)

$$A_b = \frac{\pi}{4}(1.00\text{ in.})^2$$
$$= 0.785\text{ in.}^2$$

C' = 44.7 in. (from AISC *Manual* Table 7-10)

Therefore:

$$M_{max} = \frac{84.0 \text{ ksi}}{0.90}\left(0.785 \text{ in.}^2\right)\left(44.7 \text{ in.}\right)$$
$$= 3,280 \text{ kip-in.}$$

$$t_{max} = \frac{6M_{max}}{F_y d^2} \qquad \text{(\textit{Manual} Eq. 10-3)}$$
$$= \frac{6\left(3,280 \text{ kip-in.}\right)}{\left(50 \text{ ksi}\right)\left(12.0 \text{ in.}\right)^2}$$
$$= 2.73 \text{ in.} > 2\left(\tfrac{7}{8} \text{ in.}\right) = 1.75 \text{ in.} \quad \textbf{o.k.}$$

Note that this check applies equally to both LRFD and ASD design methods.

The completed connection design is shown in Figure 6-1.

Example 6.1b—Bracing Connections for Systems not Specifically Detailed for Seismic Resistance (*R* = 3)

Given:

This example has the same geometry as Example 6.1a. Its purpose is to show the difference in the resulting designs when seismic detailing requirements can be avoided. In this example, the brace force (the required strength of the connection) is defined by brace buckling and not $R_yF_yA_g$, as was required in the previous example with $R > 3$. When designing connections for systems not specifically detailed for seismic resistance, the brace *and* the connection are designed for the same force. In a special concentrically braced frame (SCBF), the connection is designed for the expected strength of the brace ($R_yF_yA_g$).

Refer to Figure 6-9 for the final design. Design the HSS heavy bracing connection for the W16×100 beam and W14×283 column shown. Use 1-in.-diameter ASTM A490-X bolts in standard holes and 70-ksi electrodes. The shape materials are given in Figure 6-9. The brace length is 24 ft (work point-to-work point). For the single-plate connections, use ½-in.-thick plate for the gusset-to-column connection and ½-in.-thick plate for the beam-to-column connection, initially (¾-in.-thick plate is used in the final design).

The required strengths of the brace from seismic and wind loads are as follows:

LRFD	ASD
$P_u = 300$ kips (wind) $P_u = 290$ kips (seismic)	$P_a = 200$ kips (wind) $P_a = 193$ kips (seismic)

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A36
$F_y = 36$ ksi $\quad F_u = 58$ ksi

ASTM A572, Grade 50
$F_y = 50$ ksi $\quad F_u = 65$ ksi

ASTM A500, Grade B
$F_y = 46$ ksi $\quad F_u = 58$ ksi

From AISC *Manual* Table 1-1 and Table 1-12 the geometric member properties are as follows:

Beam
W16×100
$t_w = 0.585$ in. $t_f = 0.985$ in. $d = 17.0$ in. $k_{des} = 1.39$ in.

Column
W14×283
$t_w = 1.29$ in. $k_{des} = 2.67$ in. $d = 16.7$ in. $t_f = 2.07$ in.

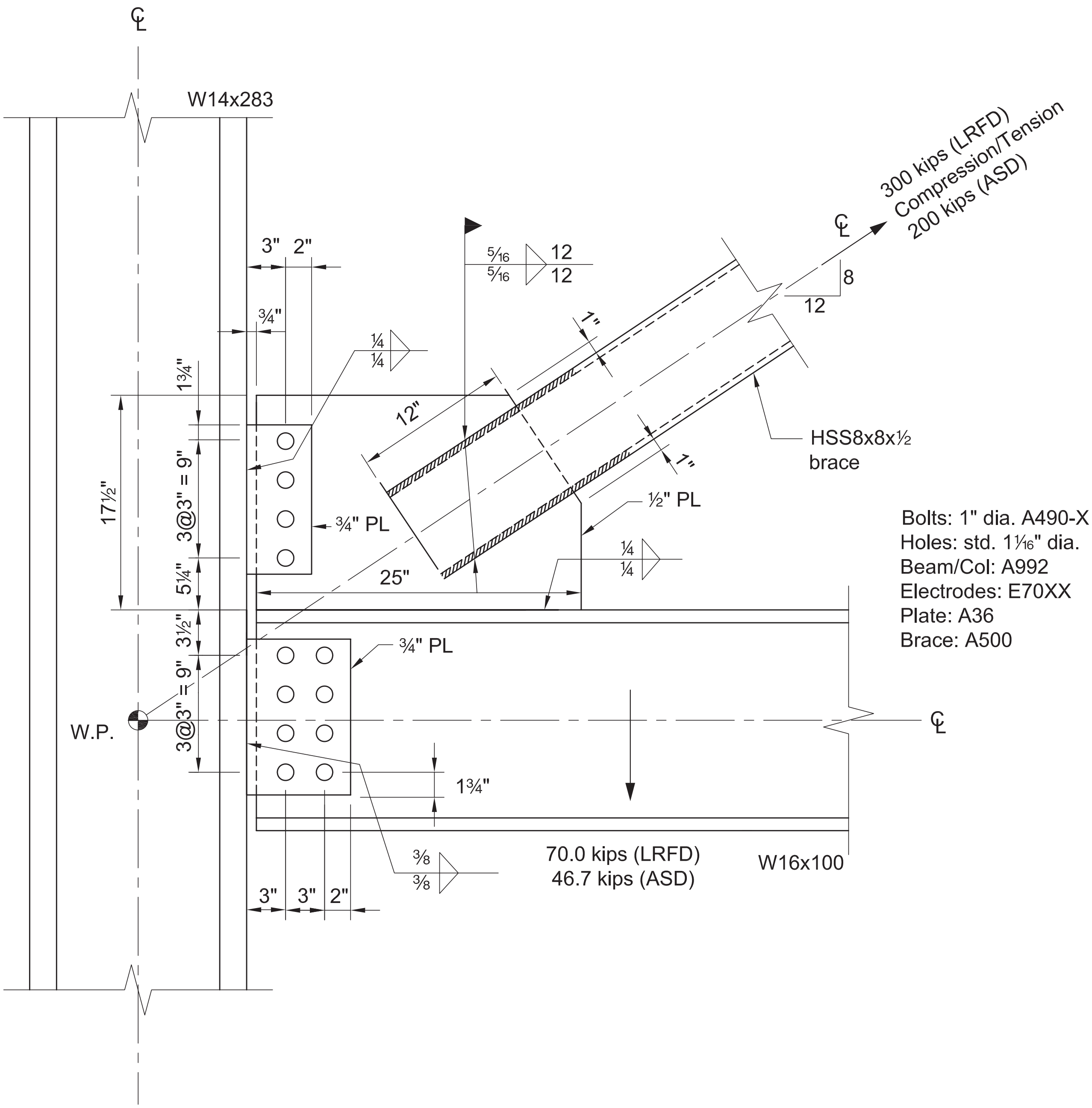


Fig. 6-9. Low-seismic connection for the same location as the connection shown in Figure 6-1.

Brace
HSS8×8×½
$t = 0.465$ in. $A_g = 13.5$ in.^{2} $b/t = 14.2$ $r = 3.04$ in.

Brace-to-Gusset Connection

Choose an HSS brace

Try an HSS8×8×½. From AISC *Manual* Table 4-4, for a 24-ft-long brace, the available compressive strength is:

LRFD	ASD
$\phi_c P_n = 306 \text{ kips (compression)}$ $> 300 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 203 \text{ kips (compression)}$ $> 200 \text{ kips}$ **o.k.**

From AISC *Manual* Table 5-5, the available tensile yielding strength is:

LRFD	ASD
$\phi_t P_n = 559 \text{ kips}$ $> 300 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 372 \text{ kips}$ $> 200 \text{ kips}$ **o.k.**

The available tensile rupture strength of the HSS effective area must be greater than the required tensile strength or reinforcement plates will be required. Assume that the slot in the brace is $\frac{1}{16}$ in. wider than the gusset plate on each side of the gusset plate. Assume a gusset plate thickness of ½ in. and verify. Start with a brace-to-gusset weld length of 12 in. From AISC *Specification* Section D3, determine the effective area:

$$A_e = A_n U \quad \text{(}Spec\text{. Eq. D3-1)}$$

$$\begin{aligned} A_n &= A_g - td_h \\ &= 13.5 \text{ in.}^2 - 2(0.465 \text{ in.})\left(\tfrac{1}{2} \text{ in.} + \tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right) \\ &= 12.9 \text{ in.}^2 \end{aligned}$$

$$U = 1 - \frac{\bar{x}}{l}$$

$$\bar{x} = \frac{B^2 + 2BH}{4(B+H)} \quad \text{(}Spec\text{. Table D3-1)}$$

$$\begin{aligned} &= \frac{(8.00 \text{ in.})^2 + 2(8.00 \text{ in.})(8.00 \text{ in.})}{4(8.00 \text{ in.} + 8.00 \text{ in.})} \\ &= 3.00 \text{ in.} \end{aligned}$$

$$\begin{aligned} U &= 1 - \frac{3.00 \text{ in.}}{12.0 \text{ in.}} \\ &= 0.75 \end{aligned}$$

$$\begin{aligned} A_e &= 0.75\left(12.9 \text{ in.}^2\right) \\ &= 9.68 \text{ in.}^2 \end{aligned}$$

The available tensile rupture strength of the column is determined from AISC *Specification* Equation D2-2:

LRFD	ASD
$\phi_t P_n = \phi_t F_u A_e$ $= 0.75(58\text{ ksi})(9.68\text{ in.}^2)$ $= 421\text{ kips} > 300\text{ kips}$ **o.k.** Therefore, no reinforcement plates are needed.	$\dfrac{P_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(58\text{ ksi})(9.68\text{ in.}^2)}{2.00}$ $= 281\text{ kips} > 200\text{ kips}$ **o.k.** Therefore, no reinforcement plates are needed.

The HSS8×8×½ is acceptable. No high or moderately ductile requirements apply.

Determine brace-to-gusset connection geometry

Choose a lap length between the brace and the gusset plate based on the limit state of shear yielding in the brace wall. By setting AISC *Specification* Equation J4-3 equal to the required shear strength, the required lap length can be determined:

LRFD	ASD
$P_u = \phi 0.60 F_y A_g$ $= \phi 0.60 F_y t(4) l_{weld}$ $300\text{ kips} = 1.00(0.60)(46\text{ ksi})(0.465\text{ in.})(4) l_{weld}$ $l_{weld} = 5.84\text{ in.}$	$P_a = \dfrac{0.60 F_y A_g}{\Omega}$ $= \dfrac{0.60 F_y t(4) l_{weld}}{\Omega}$ $200\text{ kips} = \dfrac{0.60(46\text{ ksi})(0.465\text{ in.})(4) l_{weld}}{1.50}$ $l_{weld} = 5.84\text{ in.}$

As stated previously, note that for this example, which is not specifically detailed for seismic resistance, the connection is designed for the same force as the brace. This was not the case for the SCBF connection designed in Example 6.1a.

Design weld at brace-to-gusset connection

Try a weld length of 12 in. to allow a 5⁄16-in. fillet weld. There are four lines of fillet weld between the brace and the gusset plate. From AISC *Manual* Part 8, Equation 8-2, where l = weld length:

LRFD	ASD
$\phi R_n = 1.392 Dl$ $D = \dfrac{300\text{ kips}}{4(1.392\text{ kip/in.})(12.0\text{ in.})}$ $= 4.49\text{ sixteenths}$	$\dfrac{R_n}{\Omega} = 0.928 Dl$ $D = \dfrac{200\text{ kips}}{4(0.928\text{ kip/in.})(12.0\text{ in.})}$ $= 4.49\text{ sixteenths}$

Use a 5⁄16-in. fillet weld.

Check block shear rupture on the gusset plate

Assume the gusset plate is ½ in. thick. The block shear rupture failure path will follow the fillet welds. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\text{ Eq. J4-5})$$

Shear yielding component:

$$A_{gv} = (12.0 \text{ in.})(2)(\tfrac{1}{2} \text{ in.})$$
$$= 12.0 \text{ in.}^2$$
$$0.60F_yA_{gv} = 0.60(36 \text{ ksi})(12.0 \text{ in.}^2)$$
$$= 259 \text{ kips}$$

Shear rupture component:

$$A_{nv} = A_{gv}$$
$$= (12.0 \text{ in.})(2)(\tfrac{1}{2} \text{ in.})$$
$$= 12.0 \text{ in.}^2$$
$$0.60F_uA_{nv} = 0.60(58 \text{ ksi})(12.0 \text{ in.}^2)$$
$$= 418 \text{ kips}$$

Tension rupture component:

$$A_{nt} = A_{gt}$$
$$= (8.00 \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 4.00 \text{ in.}^2$$
$$U_{bs}F_uA_{nt} = 1(58 \text{ ksi})(4.00 \text{ in.}^2)$$
$$= 232 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 418 \text{ kips} + 232 \text{ kips}$$
$$= 650 \text{ kips}$$
$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 259 \text{ kips} + 232 \text{ kips}$$
$$= 491 \text{ kips}$$

Therefore, $R_n = 491$ kips.

LRFD	ASD
$\phi R_n = 0.75(491 \text{ kips})$ $= 368 \text{ kips}$ $> 300 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{491 \text{ kips}}{2.00}$ $= 246 \text{ kips}$ $> 200 \text{ kips}$ **o.k.**

Refer to Figure 6-10:

$$d = 8.00 \text{ in.} + 1.00 \text{ in.} + 1.00 \text{ in.}$$
$$= 10.0 \text{ in.}$$
$$l = 12.0 \text{ in.}$$

Check tensile yielding on the Whitmore section

The Whitmore section is determined in accordance with AISC *Manual* Part 9. The width of the Whitmore section is:

$$l_w = 8.00 \text{ in.} + 2(12.0 \text{ in.}) \tan 30°$$
$$= 21.9 \text{ in.}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength on the Whitmore section is:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90 F_y l_w t$	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{F_y l_w t}{1.67}$

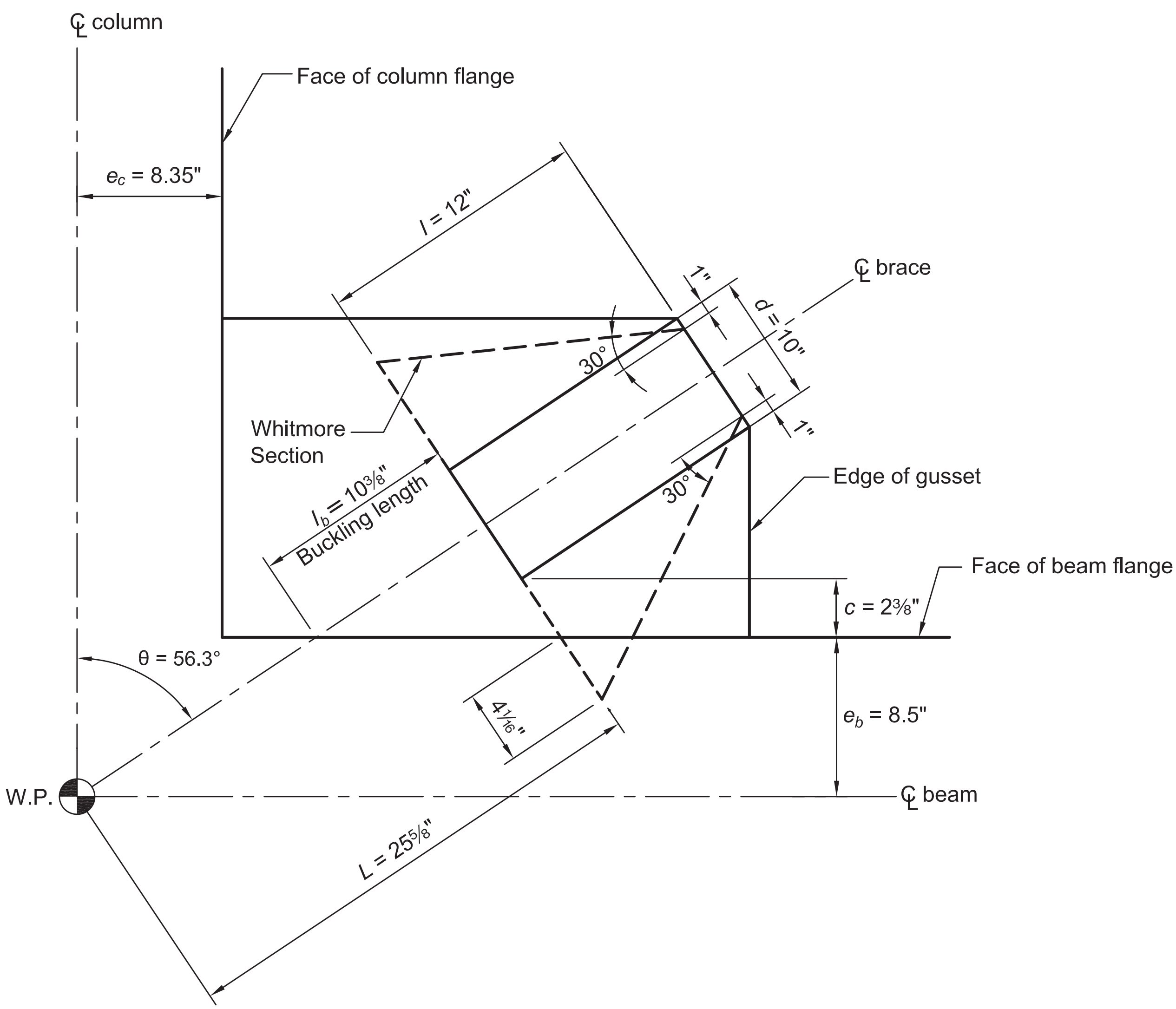


Fig. 6-10. Geometry for gusset plate in Example 6.1b.

Approximately 4 1⁄16 in. of the Whitmore width is within the beam. Because $t_w = 0.585$ in. > $t_g = ½$ in., conservatively use the gusset plate thickness across the entire Whitmore width to determine the available tensile yielding strength:

LRFD	ASD
$\phi R_n = 0.90\big[(36 \text{ ksi})(21.9 \text{ in.} - 4.06 \text{ in.})(½ \text{ in.}) + (50 \text{ ksi})(4.06 \text{ in.})(½ \text{ in.})\big]$ $= 380 \text{ kips} > 300 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1}{1.67}\big[(36 \text{ ksi})(21.9 \text{ in.} - 4.06 \text{ in.})(½ \text{ in.}) + (50 \text{ ksi})(4.06 \text{ in.})(½ \text{ in.})\big]$ $= 253 \text{ kips} > 200 \text{ kips}$ **o.k.**

Check flexural buckling in compression on the Whitmore section

From Dowswell (2006), the critical gusset plate thickness is t_β. If the actual gusset plate thickness is greater than t_β, the gusset plate will yield before it buckles and the limit state of flexural buckling does not apply.

$$t_\beta = 1.5\sqrt{\frac{F_y c^3}{E_s l_b}}$$

$$l_b = 10⅜ \text{ in. (approximately)}$$

$$c = 2⅜ \text{ in.}$$

$$t_\beta = 1.5\sqrt{\frac{(36 \text{ ksi})(2⅜ \text{ in.})^3}{(29{,}000 \text{ ksi})(10⅜ \text{ in.})}}$$

$$= 0.0600 \text{ in.} \le ½ \text{ in.; therefore, the gusset plate is compact} \quad \textbf{o.k.}$$

Because the gusset plate is considered to be compact, it need not be checked for compressive strength due to flexural buckling; however, the check is completed here to be consistent with the AISC *Specification* requirements.

With $K = 0.5$, the effective slenderness is:

$$\frac{KL_b}{r} = \frac{0.5(10⅜ \text{ in.})}{\left(\dfrac{0.50 \text{ in.}}{\sqrt{12}}\right)}$$

$$= 35.9$$

From AISC *Manual* Table 4-22, with $F_y = 36$ ksi:

LRFD	ASD
$\phi_c F_{cr} = 30.3$ ksi	$\dfrac{F_{cr}}{\Omega_c} = 20.1$ ksi

According to AISC *Specification* Section J4.4, because $KL/r > 25$, Chapter E applies. From AISC *Specification* Section E1 and E3, the available compressive strength of the gusset plate is:

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} l_w t$ $= (30.3 \text{ ksi})(21.9 \text{ in.})(½ \text{ in.})$ $= 332 \text{ kips} > 300 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr} l_w t}{\Omega_c}$ $= (20.1 \text{ ksi})(21.9 \text{ in.})(½ \text{ in.})$ $= 220 \text{ kips} > 200 \text{ kips}$ **o.k.**

Therefore, the ½-in.-thick gusset plate is adequate.

Force Distribution using the Uniform Force Method

From Figure 6-10, $L = 25\frac{5}{8}$ in. To eliminate any moment on the gusset-to-column connection interface, assume $\beta = \bar{\beta}$, where $\bar{\beta}$ is the distance from the face of the beam flange to the centroid of the gusset-to-column connection, which in this case is a bolted single plate.

From Figure 6-9 and AISC *Manual* Part 3:

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \qquad \text{(\textit{Manual} Eq. 13-1)}$$

$$e_b = 8.50 \text{ in.}$$

$$e_c = 8.35 \text{ in.}$$

$$\begin{aligned}\tan\theta &= \frac{12}{8} \\ &= 1.50\end{aligned}$$

$$\begin{aligned}\beta &= \bar{\beta} \\ &= 5\tfrac{1}{4} \text{ in.} + 3 \text{ in.} + 1\tfrac{1}{2} \text{ in.} \\ &= 9.75 \text{ in.}\end{aligned}$$

$$\begin{aligned}\alpha &= e_b \tan\theta - e_c + \beta \tan\theta \qquad \text{(from \textit{Manual} Eq. 13-1)} \\ &= (8.50 \text{ in.})(1.50) - 8.35 \text{ in.} + (9.75 \text{ in.})(1.50) \\ &= 19.0 \text{ in.}\end{aligned}$$

$$\begin{aligned}\bar{\alpha} &= \frac{25.0 \text{ in.}}{2} + \tfrac{3}{4} \text{ in.} \\ &= 13.3 \text{ in.}\end{aligned}$$

$$\begin{aligned}r &= \left[(\alpha + e_c)^2 + (\beta + e_b)^2\right]^{0.5} \qquad \text{(\textit{Manual} Eq. 13-6)} \\ &= \left[(19.0 \text{ in.} + 8.35 \text{ in.})^2 + (9.75 \text{ in.} + 8.50 \text{ in.})^2\right]^{0.5} \\ &= 32.9 \text{ in.}\end{aligned}$$

The forces at the connection interface are determined from AISC *Manual* Part 13:

LRFD	ASD
$\begin{aligned}\frac{P_u}{r} &= \frac{300 \text{ kips}}{32.9 \text{ in.}} \\ &= 9.12 \text{ kip/in.}\end{aligned}$	$\begin{aligned}\frac{P_a}{r} &= \frac{200 \text{ kips}}{32.9 \text{ in.}} \\ &= 6.08 \text{ kip/in.}\end{aligned}$

LRFD	ASD
From AISC *Manual* Equation 13-2: $V_{uc} = \beta \frac{P_u}{r}$ $= (9.75\text{ in.})(9.12\text{ kip/in.})$ $= 88.9\text{ kips}$	From AISC *Manual* Equation 13-2: $V_{ac} = \beta \frac{P_a}{r}$ $= (9.75\text{ in.})(6.08\text{ kip/in.})$ $= 59.3\text{ kips}$
From AISC *Manual* Equation 13-4: $V_{ub} = \frac{e_b P_u}{r}$ $= (8.50\text{ in.})(9.12\text{ kip/in.})$ $= 77.5\text{ kips}$	From AISC *Manual* Equation 13-4: $V_{ab} = \frac{e_b P_a}{r}$ $= (8.50\text{ in.})(6.08\text{ kip/in.})$ $= 51.7\text{ kips}$
$\sum(V_{uc} + V_{ub}) = 88.9\text{ kips} + 77.5\text{ kips}$ $= 166\text{ kips}$ $= (300\text{ kips})\cos 56.3°$ $= 166\text{ kips}$ **o.k.**	$\sum(V_{ac} + V_{ab}) = 59.3\text{ kips} + 51.7\text{ kips}$ $= 111\text{ kips}$ $= (200\text{ kips})\cos 56.3°$ $= 111\text{ kips}$ **o.k.**
From AISC *Manual* Equation 13-3: $H_{uc} = \frac{e_c P_u}{r}$ $= (8.35\text{ in.})(9.12\text{ kip/in.})$ $= 76.2\text{ kips}$	From AISC *Manual* Equation 13-3: $H_{ac} = \frac{e_c P_a}{r}$ $= (8.35\text{ in.})(6.08\text{ kip/in.})$ $= 50.8\text{ kips}$
From AISC *Manual* Equation 13-5: $H_{ub} = \alpha \frac{P_u}{r}$ $= (19.0\text{ in.})(9.12\text{ kip/in.})$ $= 173\text{ kips}$	From AISC *Manual* Equation 13-5: $H_{ab} = \alpha \frac{P_a}{r}$ $= (19.0\text{ in.})(6.08\text{ kip/in.})$ $= 116\text{ kips}$
$\sum(H_{uc} + H_{ub}) = 76.2\text{ kips} + 173\text{ kips}$ $= 249\text{ kips}$ $\approx (300\text{ kips})\sin 56.3°$ $= 250\text{ kips}$ **o.k.**	$\sum(H_{ac} + H_{ab}) = 50.8\text{ kips} + 116\text{ kips}$ $= 167\text{ kips}$ $\approx (200\text{ kips})\sin 56.3°$ $= 166\text{ kips}$ **o.k.**
From AISC *Manual* Equation 13-17: $M_{ub} = V_{ub}(\alpha - \bar{\alpha})$ $= (77.5\text{ kips})(19.0\text{ in.} - 13.3\text{ in.})$ $= 442\text{ kip-in.}$ clockwise on gusset	From AISC *Manual* Equation 13-17: $M_{ab} = V_{ab}(\alpha - \bar{\alpha})$ $= (51.7\text{ kips})(19.0\text{ in.} - 13.3\text{ in.})$ $= 295\text{ kip-in.}$ clockwise on gusset

The admissible force distribution shown in Figure 6-11 assumes a bracing configuration similar to configuration A of Figure 6-6. In this case there is a small addition of lateral force to the braced bay at each floor. This can usually be neglected in the calculations.

Configuration A will be used here, with the admissible force distribution shown in Figure 6-11.

Gusset-to-Beam Connection

The required strengths at the gusset-to-beam interface are:

LRFD	ASD
Required shear strength, H_{ub} = 173 kips	Required shear strength, H_{ab} = 115 kips
Required normal strength, V_{ub} = 77.5 kips	Required normal strength, V_{ab} = 51.7 kips
Required flexural strength, M_{ub} = 442 kip-in.	Required flexural strength, M_{ab} = 295 kip-in.

Horizontal length of gusset-to-beam connection, from Figure 6-9, is 25 in.

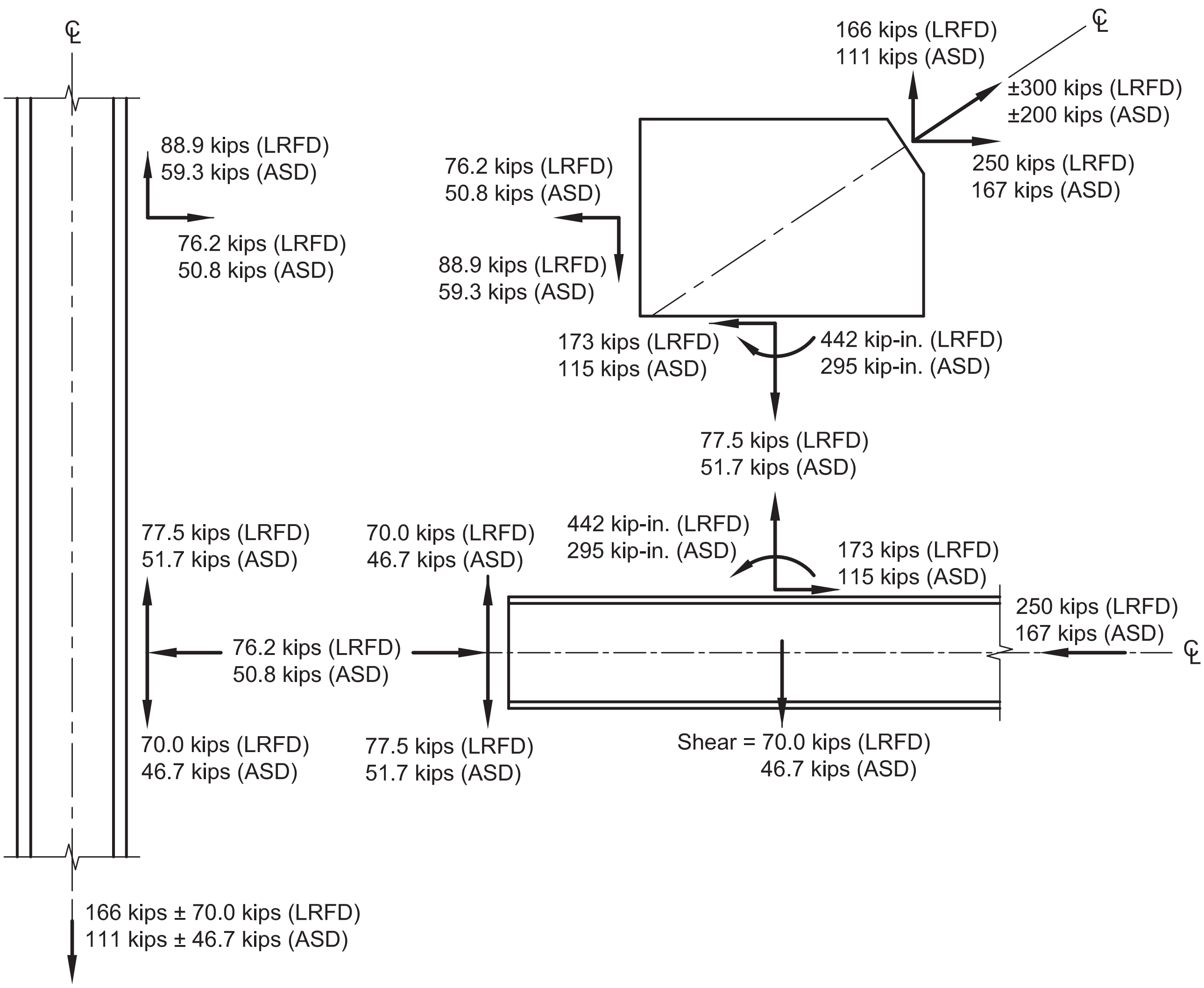


Fig. 6-11. Admissible force field for connection shown in Figure 6-9.

Check gusset plate for tension yielding and shear yielding along the beam flange

In this example, to facilitate combining the required normal strength and flexural strength, stresses will be used to check the tensile and shear yielding limit states. Tensile yielding is checked using AISC *Specification* Section J4.1:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{A_g}$ $= \dfrac{77.5 \text{ kips}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})}$ $= 6.20 \text{ ksi}$ $f_{ub} = \dfrac{M_{ub}}{Z}$ $= \dfrac{442 \text{ kip-in.}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})^2 / 4}$ $= 5.66 \text{ ksi}$ $f_{un} = f_{ua} + f_{ub}$ $= 6.20 \text{ ksi} + 5.66 \text{ ksi}$ $= 11.9 \text{ ksi} < 0.90(36 \text{ ksi}) = 32.4 \text{ ksi}$ **o.k.**	$f_{aa} = \dfrac{V_{ab}}{A_g}$ $= \dfrac{51.7 \text{ kips}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})}$ $= 4.14 \text{ ksi}$ $f_{ab} = \dfrac{M_{ab}}{Z}$ $= \dfrac{295 \text{ kip-in.}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})^2 / 4}$ $= 3.78 \text{ ksi}$ $f_{an} = f_{aa} + f_{ab}$ $= 4.14 \text{ ksi} + 3.78 \text{ ksi}$ $= 7.92 \text{ ksi} < \dfrac{36 \text{ ksi}}{1.67} = 21.6 \text{ ksi}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear yielding strength of the gusset plate is:

LRFD	ASD
$f_{uv} = \dfrac{173 \text{ kips}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})}$ $= 13.9 \text{ ksi} < 1.00(0.60)(36 \text{ ksi}) = 21.6 \text{ ksi}$ **o.k.**	$f_{av} = \dfrac{115 \text{ kips}}{(\tfrac{1}{2} \text{ in.})(25.0 \text{ in.})}$ $= 9.20 \text{ ksi} < \dfrac{0.60(36 \text{ ksi})}{1.50} = 14.4 \text{ ksi}$ **o.k.**

Design gusset-to-beam weld

The AISC *Manual* recommends an increase in design force for a welded connection between a gusset plate and member flange to account for uneven stress distribution across the weld surface. The average stress should be multiplied by a factor of 1.25 and compared to the peak stress, with the greater of the two being the required stress for the weld (Hewitt and Thornton, 2004) determined as follows:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{2l}$ $= \dfrac{77.5 \text{ kips}}{2(25.0 \text{ in.})}$ $= 1.55 \text{ kip/in.}$	$f_{aa} = \dfrac{H_{ab}}{2l}$ $= \dfrac{51.7 \text{ kips}}{2(25.0 \text{ in.})}$ $= 1.03 \text{ kip/in.}$

LRFD	ASD
$f_{ub} = \dfrac{2M_{ub}}{l^2}$ $= \dfrac{2(442\text{ kip-in.})}{(25.0\text{ in.})^2}$ $= 1.41\text{ kip/in.}$ $f_{un} = f_{ua} + f_{ub}$ $= 1.55\text{ kip/in.} + 1.41\text{ kip/in.}$ $= 2.96\text{ kip/in.}$ $f_{uv} = \dfrac{H_{ub}}{2l}$ $= \dfrac{173\text{ kips}}{2(25.0\text{ in.})}$ $= 3.46\text{ kip/in.}$ $f_{peak} = \sqrt{(f_{ua}+f_{ub})^2 + f_{uv}{}^2}$ $f_{avg} = \dfrac{1}{2}\left[\sqrt{(f_{ua}-f_{ub})^2 + f_{uv}{}^2} + \sqrt{(f_{ua}+f_{ub})^2 + f_{uv}{}^2}\right]$ $f_{peak} = \sqrt{(1.55\text{ kip/in.} + 1.41\text{ kip/in.})^2 + (3.46\text{ kip/in.})^2}$ $= 4.55\text{ kip/in.}$ $f_{avg} = \dfrac{1}{2}\left[\sqrt{(1.55\text{ kip/in.} - 1.41\text{ kip/in.})^2 + (3.46\text{ kip/in.})^2} + (4.55\text{ kip/in.})\right]$ $= 4.01\text{ kip/in.}$ $1.25\, f_{avg} = 1.25(4.02\text{ kip/in.})$ $= 5.03\text{ kip/in.} > 4.55\text{ kip/in.}$ Use $f_{weld} = 5.03\text{ kip/in.}$ $\theta = \tan^{-1}\left(\dfrac{f_{un}}{f_{uv}}\right)$ $= \tan^{-1}\left(\dfrac{2.96\text{ kip/in.}}{3.46\text{ kip/in.}}\right)$ $= 40.5°$ From AISC *Manual* Equation 8-2: $D = \dfrac{f_{weld}}{(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)}$ $= \dfrac{5.03\text{ kip/in.}}{(1.392\text{ kip/in.})(1.0 + 0.50\sin^{1.5} 40.5°)}$ $= 2.86$ sixteenths	$f_{ab} = \dfrac{2M_a}{l^2}$ $= \dfrac{2(295\text{ kip-in.})}{(25.0\text{ in.})^2}$ $= 0.944\text{ kip/in.}$ $f_{an} = f_{aa} + f_{ab}$ $= 1.03\text{ kip/in.} + 0.944\text{ kip/in.}$ $= 1.97\text{ kip/in.}$ $f_{av} = \dfrac{V_a}{2l}$ $= \dfrac{115\text{ kips}}{2(25.0\text{ in.})}$ $= 2.30\text{ kip/in.}$ $f_{peak} = \sqrt{(f_{aa}+f_{ab})^2 + f_{av}{}^2}$ $f_{avg} = \dfrac{1}{2}\left[\sqrt{(f_{aa}-f_{ab})^2 + f_{av}{}^2} + \sqrt{(f_{aa}+f_{ab})^2 + f_{av}{}^2}\right]$ $f_{peak} = \sqrt{(1.03\text{ kip/in.} + 0.944\text{ kip/in.})^2 + (2.30\text{ kip/in.})^2}$ $= 3.03\text{ kip/in.}$ $f_{avg} = \dfrac{1}{2}\left[\sqrt{(1.03\text{ kip/in.} - 0.944\text{ kip/in.})^2 + (2.30\text{ kip/in.})^2} + (3.03\text{ kip/in.})\right]$ $= 2.67\text{ kip/in.}$ $1.25\, f_{avg} = 1.25(2.67\text{ kip/in.})$ $= 3.34\text{ kip/in.} > 3.03\text{ kip/in.}$ Use $f_{weld} = 3.34\text{ kip/in.}$ $\theta = \tan^{-1}\left(\dfrac{f_{an}}{f_{av}}\right)$ $= \tan^{-1}\left(\dfrac{1.97\text{ kip/in.}}{2.30\text{ kip/in.}}\right)$ $= 40.6°$ From AISC *Manual* Equation 8-2: $D = \dfrac{f_{weld}}{(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5}\theta)}$ $= \dfrac{3.34\text{ kip/in.}}{(0.928\text{ kip/in.})(1.0 + 0.50\sin^{1.5} 40.6°)}$ $= 2.85$ sixteenths

Use a 3⁄16-in. fillet weld on both sides of the gusset plate.

Determine equivalent normal force, N_e

The concentrated force for which the beam must be designed includes both the normal force and the moment determined previously. For convenience, convert the moment into an equivalent normal force and add it to the normal force:

LRFD	ASD
$$N_e = V_{ub} + \frac{2M_{ub}}{\left(\frac{l}{2}\right)}$$ $$= 77.5 \text{ kips} + \frac{2(442 \text{ kip-in.})}{\left(\frac{25.0 \text{ in.}}{2}\right)}$$ $$= 148 \text{ kips}$$	$$N_e = V_{ab} + \frac{2M_{ab}}{\left(\frac{l}{2}\right)}$$ $$= 51.7 \text{ kips} + \frac{2(295 \text{ kip-in.})}{\left(\frac{25.0 \text{ in.}}{2}\right)}$$ $$= 98.9 \text{ kips}$$

Check beam web local yielding

The concentrated force, which is the resultant force from the gusset plate, acts at a distance of 25.0 in./2 = 12.5 in. from the end of the beam. From AISC *Specification* Section J10.2, use Equation J10-3 because the concentrated force is applied at a distance from the member end that is less than the beam depth:

LRFD	ASD
$$\phi R_n = \phi F_{yw} t_w (2.5k + l_b)$$ $$= 1.00(50 \text{ ksi})(0.585 \text{ in.})[2.5(1.39 \text{ in.}) + 25.0 \text{ in.}]$$ $$= 833 \text{ kips} > 148 \text{ kips} \quad \textbf{o.k.}$$	$$\frac{R_n}{\Omega} = \frac{F_{yw} t_w (2.5k + l_b)}{\Omega}$$ $$= \frac{(50 \text{ ksi})(0.585 \text{ in.})[2.5(1.39 \text{ in.}) + 25.0 \text{ in.}]}{1.50}$$ $$= 555 \text{ kips} > 98.9 \text{ kips} \quad \textbf{o.k.}$$

Check beam web local crippling

As calculated for web local yielding, the concentrated force acts at a distance approximately 12.5 in. from the beam end, which is greater than $d/2$. Therefore, from AISC *Specification* Equation J10-4:

LRFD	ASD
$$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw} t_f}{t_w}}$$ $$= 0.75(0.80)(0.585 \text{ in.})^2 \left[1 + 3\left(\frac{25.0 \text{ in.}}{17.0 \text{ in.}}\right)\left(\frac{0.585 \text{ in.}}{0.985 \text{ in.}}\right)^{1.5}\right]$$ $$\times \sqrt{\frac{(29{,}000 \text{ ksi})(50 \text{ ksi})(0.985 \text{ in.})}{0.585 \text{ in.}}}$$ $$= 969 \text{ kips} > 148 \text{ kips} \quad \textbf{o.k.}$$	$$\frac{R_n}{\Omega} = \frac{0.80 t_w^2}{\Omega}\left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw} t_f}{t_w}}$$ $$= \frac{0.80(0.585 \text{ in.})^2}{2.00}\left[1 + 3\left(\frac{25.0 \text{ in.}}{17.0 \text{ in.}}\right)\left(\frac{0.585 \text{ in.}}{0.985 \text{ in.}}\right)^{1.5}\right]$$ $$\times \sqrt{\frac{29{,}000 \text{ ksi}(50 \text{ ksi})(0.985 \text{ in.})}{0.585 \text{ in.}}}$$ $$= 646 \text{ kips} > 98.9 \text{ kips} \quad \textbf{o.k.}$$

Shear force at toe of beam flange to web fillet

LRFD	ASD
$\phi R_n = \phi 0.6 F_y t_w l_{req}$ $l_{req} = \dfrac{173 \text{ kips}}{1.00(0.6)(50 \text{ ksi})(0.585 \text{ in.})}$ $= 9.86 \text{ in.} < 25.0 \text{ in.}$ **o.k.** Therefore, a sufficient load path into the full section is provided.	$\dfrac{R_n}{\Omega} = \dfrac{0.6 F_y t_w l_{req}}{\Omega}$ $l_{req} = \dfrac{115 \text{ kips}(1.50)}{(0.6)(50 \text{ ksi})(0.585 \text{ in.})}$ $= 9.83 \text{ in.} < 25.0 \text{ in.}$ **o.k.** Therefore, a sufficient load path into the full section is provided.

This is less than the gusset length of 25 in. Even if l_{req} were greater than 25 in., the beam web would provide sufficient length to provide a load path into the section. A reasonable maximum length to transmit the shear force into the W16×100 would be the span divided by two, or 120 in.

Gusset-to-Column Connection

The required strengths are:

LRFD	ASD
Required normal strength: $H_{uc} = 76.2$ kips Required shear strength: $V_{uc} = 88.9$ kips	Required normal strength: $H_{ac} = 50.8$ kips Required shear strength: $V_{ac} = 59.3$ kips

Assume the single plate is ½-in.-thick ASTM A36 material.

The connection does not satisfy all of the dimensional limitations for the conventional single-plate connection given in AISC *Manual* Part 10, so it will be designed as an extended configuration according to Part 10.

Check bolt shear

From AISC *Manual* Table 7-1, the available shear strength per 1-in.-diameter ASTM A490-X bolt in single shear is:

LRFD	ASD
$\phi r_n = 49.5 \text{ kips}$	$\dfrac{r_n}{\Omega} = 33.0 \text{ kips}$

The force resultant at the gusset-to-column connection is:

LRFD	ASD
$P_{uc} = \sqrt{(76.2 \text{ kips})^2 + (88.9 \text{ kips})^2}$ $= 117 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{76.2 \text{ kips}}{88.9 \text{ kips}}\right)$ $= 40.6°$	$P_{ac} = \sqrt{(50.8 \text{ kips})^2 + (59.3 \text{ kips})^2}$ $= 78.1 \text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{50.8 \text{ kips}}{59.3 \text{ kips}}\right)$ $= 40.6°$

LRFD	ASD
From AISC *Manual* Table 7-6 with $\theta = 30°$, $e_x = 3$ in., and $n = 4, C = 2.79$. $r_u = \dfrac{P_{uc}}{C}$ $= \dfrac{117\text{ kips}}{2.79\text{ bolts}}$ $= 41.9$ kips/bolt Therefore, use 1-in.-diameter ASTM A490-X bolts. $\phi r_n = 49.5$ kips/bolt $> r_u = 41.9$ kips/bolt **o.k.**	From AISC *Manual* Table 7-6 with $\theta = 30°$, $e_x = 3.0$ in., and $n = 4, C = 2.79$. $r_a = \dfrac{P_{ac}}{C}$ $= \dfrac{78.1\text{ kips}}{2.79\text{ bolts}}$ $= 28.0$ kips/bolt Therefore, use 1-in.-diameter ASTM A490-X bolts. $r_n/\Omega = 33.0$ kips/bolt $> r_a = 28.0$ kips/bolt **o.k.**

Check bolt bearing on the gusset-to-column single plate

Check the available bearing strength using AISC *Manual* Tables 7-4 and 7-5, with 3-in. bolt spacing and 1¼-in.-edge distance:

LRFD	ASD
Based on bolt spacing: $\phi r_n = (101\text{ kip/in.})(½\text{ in.})$ $= 50.5$ kips/bolt Based on edge distance: $\phi r_n = (37.5\text{ kip/in.})(½\text{ in.})$ $= 18.8$ kips/bolt $< r_u = 41.9$ kips/bolt **n.g.**	Based on bolt spacing: $\dfrac{r_n}{\Omega} = (67.4\text{ kip/in.})(½\text{ in.})$ $= 33.7$ kips/bolt Based on edge distance: $\dfrac{r_n}{\Omega} = (25.0\text{ kip/in.})(½\text{ in.})$ $= 12.5$ kips/bolt $< r_a = 28.0$ kips/bolt **n.g.**

The available strength of the edge bolts for the limit state of bolt bearing on the single plate is less than the required shear strength per bolt. Therefore, the full bolt strength cannot be achieved on the edge bolts. Because the theory behind AISC *Manual* Table 7-6 assumes that all bolts can support a force equal to or less than the required shear strength, the theory is invalid unless this is possible. One way to satisfy this requirement is to increase the shear plate thickness:

LRFD	ASD
$t_{p\ reqd} = \dfrac{41.9\text{ kips}}{18.8\text{ kips}}(½\text{ in.})$ $= 1.11$ in.	$t_{p\ reqd} = \dfrac{28.0\text{ kips}}{12.5\text{ kips}}(½\text{ in.})$ $= 1.12$ in.

Thus, a 1⅛-in.-thick plate is required. Another option is to use a stronger plate ($F_y = 50$ ksi), which would require a thickness of:

$$t_{p\ reqd} = (1.12\text{ in.})\left(\frac{58\text{ ksi}}{65\text{ ksi}}\right)$$
$$= 1.00\text{ in.}$$

or increase the edge distance. There is room to accommodate an increased edge distance in this case, so consider increasing the edge distance to 1¾ in. (top and bottom).

With the increased edge distance, by interpolation in AISC *Manual* Table 7-5:

LRFD	ASD
$\phi r_n = (63.6 \text{ kip/in.})(\tfrac{1}{2} \text{ in.})$ $= 31.8 \text{ kips/bolt} < r_u = 41.9 \text{ kips/bolt}$ **n.g.**	$\dfrac{r_n}{\Omega} = (42.4 \text{ kip/in.})(\tfrac{1}{2} \text{ in.})$ $= 21.2 \text{ kips/bolt} < r_a = 28.0 \text{ kips/bolt}$ **n.g.**

This is still inadequate. Therefore, increase the plate thickness to ensure that the available bearing strength based on edge distance at least equals the required force per bolt. Thus, the plate thickness required is:

LRFD	ASD
$\dfrac{41.9 \text{ kips}}{31.8 \text{ kips}}(\tfrac{1}{2} \text{ in.}) = 0.659 \text{ in.}$	$\dfrac{28.0 \text{ kips}}{21.2 \text{ kips}}(\tfrac{1}{2} \text{ in.}) = 0.660 \text{ in.}$

Use a ¾-in.-thick plate.

With the ¾-in.-thick plate and 1¾-in. vertical edge distances:

LRFD	ASD
$\phi r_n = (63.6 \text{ kip/in.})(\tfrac{3}{4} \text{ in.})$ $= 47.7 \text{ kips/bolt} > 41.9 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_{n\,edge}}{\Omega} = (42.4 \text{ kip/in.})(\tfrac{3}{4} \text{ in.})$ $= 31.8 \text{ kips/bolt} > 28.0 \text{ kips/bolt}$ **o.k.**

Check bolt bearing on the gusset plate

The minimum edge distance provided for the gusset plate is 2.25 in., in the horizontal direction. Therefore, check bearing in the horizontal direction, which will control.

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(\textit{Spec}. Eq. J3-6a)}$$

The clear distance from the bolt hole edge to the edge of the material is:

$$\begin{aligned} l_c &= 2.25 \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.}) \\ &= 1.72 \text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.72 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 44.9 \text{ kips}$ $\phi 2.4 d t F_u = 0.75(2.4)(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$ $= 52.2 \text{ kips}$ Therefore, $\phi r_n = 44.9$ kips/bolt.	$1.2 l_c t F_u/\Omega = \dfrac{1.2(58 \text{ ksi})(1.72 \text{ in.})(\tfrac{1}{2} \text{ in.})}{2.00}$ $= 29.9 \text{ kips/bolt}$ $2.4 d t F_u/\Omega = 2.4(1.00 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})/2.00$ $= 34.8 \text{ kips}$ Therefore, $r_n/\Omega = 29.9$ kips/bolt.

Because the available bolt shear strength determined previously (41.9 kips for LRFD and 28.0 kips for ASD) is less than the bearing strength, the limit state of bolt shear controls the strength of the bolt group.

Check shear yielding on the single plate

The gross area subject to shear is:

$$\begin{aligned} A_{gv} &= (\text{¾ in.})\left[9.00\text{ in.} + 2(1.75\text{ in.})\right] \\ &= 9.38\text{ in}^2 \end{aligned}$$

From AISC *Specification* Section J4.2, the available shear yielding strength is:

LRFD	ASD
$\begin{aligned} \phi R_n &= \phi 0.60 F_y A_{gv} \\ &= 1.00(0.60)(36\text{ ksi})\left(9.38\text{ in.}^2\right) \\ &= 203\text{ kips} > 88.9\text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{0.60 F_y A_{gv}}{\Omega} \\ &= \frac{0.60(36\text{ ksi})\left(9.38\text{ in.}^2\right)}{1.50} \\ &= 135\text{ kips} > 59.3\text{ kips} \quad \textbf{o.k.} \end{aligned}$

Check shear rupture on the single plate

The net area subject to shear is:

$$\begin{aligned} A_{nv} &= 9.38\text{ in.}^2 - 4\left(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(\text{¾ in.}) \\ &= 6.01\text{ in.}^2 \end{aligned}$$

LRFD	ASD
$\begin{aligned} \phi R_n &= \phi 0.60 F_u A_{nv} \\ &= 0.75(0.60)(58\text{ ksi})\left(6.01\text{ in.}^2\right) \\ &= 157\text{ kips} > 88.9\text{ kips} \quad \textbf{o.k.} \end{aligned}$	$\begin{aligned} \frac{R_n}{\Omega} &= \frac{0.60 F_u A_{nv}}{\Omega} \\ &= \frac{0.60(58\text{ ksi})\left(6.01\text{ in.}^2\right)}{2.00} \\ &= 105\text{ kips} > 59.3\text{ kips} \quad \textbf{o.k.} \end{aligned}$

Use 1-in.-diameter ASTM A490-X bolts for uniformity throughout the connection. Use ASTM A36 plate as the connecting element between the gusset and the column.

Check block shear rupture on the single plate

Because the gusset is thinner than the plate and both are ASTM A36 material, block shear rupture on the gusset plate will be checked using a top edge distance of 1¾ in.

Check block shear rupture on the gusset plate

The block shear rupture failure path is assumed to follow the line of bolts and then turn perpendicular and run to the gusset plate edge at the bottom bolt. The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}.\text{ Eq. J4-5})$$

Assume a 1¾-in. top edge distance (actual edge distance is 3.25 in.), which allows the gusset plate to be shaped if required.

Shear yielding component:

$$\begin{aligned}A_{gv} &= (10.75 \text{ in.})(\tfrac{1}{2} \text{ in.})\\ &= 5.38 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}0.60F_yA_{gv} &= 0.60(36 \text{ ksi})(5.38 \text{ in.}^2)\\ &= 116 \text{ kips}\end{aligned}$$

Shear rupture component:

$$\begin{aligned}A_{nv} &= 5.38 \text{ in.}^2 - 3.5(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})\\ &= 3.41 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}0.60F_uA_{nv} &= 0.60(58 \text{ ksi})(3.41 \text{ in.}^2)\\ &= 119 \text{ kips}\end{aligned}$$

Tension rupture component:

$U_{bs} = 1$ from AISC *Specifications* Section J4.3 because the connection is uniformly loaded

$$\begin{aligned}A_{nt} &= \left[2.25 \text{ in.} - 0.5(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.})\\ &= 0.844 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}U_{bs}F_uA_{nt} &= 1(58 \text{ ksi})(0.844 \text{ in.}^2)\\ &= 49.0 \text{ kips}\end{aligned}$$

The available strength for the limit state of block shear rupture is:

$$\begin{aligned}0.60F_uA_{nv} + U_{bs}F_uA_{nt} &= 119 \text{ kips} + 49.0 \text{ kips}\\ &= 168 \text{ kips}\\ 0.60F_yA_{gv} + U_{bs}F_uA_{nt} &= 116 \text{ kips} + 49.0 \text{ kips}\\ &= 165 \text{ kips}\end{aligned}$$

Therefore, $R_n = 165$ kips.

LRFD	ASD
$\phi R_n = 0.75(165 \text{ kips})$ $= 124 \text{ kips} > 88.9 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{165 \text{ kips}}{2.00}$ $= 82.5 \text{ kips} > 59.3 \text{ kips}$ **o.k.**

The block shear rupture strength for the axial force (normal to the gusset plate edge) can also be checked and will be satisfactory.

Therefore, ¾-in.-thick ASTM A36 plate is sufficient for the connection plate and ½-in.-thick ASTM A36 plate is sufficient for the gusset plate.

Check column web local yielding

The concentrated force, which is the resultant force from the gusset plate, is not near the end of the column. From AISC *Specification* Section J10.2, use Equation J10-2 because the concentrated force is applied farther from the end of the member than its depth. The length of bearing is assumed to be the length of the connection plate, l_b = 12.5 in.

LRFD	ASD
$\phi R_n = \phi F_y t_w (5k + l_b)$ $= 1.00(50\text{ ksi})(1.29\text{ in.})[5(2.67\text{ in.}) + 12.5\text{ in.}]$ $= 1{,}670\text{ kips} > 76.2\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w}{\Omega}(5k + l_b)$ $= \dfrac{(50\text{ ksi})(1.29\text{ in.})}{1.50}[5(2.67\text{ in.}) + 12.5\text{ in.}]$ $= 1{,}110\text{ kips} > 50.8\text{ kips}$ **o.k.**

Check column web local crippling

From AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= 0.75(0.80)(1.29\text{ in.})^2 \left[1 + 3\left(\dfrac{12.5\text{ in.}}{16.7\text{ in.}}\right)\left(\dfrac{1.29\text{ in.}}{2.07\text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(2.07\text{ in.})}{1.29\text{ in.}}}$ $= 3{,}210\text{ kips} > 76.2\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2}{\Omega}\left[1 + 3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_{yw}t_f}{t_w}}$ $= \dfrac{0.80(1.29\text{ in.})^2}{2.00}\left[1 + 3\left(\dfrac{12.5\text{ in.}}{16.7\text{ in.}}\right)\left(\dfrac{1.29\text{ in.}}{2.07\text{ in.}}\right)^{1.5}\right]$ $\times \sqrt{\dfrac{(29{,}000\text{ ksi})(50\text{ ksi})(2.07\text{ in.})}{1.29\text{ in.}}}$ $= 2{,}140\text{ kips} > 50.8\text{ kips}$ **o.k.**

Check column shear

Check the column for the limit state of shear yielding (on cross section at beam flange elevation) using AISC *Specification* Sections G1 and G2. The area of the web is:

$$
\begin{aligned}
A_{gv} &= (16.7\text{ in.})(1.29\text{ in.}) \\
&= 21.5\text{ in.}^2
\end{aligned}
$$

The available shear strength is:

LRFD	ASD
$\phi_v R_n = \phi_v 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(21.5\text{ in.}^2)$ $= 645\text{ kips} > 76.2\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega_v} = \dfrac{0.60 F_y A_{gv}}{\Omega_v}$ $= \dfrac{0.60(50\text{ ksi})(21.5\text{ in.}^2)}{1.50}$ $= 430\text{ kips} > 50.8\text{ kips}$ **o.k.**

Check weld rupture at column flange

From AISC *Manual* Equation 8-2:

LRFD	ASD
$D = \dfrac{R_n}{1.392l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{117 \text{ kips}}{(1.392 \text{ kip/in.})(2)(12.5 \text{ in.})\left(1.0+0.50\sin^{1.5} 40.7°\right)}$ $= 2.67$ sixteenths	$D = \dfrac{R_n}{0.928l\left(1.0+0.50\sin^{1.5}\theta\right)}$ $= \dfrac{78.1 \text{ kips}}{(0.928 \text{ kip/in.})(2)(12.5 \text{ in.})\left(1.0+0.50\sin^{1.5} 40.7°\right)}$ $= 2.66$ sixteenths

Beam-to-Column Connection

The required strengths of the connection are:

LRFD	ASD
Required normal strength: $H_u = 76.2$ kips	Required normal strength: $H_a = 50.8$ kips
Required shear strength: $V_u = 77.5$ kips $+70.0$ kips	Required shear strength: $V_a = 51.7$ kips $+ 46.7$ kips
$= 148$ kips	$= 98.4$ kips

Assume shear plate is ½-in. thick.

Check block shear rupture on single plate

Determine the block shear rupture strength using AISC *Specification* Equation J4-5 assuming the rupture to occur is due to the shear force. The controlling failure path is assumed to follow the bolt line nearest to the column flange to the bottom row of bolts and then run perpendicular to the plate edge connected to the beam.

Shear yielding component:

$$A_{gv} = (9.00 \text{ in.} + 3\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 6.25 \text{ in.}^2$$

$$0.60F_yA_{gv} = 0.60(36 \text{ ksi})(6.25 \text{ in.}^2)$$
$$= 135 \text{ kips}$$

Shear rupture component:

$$A_{nv} = 6.25 \text{ in.}^2 - 3.5(\tfrac{1}{2} \text{ in.})(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})$$
$$= 4.28 \text{ in.}^2$$

$$0.60F_uA_{nv} = 0.60(58 \text{ ksi})(4.28 \text{ in.}^2)$$
$$= 149 \text{ kips}$$

Tensile rupture component:

$$A_{nt} = \left[5.00 \text{ in.} - 1.5(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 1.66 \text{ in.}^2$$

$$U_{bs}F_uA_{nt} = 0.5(58 \text{ ksi})(1.66 \text{ in.}^2)$$
$$= 48.1 \text{ kips}$$

The available strength for the limit state of block shear rupture is:

$$0.60F_uA_{nv} + U_{bs}F_uA_{nt} = 149 \text{ kips} + 48.1 \text{ kips}$$
$$= 197 \text{ kips}$$
$$0.60F_yA_{gv} + U_{bs}F_uA_{nt} = 135 \text{ kips} + 48.1 \text{ kips}$$
$$= 183 \text{ kips}$$

Therefore, $R_n = 183$ kips.

LRFD	ASD
$\phi R_n = 0.75(183 \text{ kips})$ $= 137 \text{ kips} < 148 \text{ kips}$ **n.g.** Try ¾-in.-thick ASTM A36 plate: $\phi R_n = (137 \text{ kips})\left(\dfrac{\text{¾ in.}}{\text{½ in.}}\right)$ $= 206 \text{ kips} > 148 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{183 \text{ kips}}{2.00}$ $= 91.5 \text{ kips} < 98.4 \text{ kips}$ **n.g.** Try ¾-in.-thick ASTM A36 plate: $\dfrac{R_n}{\Omega} = (91.5 \text{ kips})\left(\dfrac{\text{¾ in.}}{\text{½ in.}}\right)$ $= 137 \text{ kips} > 98.4 \text{ kips}$ **o.k.**

A conservative way to check block shear rupture for both shear and axial forces is to treat the resultant as a shear force. The resultant is:

LRFD	ASD
$R_u = \sqrt{(76.2 \text{ kips})^2 + (148 \text{ kips})^2}$ $= 166 \text{ kips} < 206 \text{ kips}$ **o.k.**	$R_a = \sqrt{(50.8 \text{ kips})^2 + (98.4 \text{ kips})^2}$ $= 111 \text{ kips} < 137 \text{ kips}$ **o.k.**

Check bolt shear strength

The resultant load is determined in the previous calculation. The resultant load angle is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{76.2 \text{ kips}}{148 \text{ kips}}\right)$ $= 27.2°$ Use AISC *Manual* Table 7-7 for $\theta = 15°$, $e_x = 4.5$, $n = 4$, and $C = 4.53$. The required strength per bolt is: $r_u = \dfrac{R_u}{C}$ $= \dfrac{166 \text{ kips}}{4.53 \text{ bolts}}$ $= 36.6 \text{ kips/bolt}$ Therefore, use 1-in.-diameter ASTM A490-X bolts. From AISC *Manual* Table 7-1, the available bolt shear strength is: $\phi r_{nv} = 49.5 \text{ kips/bolt} > r_u = 36.9 \text{ kips/bolt}$ **o.k.**	$\theta = \tan^{-1}\left(\dfrac{50.8 \text{ kips}}{98.4 \text{ kips}}\right)$ $= 27.3°$ Use AISC *Manual* Table 7-7 for $\theta = 15°$, $e_x = 4.5$, $n = 4$, and $C = 4.53$. The required strength per bolt is: $r_a = \dfrac{R_a}{C}$ $= \dfrac{111 \text{ kips}}{4.53 \text{ bolts}}$ $= 24.5 \text{ kips/bolt}$ Therefore, use 1-in.-diameter ASTM A490-X bolts. From AISC *Manual* Table 7-1, the available bolt shear strength is: $\dfrac{r_{nv}}{\Omega} = 33.0 \text{ kips/bolt} > r_a = 24.5 \text{ kips/bolt}$ **o.k.**

Check bolt bearing on the beam web

Assuming that deformation at the bolt hole is a design consideration, use AISC *Specification* Equation J3-6a to determine the bearing and tearout strength:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad (Spec.\ \text{Eq. J3-6a})$$

The available strength is determined as follows:

LRFD	ASD
Tearout strength based on bolt spacing: $\phi 1.2 l_c t F_u = 0.75(1.2)(3.00\text{ in.} - 1\tfrac{1}{16}\text{ in.})(0.585\text{ in.})(65\text{ ksi})$ $= 66.3\text{ kips/bolt}$ Tearout strength based on edge distance: $\phi 1.2 l_c t F_u = 0.75(1.2)\left[2.25\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.})\right]$ $\times(0.585\text{ in.})(65\text{ ksi})$ $= 58.8\text{ kips/bolt}$ Bearing strength: $\phi 2.4 d t F_u = 0.75(2.4)(1.00\text{ in.})(0.585\text{ in.})(65\text{ ksi})$ $= 68.4\text{ kips/bolt}$	Tearout strength based on bolt spacing: $1.2 l_c t F_u / \Omega = 1.2(3.00\text{ in.} - 1\tfrac{1}{16}\text{ in.})(0.585\text{ in.})(65\text{ ksi})/2.00$ $= 44.2\text{ kips/bolt}$ Tearout strength based on edge distance: $1.2 l_c t F_u / \Omega = \left\{\begin{matrix} 1.2\left[2.25\text{ in.} - 0.5(1\tfrac{1}{16}\text{ in.})\right] \\ \times(0.585\text{ in.})(65\text{ ksi}) \end{matrix}\right\} \Big/ 2.00$ $= 39.2\text{ kips/bolt}$ Bearing strength: $2.4 d t F_u / \Omega = 2.4(1.00\text{ in.})(0.585\text{ in.})(65\text{ ksi})/2.00$ $= 45.6\text{ kips/bolt}$

Note that since 58.8 kips/bolt, LRFD (39.2 kips/bolt, ASD) > 36.9 kips/bolt, LRFD (24.5 kips/bolt, ASD), all bolts are fully effective in bearing in carrying their shear load. If $\phi r_{n\ edge}$ were less than the bolt shear strength of 49.5 kips, LRFD (33.0 kips, ASD), the edge bolts would have a reduced strength and the reduced strength value would have to be used for all of the bolts in order to satisfy the basis of AISC *Manual* Table 7-8. The bolt shear controls the strength of the bolt group, thus:

LRFD	ASD
$\phi R_n = \phi r_n C$ $= (49.5\text{ kips/bolt})(4.53\text{ bolts})$ $= 224\text{ kips} > 166\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{r_n}{\Omega} C$ $= (33.0\text{ kips/bolt})(4.53\text{ bolts})$ $= 149\text{ kips} > 111\text{ kips}$ **o.k.**

Check bolt bearing on the single plate

Determine the available bearing and tearout strength of the single plate at the beam-to-column connection. With 3-in. bolt spacing and 1¾-in. edge distance, use AISC *Manual* Tables 7-4 and 7-5:

LRFD	ASD
Based on bolt spacing: $\phi r_n = (101\text{ kip/in.})(\tfrac{3}{4}\text{ in.})$ $= 75.8\text{ kips/bolt}$ Based on edge distance: $\phi r_{n\ edge} = (63.6\text{ kip/in.})(\tfrac{3}{4}\text{ in.})$ (by interpolation) $= 47.7\text{ kips/bolt} > 36.9\text{ kips}$ **o.k.**	Based on bolt spacing: $\dfrac{r_n}{\Omega} = (67.4\text{ kip/in.})(\tfrac{3}{4}\text{ in.})$ $= 50.6\text{ kips/bolt}$ Based on edge distance: $\dfrac{r_{n\ edge}}{\Omega} = (42.4\text{ kip/in.})(\tfrac{3}{4}\text{ in.})$ (by interpolation) $= 31.8\text{ kips/bolt} > 24.5\text{ kips}$ **o.k.**

Check shear yielding and shear rupture on single plate

The available shear yielding strength is determined from AISC *Specification* Section J4.2(a). The gross area subject to shear is:

$$\begin{aligned}A_{gv} &= (\tfrac{3}{4}\text{ in.})\left[9.00\text{ in.} + 2(1\tfrac{3}{4}\text{ in.})\right]\\ &= 9.38\text{ in.}^2\end{aligned}$$

LRFD	ASD
$\begin{aligned}\phi R_n &= \phi 0.60 F_y A_{gv}\\ &= 1.00(0.60)(36\text{ ksi})(9.38\text{ in.}^2)\\ &= 203\text{ kips} > 148\text{ kips}\quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= 0.60 F_y A_{gv}/\Omega\\ &= 0.60(36\text{ ksi})(9.38\text{ in.}^2)/1.50\\ &= 135\text{ kips} > 98.4\text{ kips}\quad \textbf{o.k.}\end{aligned}$

The available shear rupture strength is determined from AISC *Specification* Section J4.2(b). The net area subject to shear is:

$$\begin{aligned}A_{nv} &= 9.38\text{ in.}^2 - 4(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{4}\text{ in.})\\ &= 6.00\text{ in.}^2\end{aligned}$$

LRFD	ASD
$\begin{aligned}\phi R_n &= \phi 0.60 F_u A_{nv}\\ &= 0.75(0.60)(58\text{ ksi})(6.00\text{ in.}^2)\\ &= 157\text{ kips} > 148\text{ kips}\quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= \frac{0.60 F_u A_{nv}}{\Omega}\\ &= 0.60(58\text{ ksi})(6.00\text{ in.}^2)/2.00\\ &= 104\text{ kips} > 98.4\text{ kips}\quad \textbf{o.k.}\end{aligned}$

Use a ¾-in.-thick ASTM A36 plate.

Check tension yielding and tension rupture on the single plate

When the brace is in compression, the beam-to-column axial force results in tension on the single plate.

The available tensile yielding strength of the plate is determined from AISC *Specification* Section J4.1(a):

LRFD	ASD
$\begin{aligned}\phi R_n &= \phi F_y A_g\\ &= 0.90(36\text{ ksi})(\tfrac{3}{4}\text{ in.})\left[9.00\text{ in.} + 2(1\tfrac{3}{4}\text{ in.})\right]\\ &= 304\text{ kips} > 76.2\text{ kips}\quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= \frac{F_y A_g}{\Omega}\\ &= \frac{(36\text{ ksi})(\tfrac{3}{4}\text{ in.})\left[9.00\text{ in.} + 2(1\tfrac{3}{4}\text{ in.})\right]}{1.67}\\ &= 202\text{ kips} > 50.8\text{ kips}\quad \textbf{o.k.}\end{aligned}$

The available tensile rupture strength of the plate is determined from AISC *Specification* Section J4.1(b), with $A_e = A_n$:

LRFD	ASD
$\begin{aligned}\phi R_n &= \phi F_u A_e\\ &= 0.75(58\text{ ksi})(\tfrac{3}{4}\text{ in.})\left[12.5\text{ in.} - 4(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right]\\ &= 261\text{ kips} > 76.2\text{ kips}\quad \textbf{o.k.}\end{aligned}$	$\begin{aligned}\frac{R_n}{\Omega} &= \frac{F_u A_e}{\Omega}\\ &= \frac{(58\text{ ksi})(\tfrac{3}{4}\text{ in.})\left[12.5\text{ in.} - 4(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right]}{2.00}\\ &= 174\text{ kips} > 50.8\text{ kips}\quad \textbf{o.k.}\end{aligned}$

Therefore, use a ¾-in.-thick ASTM A36 plate for the beam-to-column single-plate connection.

Check beam shear

From AISC *Manual* Table 3-6, the available shear strength of the beam is:

LRFD	ASD
$\phi_v V_n = 298 \text{ kips} > 148 \text{ kips} \quad \textbf{o.k.}$	$\dfrac{V_n}{\Omega_v} = 199 \text{ kips} > 98.7 \text{ kips} \quad \textbf{o.k.}$

Check compressive strength of single plate

When the brace is in tension, the beam-to-column axial force is compressive.

From AISC *Specification* Section J4.4, the available compressive strength of the plate, based on buckling between the weld to the column and the first line of bolts, is:

LRFD	ASD
$\dfrac{KL}{r} = \dfrac{0.65(3 \text{ in.})}{\left(\dfrac{\text{¾ in.}}{\sqrt{12}}\right)} = 9.00 < 25$ Therefore: $\phi P_n = \phi F_y A_g = \phi F_y t_p l = 0.90(36 \text{ ksi})(\text{¾ in.})(12.5 \text{ in.}) = 304 \text{ kips} > 76.2 \text{ kips} \quad \textbf{o.k.}$	$\dfrac{KL}{r} = \dfrac{0.65(3 \text{ in.})}{\left(\dfrac{\text{¾ in.}}{\sqrt{12}}\right)} = 9.00 < 25$ Therefore: $\dfrac{P_n}{\Omega} = \dfrac{F_y A_g}{\Omega} = \dfrac{F_y t_p l}{\Omega} = \dfrac{(36 \text{ ksi})(\text{¾ in.})(12.5 \text{ in.})}{1.67} = 202 \text{ kips} > 50.8 \text{ kips} \quad \textbf{o.k.}$

Check beam web local yielding and web local crippling

Check in the same manner as for the gusset-to-column connection.

Design weld at column flange

Determine the weld size using AISC *Manual* Equation 8-2 and incorporating the directional weld strength provided in AISC *Specification* Section J2.4:

LRFD	ASD
$D = \dfrac{R_u}{1.392l\left(1.0 + 0.50\sin^{1.5}\theta\right)} = \dfrac{166 \text{ kips}}{(1.392 \text{ kip/in.})(2)(12.5 \text{ in.})\left(1.0 + 0.50\sin^{1.5} 27.2°\right)} = 4.13 \text{ sixteenths}$ Therefore, use a 5⁄16-in. fillet weld.	$D = \dfrac{R_a}{0.928l\left(1.0 + 0.50\sin^{1.5}\theta\right)} = \dfrac{111 \text{ kips}}{(0.928 \text{ kip/in.})(2)(12.5 \text{ in.})\left(1.0 + 0.50\sin^{1.5} 27.3°\right)} = 4.14 \text{ sixteenths}$ Therefore, use a 5⁄16-in. fillet weld.

The completed design is shown in Figure 6-9**.**

DISCUSSION: COMPARISON OF HIGH- AND LOW-SEISMIC CONNECTIONS

The connection designs shown in Figures 6-1 and 6-9 are subjected to exactly the same loads from wind and seismic events. In the design of the connection shown in Figure 6-1, the requirements of the AISC *Seismic Provisions* are invoked, in addition to those of the AISC *Specification*. In the design of the connection shown in Figure 6-9, only the AISC *Specification* requirements are used. The effect of the *Seismic Provisions* can clearly be seen when these two figures are compared. If the *Seismic Provisions* are required because of building type or usage, system choice, or because of owner preference, all parties should be aware of the cost involved. The requirements of the *Seismic Provisions* have been known to increase the structural steel cost by 30 to 40%.

APPENDIX A
Derivation and Generalization of the Uniform Force Method

When properly applied, the uniform force method provides a standardized procedure to arrive at a force distribution that is consistent with the boundary conditions and will satisfy equilibrium. It is also thought to result in a force distribution that will produce the most economical connection. However, in some cases it is not possible to satisfy all of the constraints inherent in the uniform force method. In such instances, it is useful to have a generalized method that can be utilized to obtain consistent force distributions.

The uniform force method produces connection interfaces free of moments by placing constraints on the connection geometry and the forces distributed to the connection interfaces. A force distribution is produced such that the line of action of the gusset-to-column force passes through the column control point, the line of action of the gusset-to-beam force passes through the beam control point, and the gusset-to-column, gusset-to-beam, and the brace forces all intersect at a common point.

A.1 GENERAL METHOD

The following is a derivation of the method along with several modifications to ease its use and expand its scope of applicability.

A.1.1 Beam Control Point

Three forces (see Figure A-1) are applied to the beam: H, $\sqrt{V_b^2 + H_c^2}$, and $\sqrt{V_b^2 + H_b^2}$,

where

H = horizontal component of the required axial force

$H_b = H_{ub}$ or H_{ab}, required shear force on the gusset-to-beam connection

$H_c = H_{uc}$ or H_{ac}, required axial force on the gusset-to-column connection

$V_b = V_{ub}$ or V_{ab}, required axial force on the gusset-to-beam connection

H is assumed to act at the centerline of the beam. $\sqrt{V_b^2 + H_c^2}$ acts at the beam-to-column connection located at $(e_c, 0)$, and $\sqrt{V_b^2 + H_b^2}$ acts at the beam-to-gusset connection located at $\left[(e_c + \alpha), e_b\right]$.

H and $\sqrt{V_b^2 + H_c^2}$ intersect at $(e_c, 0)$; therefore $\sqrt{V_b^2 + H_b^2}$ must also pass through the coordinates $(e_c, 0)$. This means that the point $(e_c, 0)$ must be the location of the beam control point V (point A of Figure A-1) and $\dfrac{V_b}{H_b} = \dfrac{(e_b - 0)}{(e_c + \alpha - e_c)} = \dfrac{e_b}{\alpha}$.

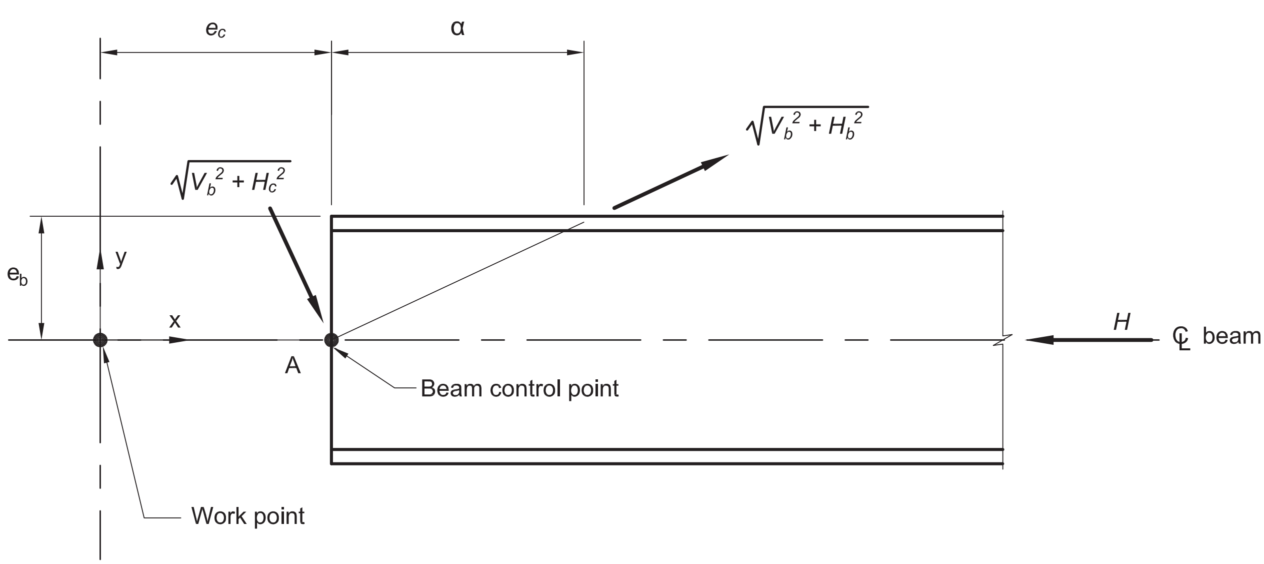


Fig. A-1. Free body diagram of beam.

A.1.2 Gusset Control Point

Three forces are applied to the gusset (see Figure A-2): P, $\sqrt{V_b^2+H_b^2}$, and $\sqrt{V_c^2+H_c^2}$. P acts through the work point at the angle θ. We know the slope of the line of action of $\sqrt{V_b^2+H_b^2}$ from the discussion above is $\dfrac{V_b}{H_b}=\dfrac{e_b}{\alpha}$.

The equations for the line of action of the three vectors are determined as follows:

For force P:

$$y-0=\frac{1}{\tan\theta}(x-0)$$

Solving for y:

$$y=\frac{x}{\tan\theta} \tag{A-1}$$

For $\sqrt{V_b^2+H_b^2}$:

$$y-e_b=\frac{V_b}{H_b}\left[x-(e_c+\alpha)\right]$$

Solving for y and substituting $\dfrac{V_b}{H_b}=\dfrac{e_b}{\alpha}$:

$$y=\frac{e_b}{\alpha}(x-e_c) \tag{A-2}$$

For $\sqrt{V_c^2+H_c^2}$:

$$y-(e_b+\beta)=\frac{V_c}{H_c}(x-e_c)$$

Solving for y:

$$y=\frac{V_c}{H_c}(x-e_c)+e_b+\beta \tag{A-3}$$

Substituting $m_c=\dfrac{V_c}{H_c}$ into Equation A-3 results in:

$$y=m_c(x-e_c)+e_b+\beta \tag{A-3a}$$

Setting Equations A-1 and A-3a equal and solving for x results in:

$$\frac{x}{\tan\theta}=m_c(x-e_c)+e_b+\beta$$

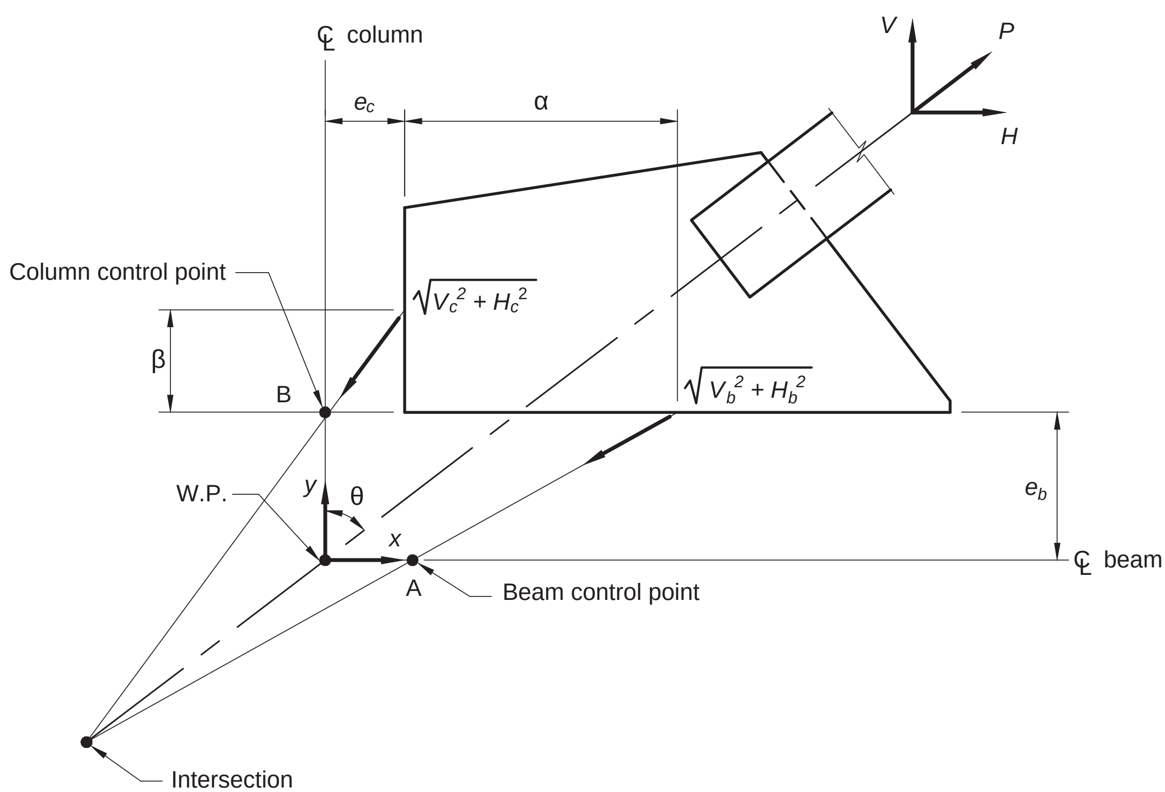


Fig. A-2. Free body diagram of gusset.

Solving for x:

$$x = \left(\frac{m_c e_c - e_b - \beta}{m_c \tan\theta - 1}\right)\tan\theta \qquad \text{(A-4)}$$

Setting Equations A-2 and A-3a equal and solving for x results in:

$$\frac{e_b}{\alpha}\left(x - e_c\right) = m_c\left(x - e_c\right) + e_b + \beta$$

Solving for x:

$$x = \frac{\alpha\left(m_c e_c - \beta - e_b\right) - e_b e_c}{m_c \alpha - e_b} \qquad \text{(A-5)}$$

Setting Equations A-4 and A-5 equal and solving for m_c:

$$\left(\frac{m_c e_c - e_b - \beta}{m_c \tan\theta - 1}\right)\tan\theta = \frac{\alpha\left(m_c e_c - \beta - e_b\right) - e_b e_c}{m_c \alpha - e_b}$$

Solving for m_c:

$$m_c = \frac{\beta}{e_c}\left\{\frac{e_b}{\beta}\left[1 - \frac{\left(e_b + \beta\right)\tan\theta - e_c}{\alpha}\right] + 1\right\} \qquad \text{(A-6)}$$

The location of the gusset control point is determined as follows.

Setting Equation A-1 equal to Equation A-2:

$$\frac{x}{\tan\theta} = \frac{e_b}{\alpha}\left(x - e_c\right)$$

Solving for x, where $x = x_{gcp}$:

$$x_{gcp} = \frac{e_b e_c \tan\theta}{e_b \tan\theta - \alpha} \qquad \text{(A-7)}$$

Using this value in Equation A-1, solve for the y-coordinate:

$$y_{gcp} = \frac{e_b e_c}{e_b \tan\theta - \alpha} \qquad \text{(A-8)}$$

Now find the location of the column control point, point B (see Figure A-3):

The y-coordinate of the intersection of $\sqrt{V_c^2 + H_c^2}$ with the centerline of the column can be found by substituting $x = 0$ into Equation A-3:

$$y = e_b\,\frac{e_b \tan\theta + \beta\tan\theta - e_c}{\alpha} \qquad \text{(A-9)}$$

The slope of $\sqrt{V_b^2 + H_c^2}$ is:

$$\frac{V_b}{H_c} = \frac{e_b\,\dfrac{e_b \tan\theta + \beta\tan\theta - e_c}{\alpha} - 0}{0 + e_c} = \frac{e_b}{e_c}\,\frac{\left(e_b + \beta\right)\tan\theta - e_c}{\alpha} \qquad \text{(A-10)}$$

We have now derived all of the requisite geometry to ensure that there are no moments at any connection interfaces. It may be noted that the relationship between α and β has not yet been established. The relationship given in the body of this Design Guide and from AISC *Manual* Equation 13-1 is $\alpha = (e_b + \beta)\tan\theta - e_c$. This relationship is not necessary to ensure that moments do not exist at the interfaces and is the result of using the point (0, e_b) as a control point.

A.1.3 Determination of Forces

Having established the requisite geometry, the forces can now be obtained. The easiest place to start is with the equal and opposite horizontal forces applied at the column. To satisfy equilibrium, H_c must be:

$$H_c = \frac{P e_c \cos\theta}{e_b + \beta} \qquad \text{(A-11)}$$

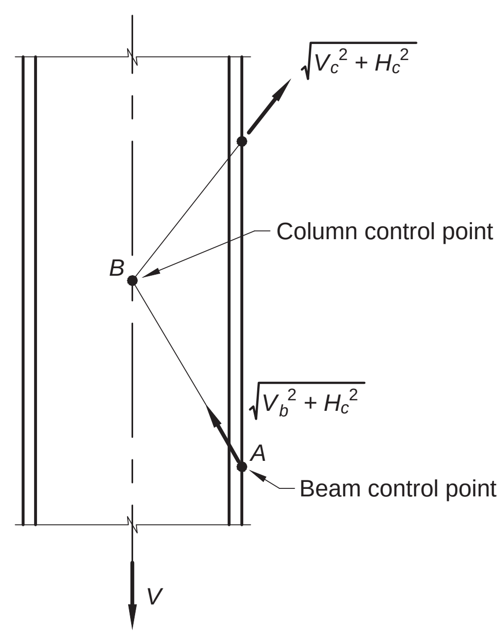


Fig. A-3. Free body diagram of column.

From this, determine the vertical force applied at the beam-to-column connection:

$$V_b = H_c \left[\frac{e_b}{e_c} \frac{(e_b + \beta)\tan\theta - e_c}{\alpha} \right]$$

$$= P \left\{ \frac{e_b \left[(e_b + \beta)\sin\theta - e_c \cos\theta \right]}{\alpha (e_b + \beta)} \right\} \qquad \text{(A-12)}$$

The remaining forces are simply found as:

$$\sum F_x = 0$$

$$V_c = P\cos\theta - V_b \qquad \text{(A-13)}$$

and

$$\sum F_y = 0$$

$$H_b = P\sin\theta - H_c \qquad \text{(A-14)}$$

It should be noted that since there is no defined relationship between α and β, the equations above will produce a force distribution free of moments for any connection geometry. This eliminates the need to include the variables $\bar{\alpha}$ and $\bar{\beta}$ in this procedure, where $\bar{\alpha}$ is the actual distance from the face of the column flange or web to the centroid of the gusset-to-beam connection, and $\bar{\beta}$ is the actual distance from the face of the beam flange to the centroid of the gusset-to-column connection. However, there still may be a need to redistribute the vertical load between the beam-to-column and gusset-to-column connections. This redistribution will obviously result in a moment. Because the beam-to-gusset connection will generally be welded and the gusset-to-column connection is generally bolted, the moment will be assumed to act at the stiffer beam-to-gusset interface. The moment can be calculated as:

$$M_b = H_b e_b - \overline{V_b}\alpha \qquad \text{(A-15)}$$

where

$$\overline{V_b} = V_b - \Delta V_b \qquad \text{(A-16)}$$

This completes the derivation of the uniform force method in its most general form. If the relationship $\alpha = (e_b + \beta)\tan\theta - e_c$ is imposed, the more familiar equations are obtained:

$$V_c = \frac{\beta}{r} P \qquad (\textit{Manual}\ \text{Eq. 13-2})$$

$$H_c = \frac{e_c}{r} P \qquad (\textit{Manual}\ \text{Eq. 13-3})$$

$$V_b = \frac{e_b}{r} P \qquad (\textit{Manual}\ \text{Eq. 13-4})$$

$$H_b = \frac{\alpha}{r} P \qquad (\textit{Manual}\ \text{Eq. 13-5})$$

$$r = \sqrt{(\alpha + e_c)^2 + (e_b + \beta)^2} \qquad (\textit{Manual}\ \text{Eq. 13-6})$$

A.1.4 Accounting for the Beam Reaction

The uniform force method is intended to optimize the connection design by eliminating moments at the connection interfaces. In most presentations of the uniform force method, the beam reaction is neglected. It has therefore been standard practice to deliver all of the beam reaction to the face of the column through the beam-to-column connection. The eccentricity measured from the center of the column to the beam connection is ignored, as is common practice for a beam without vertical bracing. In most instances, this treatment is acceptable. However, in some instances redistributing some of the beam reaction to the gusset-to-column connection can greatly increase economy. This is especially true when the beam reaction is applied eccentric to the column. As an example, buildings are often designed with moment frames along the strong axis of the columns and vertical bracing along the weak axis of the columns. Since moment connections will often require the use of stiffeners, it can become difficult to erect the weak-axis bracing. One solution is to use extended single-plate connections, sometimes called extended shear tabs, to connect the braces. The uniform force method is particularly well suited to analyzing this type of connection. Improved economy is obtained by assuming e_c is the distance from the work-point to the center of the bolt group in the extended shear tab. This is contrary to the typical assumption that e_c is equal to zero, when vertical bracing frames to a column web. By analyzing the connection in this way, the eccentricity that would normally be resisted by the bolt group can now be taken as a couple, H_c. The uniform force method can be applied to the connection, with the forces normally associated with the column now associated with the single-plate connection. As long as the forces derived from the uniform force method are used without the introduction of ΔV_b, no moments will need to be considered in any of the connection interfaces. However, the single-plate connection must now be checked to transfer H_c as a horizontal shear and also an internal moment that will not appear at any of the interfaces.

This procedure will eliminate moments at the connections due to the brace force, but a moment will still have to be resisted by the bolts at the beam-to-column connection. It is impossible to eliminate this moment entirely, since the beam reaction does not have a line of action that passes through the work-point. However, it is possible to move the moment from the beam-to-column connection to the beam-to-gusset connection. It is usually more economical to resist this moment at the beam-to-gusset connection since the beam-to-gusset connection is usually welded and is not limited by the depth of the beam. In order to accomplish this, some of the beam reaction is distributed to the gusset-to-column connection through the beam-to-gusset connection. Though a model can be established that satisfies equilibrium without redistributing a portion of the vertical beam end reaction into the gusset, the model presented here has the advantage that it will result in no moment in the single plate at a section cut at the elevation of the beam flange. In order to avoid coping of the beam, the single plate is typically necked-down local to the beam flange, and in some instances, as shown in Figure A-4, stiffeners may intersect the tab plate requiring welding to transfer the H_c force. Due to the reduced width of the single plate at these locations, additional moment can lead to larger welds and/or thicker single plates. The following resulting force distribution can be superimposed on the forces resulting from the uniform force method:

Gusset-to-Column Connection:

$$H_{cR} = R\left(\frac{e_c}{e_b+\beta}\right) \tag{A-17}$$

$$V_{cR} = R\left(\frac{\beta}{e_b+\beta}\right) \tag{A-18}$$

Gusset-to-Beam Connection:

$$H_{bR} = -R\left(\frac{e_c}{e_b+\beta}\right) \tag{A-19}$$

$$V_{bR} = R\left(\frac{e_b}{e_b+\beta}-1\right) \tag{A-20}$$

$$M_{bR} = -R\left(e_c\right) \tag{A-21}$$

Beam-to-Column Connection:

$$V_{bcR} = R\left(1-\frac{\beta}{e_b+\beta}\right) \tag{A-22}$$

$$H_{bcR} = H_{cR} \tag{A-23}$$

Example A.1—Vertical Brace-to-Column Web Connection Using an Extended Single Plate

Given:

Verify the bracing connection design shown in Figure A-4 using the general uniform force method. The loading, member sizes, material strengths, bolt size and type, hole size, and connection element sizes are as given in Figure A-4. Use 70-ksi electrodes.

The required strengths are:

LRFD	ASD
Brace: $\pm P_u = 225$ kips Beam: $V_u = 30$ kips	Brace: $\pm P_a = 150$ kips Beam: $V_a = 20$ kips

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi $\quad$ $F_u = 65$ ksi

ASTM A36
$F_y = 36$ ksi $\quad$ $F_u = 58$ ksi

ASTM A500 Grade B
$F_y = 46$ ksi $\quad F_u = 58$ ksi

From AISC *Manual* Tables 1-1 and 1-8, the member geometric properties are as follows:

Beam
W16×26
$A_g = 7.68$ in.^{2} $\quad t_w = 0.250$ in. $\quad d = 15.7$ in. $\quad b_f = 5.50$ in. $\quad t_f = 0.345$ in. $\quad k_{des} = 0.747$ in.

Column
W14×145
$t_w = 0.680$ in. $\quad d = 14.8$ in. $\quad b_f = 15.5$ in. $\quad t_f = 1.09$ in. $\quad h/t_w = 16.8$

Brace
HSS10×10×5⁄16
$A_g = 11.1$ in.^{2} $\quad r = 3.94$ in. $\quad t_{des} = 0.291$ in.

From AISC *Specification* Table J3.3, the hole dimensions for ¾-in.-diameter bolts are as follows:

Brace, column and plates
Standard: d_h = 13⁄16 in. diameter

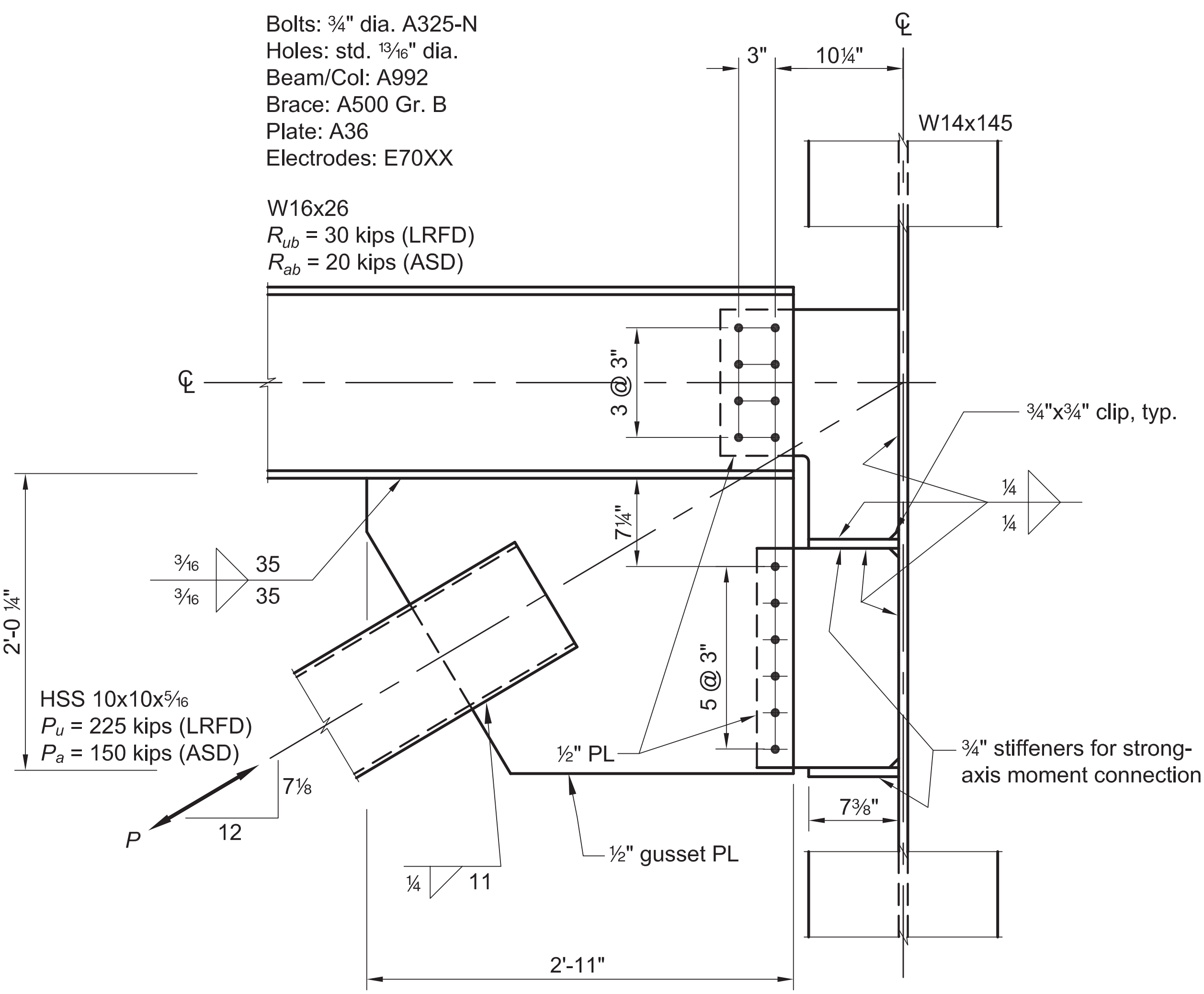


Fig. A-4. Extended single-plate connection example.

Brace-to-Gusset Connection

Design the weld between the brace and the gusset

The weld will be assumed to be ¼ in. This is done to maintain a single-pass 5⁄16-in. fillet weld, while accounting for the gap between the tube wall and the gusset plate. Using AISC *Manual* Equation 8-2, and solving for l, the required length of ¼-in. weld for four lines of weld is:

LRFD	ASD
$l = \dfrac{P_u}{4(1.392 \text{ kip/in.})D}$ $= \dfrac{225 \text{ kips}}{4(1.392 \text{ kip/in.})(4 \text{ sixteenths})}$ $= 10.1 \text{ in.}$	$l = \dfrac{P_a}{4(0.928 \text{ kip/in.})D}$ $= \dfrac{150 \text{ kips}}{4(0.928 \text{ kip/in.})(4 \text{ sixteenths})}$ $= 10.1 \text{ in.}$

Therefore, use an 11-in. fillet weld.

Check shear rupture on the HSS wall local to the weld

The available shear rupture strength is determined from AISC *Specification* Section J4.2, Equation J4-4:

LRFD	ASD
$\phi R_n = 4(\phi 0.60 F_u l t_{des})$ $= 4[(0.75)(0.60)(58 \text{ ksi})(11.0 \text{ in.})(0.291 \text{ in.})]$ $= 334 \text{ kips} > 225 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 4\left(\dfrac{0.60 F_u l t_{des}}{2.00}\right)$ $= 4\left[\dfrac{0.60(58 \text{ ksi})(11.0 \text{ in.})(0.291 \text{ in.})}{2.00}\right]$ $= 223 \text{ kips} > 150 \text{ kips}$ **o.k.**

Check tension yielding on the brace

The available tensile yielding strength is determined from AISC *Specification* Section D2, Equation D2-1. Alternatively, AISC *Manual* Table 5-5 may be used:

LRFD	ASD
$\phi_t P_n = \phi_t F_y A_g$ $= 0.90(46 \text{ ksi})(11.1 \text{ in.}^2)$ $= 460 \text{ kips} > 225 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = \dfrac{F_y A_g}{\Omega_t}$ $= \dfrac{(46 \text{ ksi})(11.1 \text{ in.}^2)}{1.67}$ $= 306 \text{ kips} > 150 \text{ kips}$ **o.k.**

Check tensile rupture on the brace

For preliminary calculations, assume a ½-in.-thick gusset plate. Assume that the slot in the brace is ⅛ in. wider than the gusset plate. The effective net area is:

$$A_e = A_n U \qquad \text{(}\textit{Spec}\text{. Eq. D3-1)}$$

where

$$\begin{aligned}A_n &= A_g - 2\left(t_g + \tfrac{1}{8}\text{ in.}\right)t_{des}\\ &= 11.1\text{ in.}^2 - 2\left(\tfrac{1}{2}\text{ in.} + \tfrac{1}{8}\text{ in.}\right)\left(0.291\text{ in.}\right)\\ &= 10.7\text{ in.}^2\end{aligned}$$

The shear lag factor, U, is determined from AISC *Specification* Table D3.1, Case 6, with a single concentric gusset plate, assuming $l \geq H$:

$$\begin{aligned}\bar{x} &= \frac{B^2 + 2BH}{4\left(B+H\right)} \qquad \left(B = H = 10.0\text{ in.}\right)\\ &= \frac{\left(10.0\text{ in.}\right)^2 + 2\left(10.0\text{ in.}\right)\left(10.0\text{ in.}\right)}{4\left(10.0\text{ in.} + 10.0\text{ in.}\right)}\\ &= 3.75\text{ in.}\end{aligned}$$

The equation in AISC *Specification* Table D3.1, Case 6, with a single concentric gusset plate can be used because the length of weld, l, exceeds the wall of the HSS, H:

$$\begin{aligned}U &= 1 - \left(\frac{\bar{x}}{l}\right)\\ &= 1 - \left(\frac{3.75\text{ in.}}{11.0\text{ in.}}\right)\\ &= 0.659\end{aligned}$$

Therefore, the effective net area is:

$$\begin{aligned}A_e &= A_nU \qquad \text{(}\textit{Spec}\text{. Eq. D3-1)}\\ &= \left(10.7\text{ in.}^2\right)\left(0.659\right)\\ &= 7.05\text{ in.}^2\end{aligned}$$

From AISC *Specification* Section D2(b), Equation D2-2, the available strength for the limit state of tensile rupture is:

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75\left(58\text{ ksi}\right)\left(7.05\text{ in.}^2\right)$ $= 307\text{ kips} > 225\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{\left(58\text{ ksi}\right)\left(7.05\text{ in.}^2\right)}{2.00}$ $= 205\text{ kips} > 150\text{ kips}$ **o.k.**

Proceed with a ½-in.-thick ASTM A36 gusset plate and check block shear, tension on the Whitmore section, and buckling of the gusset plate.

Check block shear rupture on the gusset plate

The available strength for the limit state of block shear rupture is given in AISC *Specification* Section J4.3 as follows:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(}\textit{Spec}\text{. Eq. J4-5)}$$

Shear yielding component:

$$
\begin{aligned}
A_{gv} &= A_{nv} \\
&= 2t_g l \\
&= 2(\tfrac{1}{2}\text{ in.})(11.0\text{ in.}) \\
&= 11.0\text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
0.60F_y A_{gv} &= 0.60(36\text{ ksi})(11.0\text{ in.}^2) \\
&= 238\text{ kips}
\end{aligned}
$$

Shear rupture component:

$$
\begin{aligned}
0.60F_u A_{nv} &= 0.60(58\text{ ksi})(11.0\text{ in.}^2) \\
&= 383\text{ kips}
\end{aligned}
$$

Tension rupture component:

U_{bs} is 1 from AISC *Specification* Section J4.3 because the connection is uniformly loaded.

$$
\begin{aligned}
A_{gt} &= A_{nt} \\
&= t_g B \\
&= (\tfrac{1}{2}\text{ in.})(10.0\text{ in.}) \\
&= 5.00\text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
U_{bs}F_u A_{nt} &= 1(58\text{ ksi})(5.00\text{ in.}^2) \\
&= 290\text{ kips}
\end{aligned}
$$

The available strength for the limit state of block shear rupture is determined as follows:

$$
\begin{aligned}
0.60F_u A_{nv} + U_{bs}F_u A_{nt} &= 383\text{ kips} + 290\text{ kips} \\
&= 673\text{ kips} \\
0.60F_y A_{gv} + U_{bs}F_u A_{nt} &= 238\text{ kips} + 290\text{ kips} \\
&= 528\text{ kips}
\end{aligned}
$$

Therefore, $R_n = 528$ kips.

LRFD	ASD
$\phi R_n = 0.75(528\text{ kips})$ $= 396\text{ kips} > 225\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{528\text{ kips}}{2.00}$ $= 264\text{ kips} > 150\text{ kips}$ **o.k.**

Check the gusset plate for tension yielding on the Whitmore section

From AISC *Manual* Part 9, the width of the Whitmore Section is:

$$
\begin{aligned}
l_w &= 10.0\text{ in.} + 2(11.0\text{ in.})\tan 30^\circ \\
&= 22.7\text{ in.}
\end{aligned}
$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is:

LRFD	ASD
$\phi R_n = \phi F_y A_w$ $= 0.90(36\text{ ksi})(22.7\text{ in.})(\tfrac{1}{2}\text{ in.})$ $= 368\text{ kips} > 225\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_w}{\Omega}$ $= \dfrac{(36\text{ ksi})(22.7\text{ in.})(\tfrac{1}{2}\text{ in.})}{1.67}$ $= 245\text{ kips} > 150\text{ kips}$ **o.k.**

Check the gusset plate for compression buckling on the Whitmore Section

Assume $K = 0.65$ and the unbraced length of the gusset plate is 19.6 in.:

$$\begin{aligned}\frac{KL}{r} &= \frac{0.65(19.6\text{ in.})}{\tfrac{1}{2}\text{ in.}/\sqrt{12}}\\ &= 88.3\end{aligned}$$

Because $KL/r > 25$, use AISC *Manual* Table 4-22 to determine the available critical stress for the 36-ksi gusset plate. Then the available strength can be determined from AISC *Specification* Sections E1 and E3. With $A_w = (22.7\text{ in.})(\tfrac{1}{2}\text{ in.}) = 11.4\text{ in.}^2$:

LRFD	ASD
$\phi_c F_{cr} = 21.5\text{ ksi}$ $\phi P_n = \phi_c F_{cr} A_w$ $= (21.5\text{ ksi})(11.4\text{ in.}^2)$ $= 245\text{ kips} > 225\text{ kips}$ **o.k.**	$F_{cr}/\Omega_c = 14.3\text{ ksi}$ $\dfrac{P_n}{\Omega} = \dfrac{F_{cr} A_w}{\Omega_c}$ $= (14.3\text{ ksi})(11.4\text{ in.}^2)$ $= 163\text{ kips} > 150\text{ kips}$ **o.k.**

The remaining checks will assume a gusset plate that is ½ in. thick, 2 ft 11 in. long, and 2 ft ¼ in. deep.

Connection Interface Forces

The forces at the gusset-to-beam and gusset-to-column interfaces are determined using the general case of the UFM as discussed in AISC *Manual* Part 13 and discussed in this Appendix. The geometric properties of the connection configuration given in Figure A-5 are as follows:

Assume e_c is equal to the distance from the face of the support to the center of the bolt group from the gusset to the column:

$$e_c = 10\tfrac{1}{4}\text{ in.}$$

$$\begin{aligned}e_b &= \frac{d}{2}\\ &= \frac{15.7\text{ in.}}{2}\\ &= 7.85\text{ in.}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{12}{7\tfrac{1}{8}}\right)\\ &= 59.3^\circ\end{aligned}$$

Because this approach assumes that $e_c = 10\frac{1}{4}$ in. rather than 0 in., then α is calculated as:

$$\alpha = \frac{35 \text{ in.}}{2} - 1\tfrac{3}{4} \text{ in.}$$
$$= 15.75 \text{ in.}$$

$$\beta = 7\tfrac{1}{4} \text{ in.} + 7\tfrac{1}{2} \text{ in.}$$
$$= 14.75 \text{ in.}$$

Using a modified uniform force method approach, eccentricity from both the brace force and the beam reaction will be resisted by equal and opposite horizontal forces applied at the beam-to-column and the gusset-to-column connections.

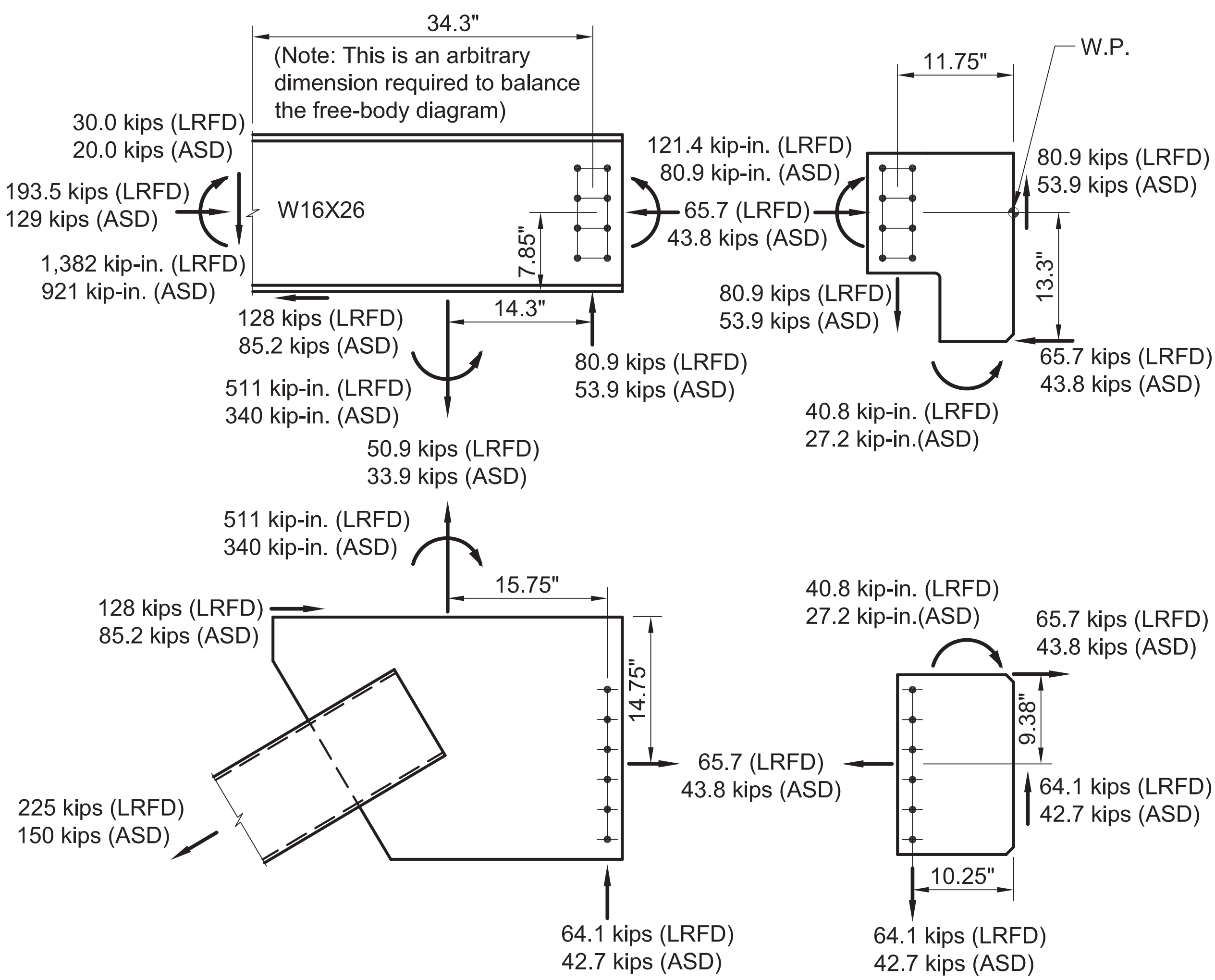


Fig. A-5. Admissible force field extended single-plate connection example of Figure A-4.

Gusset-to-column connection forces

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>

$$H_{uc}+H_{ucR}=P_u\left(\frac{e_c\cos\theta}{e_b+\beta}\right)+R_{ub}\left(\frac{e_c}{e_b+\beta}\right)$$

$$=(225\text{ kips})\left[\frac{(10.25\text{ in.})\cos 59.3°}{7.85\text{ in.}+14.75\text{ in.}}\right]$$

$$+(30\text{ kips})\left(\frac{10.25\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=65.7\text{ kips}$$

$$V_{uc}+V_{ucR}=P_u\left\{\cos\theta-\frac{e_b\left[(e_b+\beta)\sin\theta-e_c\cos\theta\right]}{\alpha(e_b+\beta)}\right\}+R_{ub}\left(\frac{\beta}{e_b+\beta}\right)$$

$$=(225\text{ kips})\left\{\cos 59.3° - \frac{(7.85\text{ in.})\left[(7.85\text{ in.}+14.75\text{ in.})\sin 59.3°-(10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.}+14.75\text{ in.})}\right\}$$

$$+(30\text{ kips})\left(\frac{14.75\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=64.1\text{ kips}$$

</td><td>

$$H_{ac}+H_{acR}=P_a\left(\frac{e_c\cos\theta}{e_b+\beta}\right)+R_{ab}\left(\frac{e_c}{e_b+\beta}\right)$$

$$=(150\text{ kips})\left[\frac{(10.25\text{ in.})\cos 59.3°}{7.85\text{ in.}+14.75\text{ in.}}\right]$$

$$+(20\text{ kips})\left(\frac{10.25\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=43.8\text{ kips}$$

$$V_{ac}+V_{acR}=P_a\left\{\cos\theta-\frac{e_b\left[(e_b+\beta)\sin\theta-e_c\cos\theta\right]}{\alpha(e_b+\beta)}\right\}+R_{ab}\left(\frac{\beta}{e_b+\beta}\right)$$

$$=(150\text{ kips})\left\{\cos 59.3° - \frac{(7.85\text{ in.})\left[(7.85\text{ in.}+14.75\text{ in.})\sin 59.3°-(10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.}+14.75\text{ in.})}\right\}$$

$$+(20\text{ kips})\left(\frac{14.75\text{ kips}}{7.85\text{ kips}+14.75\text{ kips}}\right)$$

$$=42.7\text{ kips}$$

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</table>

Gusset-to-beam connection forces

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<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>

$$H_{ub}+H_{ubR}=P_u\left(\sin\theta-\frac{e_c\cos\theta}{e_b+\beta}\right)-R_u\left(\frac{e_c}{e_b+\beta}\right)$$

$$=(225\text{ kips})\left[\sin 59.3°-\frac{(10.25\text{ in.})\cos 59.3°}{7.85\text{ in.}+14.75\text{ in.}}\right]$$

$$-(30\text{ kips})\left(\frac{10.25\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=128\text{ kips}$$

$$V_{ub}+V_{ubR}=P_u\left\{\frac{e_b\left[(e_b+\beta)\sin\theta-e_c\cos\theta\right]}{\alpha(e_b+\beta)}\right\}-R_u\left(\frac{\beta}{e_b+\beta}\right)$$

$$=(225\text{ kips})$$

$$\times\left\{\frac{(7.85\text{ in.})\left[(7.85\text{ in.}+14.75\text{ in.})\sin 59.3°-(10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.}+14.75\text{ in.})}\right\}$$

$$-(30\text{ kips})\left(\frac{14.75\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=50.9\text{ kips}$$

</td><td>

$$H_{ab}+H_{abR}=P_a\left(\sin\theta-\frac{e_c\cos\theta}{e_b+\beta}\right)-R_a\left(\frac{e_c}{e_b+\beta}\right)$$

$$=(150\text{ kips})\left[\sin 59.3°-\frac{(10.25\text{ in.})\cos 59.3°}{7.85\text{ in.}+14.75\text{ in.}}\right]$$

$$-(20\text{ kips})\left(\frac{10.25\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=85.2\text{ kips}$$

$$V_{ab}+V_{abR}=P_a\left\{\frac{e_b\left[(e_b+\beta)\sin\theta-e_c\cos\theta\right]}{\alpha(e_b+\beta)}\right\}-R_a\left(\frac{\beta}{e_b+\beta}\right)$$

$$=(150\text{ kips})$$

$$\times\left\{\frac{(7.85\text{ in.})\left[(7.85\text{ in.}+14.75\text{ in.})\sin 59.3°-(10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.}+14.75\text{ in.})}\right\}$$

$$-(20\text{ kips})\left(\frac{14.75\text{ in.}}{7.85\text{ in.}+14.75\text{ in.}}\right)$$

$$=33.9\text{ kips}$$

</td></tr>
</table>

LRFD	ASD
$M_{ub} = \alpha(V_{ub} + V_{ubR}) - e_b(H_{ub} + H_{ubR}) - e_c R_u$ $= (15.75\text{ in.})(50.9\text{ kips}) - (7.85\text{ in.})(128\text{ kips})$ $-(10.25\text{ in.})(30\text{ kips})$ $= -511\text{ kip-in.}$	$M_{ab} = \alpha(V_{ab} + V_{abR}) - e_b(H_{ab} + H_{abR}) - e_c R_a$ $= (15.75\text{ in.})(33.9\text{ kips}) - (7.85\text{ in.})(85.2\text{ kips})$ $-(10.25\text{ in.})(20\text{ kips})$ $= -340\text{ kip-in.}$

Determine beam-to-column forces

LRFD	ASD
$V_{ub} + V_{ubcR} = P_u \left[\dfrac{e_b\left[(e_b+\beta)\sin\theta - e_c\cos\theta\right]}{\alpha(e_b+\beta)} \right] + R_u\left(1 - \dfrac{\beta}{e_b+\beta}\right)$ $= (225\text{ kips})$ $\times\left\{\dfrac{(7.85\text{ in.})\left[(7.85\text{ in.} + 14.75\text{ in.})\sin 59.3° - (10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.} + 14.75\text{ in.})}\right\}$ $+(30\text{ kips})\left(1 - \dfrac{14.75\text{ in.}}{7.85\text{ in.} + 14.75\text{ in.}}\right)$ $= 80.9\text{ kips}$ As determined previously: $H_{ucR} + H_{uc} = 65.7\text{ kips}$	$V_{ab} + V_{abcR} = P_a \left\{ \dfrac{e_b\left[(e_b+\beta)\sin\theta - e_c\cos\theta\right]}{\alpha(e_b+\beta)} \right\} + R_a\left(1 - \dfrac{\beta}{e_b+\beta}\right)$ $= (150\text{ kips})$ $\times\left\{\dfrac{(7.85\text{ in.})\left[(7.85\text{ in.} + 14.75\text{ in.})\sin 59.3° - (10.25\text{ in.})\cos 59.3°\right]}{(15.75\text{ in.})(7.85\text{ in.} + 14.75\text{ in.})}\right\}$ $+(20\text{ kips})\left(1 - \dfrac{14.75\text{ in.}}{7.85\text{ in.} + 14.75\text{ in.}}\right)$ $= 53.9\text{ kips}$ As determined previously: $H_{acR} + H_{ac} = 43.8\text{ kips}$

Gusset-to-Beam Connection

Check the gusset plate for shear yielding along the beam flange

Determine the available shear strength of the gusset plate using AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(35.0\text{ in.})(½\text{ in.})$ $= 378\text{ kips} > 128\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{1.50}$ $= \dfrac{0.60(36\text{ ksi})(35.0\text{ in.})(½\text{ in.})}{1.50}$ $= 252\text{ kips} > 85.2\text{ kips}$ **o.k.**

Check the gusset plate for tensile yielding along the beam flange

Determine the available tensile yielding strength of the gusset plate from AISC *Specification* Section J4.1, Equation J4-1, as follows:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(35.0\text{ in.})(½\text{ in.})$ $= 567\text{ kips}$	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{(36\text{ ksi})(35.0\text{ in.})(½\text{ in.})}{1.67}$ $= 377\text{ kips}$

LRFD	ASD
$R_u = V_{ub} + \dfrac{4M_{ub}}{L_g}$ $= 50.9\text{ kips} + \dfrac{4(511\text{ kip-in.})}{35.0\text{ in.}}$ $= 109\text{ kips} < 567\text{ kips}$ **o.k.**	$R_a = V_{ab} + \dfrac{4M_{ab}}{L_g}$ $= 33.9\text{ kips} + \dfrac{4(340\text{ kip-in.})}{35.0\text{ in.}}$ $= 72.8\text{ kips} < 377\text{ kips}$ **o.k.**

Design weld at gusset-to-beam flange connection

The strength determination of fillet welds defined in AISC *Specification* Section J2.4 can be simplified as explained in AISC *Manual* Part 8 to AISC *Manual* Equations 8-2a and 8-2b. Also, incorporating the increased strength due to the load angle from AISC *Specification* Equation J2-5, the required weld size is determined as follows:

LRFD	ASD
Resultant load: $R = \sqrt{(109\text{ kips})^2 + (128\text{ kips})^2}$ $= 168\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{109\text{ kips}}{128\text{ kips}}\right)$ $= 40.4°$ $D_{req} = \dfrac{R_u}{2L_g(1.392\text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{168}{2(35.0\text{ in.})(1.392\text{ kip/in.})(1.0 + 0.5\sin^{1.5}40.4°)}$ $= 1.37\text{ sixteenths}$	Resultant load: $R = \sqrt{(72.8\text{ kips})^2 + (85.2\text{ kips})^2}$ $= 112\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{72.8\text{ kips}}{85.2\text{ kips}}\right)$ $= 40.5°$ $D_{req} = \dfrac{R_a}{2L_g(0.928\text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{112}{2(35.0\text{ in.})(0.928\text{ kip/in.})(1.0 + 0.5\sin^{1.5}40.5°)}$ $= 1.37\text{ sixteenths}$

Therefore, use a 3⁄16-in. fillet weld (AISC minimum size).

Check beam web local yielding

The normal force is applied at a distance α = 15.75 in. from the beam end, which is approximately equal to the depth of the W16×26 beam. Therefore, use AISC *Specification* Equation J10-3 to determine the available strength due to beam web local yielding:

LRFD	ASD
$\phi R_n = \phi F_y t_w (2.5k + L_g)$ $= 1.00(50\text{ ksi})(0.250\text{ in.})[2.5(0.747\text{ in.}) + 35.0\text{ in.}]$ $= 461\text{ kips} > 109\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y t_w (2.5k + L_g)}{1.50}$ $= \dfrac{(50\text{ ksi})(0.250\text{ in.})[2.5(0.747\text{ in.}) + 35.0\text{ in.}]}{1.50}$ $= 307\text{ kips} > 72.8\text{ kips}$ **o.k.**

Check beam web local crippling

The normal force is applied at a distance $\alpha = 15.75$ in. from the beam end, which is greater than $d/2$, where d is the depth of the W16×26 beam. Therefore, determine the beam web local crippling strength from AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1 + 3\left(\frac{L_g}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}$ $= 0.75(0.80)(0.250\text{ in.})^2\left[1 + 3\left(\frac{35.0\text{ in.}}{15.7\text{ in.}}\right)\left(\frac{0.250\text{ in.}}{0.345\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\frac{(29{,}000\text{ ksi})(50\text{ ksi})(0.345\text{ in.})}{0.250\text{ in.}}}$ $= 272\text{ kips} > 109\text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = \frac{0.80 t_w^2 \left[1 + 3\left(\frac{L_g}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}}{2.00}$ $= \frac{1}{2.00}0.80(0.250\text{ in.})^2\left[1 + 3\left(\frac{35.0\text{ in.}}{15.7\text{ in.}}\right)\left(\frac{0.250\text{ in.}}{0.345\text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\frac{(29{,}000\text{ ksi})(50\text{ ksi})(0.345\text{ in.})}{0.250\text{ in.}}}$ $= 181\text{ kips} > 72.8\text{ kips}$ **o.k.**

Beam-to-Column Connection

Determine resultant force

LRFD	ASD
$R_{ubc} = \sqrt{V_{ubc}^2 + H_{ubc}^2}$ $= \sqrt{(80.9\text{ kips})^2 + (65.7\text{ kips})^2}$ $= 104\text{ kips}$	$R_{abc} = \sqrt{V_{abc}^2 + H_{abc}^2}$ $= \sqrt{(53.9\text{ kips})^2 + (43.8\text{ kips})^2}$ $= 69.5\text{ kips}$

Design bolts at beam-to-column connection

From AISC *Manual* Table 7-1, the available shear strength for a ¾-in.-diameter ASTM A325-N bolt is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips/bolt}$	$\frac{r_n}{\Omega} = 11.9\text{ kips/bolt}$

Check bolt bearing on beam web

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad (Spec.\ \text{Eq. J3-6a})$$

Conservatively assume a 1½-in. distance to the edge of the beam:

$$\begin{aligned} l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5 d_h \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5(\tfrac{13}{16}\text{ in.}) \\ &= 1.09\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t_w F_u = 0.75(1.2)(1.09\text{ in })(0.250\text{ in.})(65\text{ ksi})$ $= 15.9\text{ kips}$ $\phi 2.4 d_b t_w F_u = 0.75(2.4)(\tfrac{3}{4}\text{ in.})(0.250\text{ in.})(65\text{ ksi})$ $= 21.9\text{ kips}$ Therefore, $\phi r_n = 15.9$ kips. $\phi r_n = 17.9\text{ kips/bolt} > \phi r_n = 15.9\text{ kips/bolt}$ Therefore, the limit state of tearout controls.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.09\text{ in.})(0.250\text{ in.})(65\text{ ksi})}{2.00}$ $= 10.6\text{ kips}$ $\dfrac{2.4 d_b t_w F_u}{\Omega} = \dfrac{2.4(\tfrac{3}{4}\text{ in.})(0.250\text{ in.})(65\text{ ksi})}{2.00}$ $= 14.6\text{ kips}$ Therefore, $r_n/\Omega = 10.6$ kips. $\dfrac{r_n}{\Omega} = 11.9\text{ kips/bolt} > \dfrac{r_n}{\Omega} = 10.6\text{ kips/bolt}$ Therefore, the limit state of tearout controls.

Check bolt bearing on single plate

Assume a ½-in.-thick tab plate:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(}\textit{Spec}\text{. Eq. J3-6a)}$$

$$\begin{aligned} l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5 d_h \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5(\tfrac{13}{16}\text{ in.}) \\ &= 1.09\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.09\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 28.4\text{ kips}$ $\phi 2.4 d_b t_w F_u = 0.75(2.4)(\tfrac{3}{4}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})$ $= 39.2\text{ kips}$ Therefore, $\phi r_n = 28.4$ kips. $\phi r_n = 17.9\text{ kips/bolt} < \phi r_n = 28.4\text{ kips/bolt}$ Therefore, the limit state of bolt shear controls.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.09\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00}$ $= 19.0\text{ kips}$ $\dfrac{2.4 d_b t_w F_u}{\Omega} = \dfrac{2.4(\tfrac{3}{4}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00}$ $= 26.1\text{ kips}$ Therefore, $r_n/\Omega = 19.0$ kips. $\dfrac{r_n}{\Omega} = 11.9\text{ kips/bolt} < \dfrac{r_n}{\Omega} = 19.0\text{ kips/bolt}$ Therefore, the limit state of bolt shear controls.

Because tearout of the beam web controls, the available strength of a bolt is 15.9 kips (LRFD) or 10.6 kips (ASD). Note that since the tearout controlled using the conservative edge distance assumption, a more rigorous analysis could be performed. However, in this case there is no advantage to performing the more rigorous analysis, so the more conservative tearout value will be used. Since e_c was assumed to be located 10¼ in. from the support, and the bolt group is 11¾ in. from the face of the support, a 1½-in. eccentricity must be considered.

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{H_{ubc}}{V_{ubc}}\right)$ $= \tan^{-1}\left(\dfrac{65.7 \text{ kips}}{80.9 \text{ kips}}\right)$ $= 39.1°$	$\theta = \tan^{-1}\left(\dfrac{H_{abc}}{V_{abc}}\right)$ $= \tan^{-1}\left(\dfrac{43.8 \text{ kips}}{53.9 \text{ kips}}\right)$ $= 39.1°$

From AISC *Manual* Table 7-8, with $\theta = 30°$, $e_x = 1.50$ in., $n = 4$, the coefficient $C = 6.90$. Therefore, the available strength of the bolt group is:

LRFD	ASD
$\phi R_n = (15.9 \text{ kips/bolt})(6.90)$ $= 110 \text{ kips} > 104 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (10.6 \text{ kips/bolt})(6.90)$ $= 73.1 \text{ kips} > 69.5 \text{ kips}$ **o.k.**

Design beam single plate-to-column weld

The strength determination of fillet welds defined in AISC *Specification* Section J2.4 can be simplified as explained in AISC *Manual* Part 8 to AISC *Manual* Equations 8-2a and 8-2b, and the required weld size is determined as follows:

Note that the column only sees the vertical shear load. Conservatively, a length equal to the length of the bolted portion of the single plate is used.

LRFD	ASD
$D_{req} = \dfrac{V_{ubc}}{2(1.392 \text{ kip/in.})l}$ $= \dfrac{80.9 \text{ kips}}{2(1.392 \text{ kip/in.})(12 \text{ in.})}$ $= 2.42$ sixteenths	$D_{req} = \dfrac{V_{abc}}{2(0.928 \text{ kip/in.})l}$ $= \dfrac{53.9 \text{ kips}}{2(0.928 \text{ kip/in.})(12 \text{ in.})}$ $= 2.42$ sixteenths

Use a 3⁄16-in. fillet weld. This also satisfies the minimum required weld size from AISC *Specification* Table J2.4.

Check shear strength of the single plate

To determine the available shear strength of the single plate, use AISC *Specification* Section J4.2.

For shear yielding of the single plate, use AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(12.0 \text{ in.})(½ \text{ in.})$ $= 130 \text{ kips} > 80.9 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{1.50}$ $= \dfrac{0.60(36 \text{ ksi})(12.0 \text{ in.})(½ \text{ in.})}{1.50}$ $= 86.4 \text{ kips} > 53.9 \text{ kips}$ **o.k.**

For shear rupture of the single plate, use AISC *Specification* Equation J4-4:

$$A_{nv} = t_p\left[L - n\left(d_h + 1/16\right)\right]$$
$$= \left(1/2 \text{ in.}\right)\left[12.0 \text{ in.} - 4\left(0.875 \text{ in.}\right)\right]$$
$$= 4.25 \text{ in.}^2$$

LRFD	ASD
$\phi R_n = \phi 0.6 F_u A_{nv}$ $= 0.75(0.6)(58 \text{ ksi})\left(4.25 \text{ in.}^2\right)$ $= 111 \text{ kips} > 80.9 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.6 F_u A_{nv}}{2.00}$ $= \dfrac{0.6(58 \text{ ksi})\left(4.25 \text{ in.}^2\right)}{2.00}$ $= 74.0 \text{ kips} > 53.9 \text{ kips}$ **o.k.**

Check tensile strength of the single plate

To determine the available tensile strength of the single plate, use AISC *Specification* Section J4.1.

For tensile yielding of the single plate, use AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(12.0 \text{ in.})(1/2 \text{ in.})$ $= 194 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{(36 \text{ ksi})(12.0 \text{ in.})(1/2 \text{ in.})}{1.67}$ $= 129 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

For tensile rupture of the single plate, use AISC *Specification* Equation J4-2:

$$A_e = t_p\left[L - n\left(d_h + 1/16\right)\right]$$
$$= \left(1/2 \text{ in.}\right)\left[12.0 \text{ in.} - 4\left(0.875 \text{ in.}\right)\right]$$
$$= 4.25 \text{ in.}^2$$

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75(58 \text{ ksi})\left(4.25 \text{ in.}^2\right)$ $= 185 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{2.00}$ $= \dfrac{(58 \text{ ksi})\left(4.25 \text{ in.}^2\right)}{2.00}$ $= 123 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

Check compression buckling of beam-to-column single plate

Check the compression buckling strength of the plate due to the horizontal load. Conservatively assume $K = 1.2$:

$$
\begin{aligned}
r &= \frac{t_p}{\sqrt{12}} \\
&= \frac{\text{½ in.}}{\sqrt{12}} \\
&= 0.144 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
\frac{KL}{r} &= \frac{1.2(10.25 \text{ in.})}{0.144 \text{ in.}} \\
&= 85.4
\end{aligned}
$$

As specified in AISC *Specification* Section J4.4, the provisions of Chapter E apply, because $KL/r > 25$.

From AISC *Manual* Table 4-22, the available critical stress is:

LRFD	ASD
$\phi F_{cr} = 22.1 \text{ ksi}$ The available compressive strength can then be determined by: $\phi R_n = \phi F_{cr} t_p L$ $= (22.1 \text{ ksi})(\text{½ in.})(12.0 \text{ in.})$ $= 133 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = 14.7 \text{ ksi}$ The available compressive strength can then be determined by: $\dfrac{R_n}{\Omega} = \dfrac{F_{cr} t_p L}{\Omega}$ $= (14.7 \text{ ksi})(\text{½ in.})(12.0 \text{ in.})$ $= 88.2 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

By inspection, block shear rupture will not control and therefore will not be checked.

Gusset-to-Column Connection

Determine resultant force

LRFD	ASD
$R_{uc} = \sqrt{V_{uc}^2 + H_{uc}^2}$ $= \sqrt{(64.1 \text{ kips})^2 + (65.7 \text{ kips})^2}$ $= 91.8 \text{ kips}$	$R_{ac} = \sqrt{V_{ac}^2 + H_{ac}^2}$ $= \sqrt{(42.7 \text{ kips})^2 + (43.8 \text{ kips})^2}$ $= 61.2 \text{ kips}$

Design bolts at gusset-to-column connection

As previously determined, the shear per bolt for a ¾-in.-diameter A325-N bolt:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 11.9 \text{ kips/bolt}$

Check bolt bearing on gusset plate and single plate

According to the User Note in AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength is the lesser of the fastener shear strength and the bearing strength. Assuming that deformation at the bolt hole at service load is a design consideration, use AISC *Specification* Equation J3-6a for the nominal bearing strength:

$$R_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad (Spec.\ \text{Eq. J3-6a})$$

Conservatively, assume a 1½-in. edge distance:

$$\begin{aligned} l_c &= 1\tfrac{1}{2}\text{ in.} - 0.5 d_h \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5\left(\tfrac{13}{16}\text{ in.}\right) \\ &= 1.09\text{ in.} \end{aligned}$$

LRFD	ASD
$\phi 1.2 l_c t F_u = 0.75(1.2)(1.09\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) = 28.5\text{ kips}$ $\phi 2.4 d_b t_w F_u = 0.75(2.4)(\tfrac{3}{4}\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) = 39.2\text{ kips}$ Therefore, $\phi r_n = 28.5$ kips/bolt. $\phi r_n = 17.9\text{ kips/bolt} < \phi r_n = 28.5\text{ kips/bolt}$ Therefore, the limit state of bolt shear controls.	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(1.09\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi})}{2.00} = 19.0\text{ kips}$ $\dfrac{2.4 F_u d_b t_w}{\Omega} = \dfrac{2.4(58\text{ ksi})(\tfrac{3}{4}\text{ in.})(\tfrac{1}{2}\text{ in.})}{2.00} = 26.1\text{ kips}$ Therefore, $\dfrac{r_n}{\Omega} = 19.0$ kips/bolt. $\dfrac{r_n}{\Omega} = 11.9\text{ kips/bolt} < \dfrac{r_n}{\Omega} = 19.0\text{ kips/bolt}$ Therefore, the limit state of bolt shear controls.

The available strength of one bolt is 17.9 kips (LRFD) or 11.9 kips (ASD). Note that if tearout controlled using the conservative edge distance assumption, a more rigorous analysis could be performed.

The total available bolt strength on the gusset plate and single plate is:

LRFD	ASD
$\phi R_b = (17.9\text{ kips/bolt})(6\text{ bolts}) = 107\text{ kips} > 91.8\text{ kips}$ **o.k.**	$\dfrac{R_b}{\Omega} = (11.9\text{ kips/bolt})(6\text{ bolts}) = 71.4\text{ kips} > 61.2\text{ kips}$ **o.k.**

Design the gusset plate-to-column weld

The strength determination of fillet welds defined in AISC *Specification* Section J2.4 can be simplified as explained in AISC *Manual* Part 8 to AISC *Manual* Equations 8-2a and 8-2b, and the required weld size is determined as follows. (Note that the column only sees the vertical shear load. Conservatively, a length equal to the length of the bolted portion of the single plate is used.)

LRFD	ASD
$D_{req} = \dfrac{V_{uc}}{2(1.392 \text{ kip/in.})L}$ $= \dfrac{64.1 \text{ kips}}{2(1.392 \text{ kip/in.})(18.0 \text{ in.})}$ $= 1.28 \text{ sixteenths}$	$D_{req} = \dfrac{V_{ac}}{2(0.928 \text{ kip/in.})L}$ $= \dfrac{42.7 \text{ kips}}{2(0.928 \text{ kip/in.})(18.0 \text{ in.})}$ $= 1.28 \text{ sixteenths}$

Therefore, use a 3⁄16-in. fillet weld. This also satisfies the minimum weld size required by AISC *Specification* Table J2.4.

Check shear strength of the single plate

To determine the available shear strength of the single plate, use AISC *Specification* Section J4.2.

For shear yielding of the single plate, use AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(18.0 \text{ in.})(\tfrac{1}{2} \text{ in.})$ $= 194 \text{ kips} > 64.1 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{1.50}$ $= \dfrac{0.60(36 \text{ ksi})(18.0 \text{ in.})(\tfrac{1}{2} \text{ in.})}{1.50}$ $= 130 \text{ kips} > 42.7 \text{ kips}$ **o.k.**

For shear rupture of the single plate, use AISC *Specification* Equation J4-4:

$$
\begin{aligned}
A_{nv} &= t_p\left[L - n\left(d_h + \tfrac{1}{16}\right)\right] \\
&= \left(\tfrac{1}{2} \text{ in.}\right)\left[18.0 \text{ in.} - 6(0.875 \text{ in.})\right] \\
&= 6.38 \text{ in.}^2
\end{aligned}
$$

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(6.38 \text{ in.}^2)$ $= 167 \text{ kips} > 64.1 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{2.00}$ $= \dfrac{0.60(58 \text{ ksi})(6.38 \text{ in.}^2)}{2.00}$ $= 111 \text{ kips} > 42.7 \text{ kips}$ **o.k.**

Check tensile strength of the single plate

To determine the available tensile strength of the single plate, use AISC *Specification* Section J4.1.

For tensile yielding of the single plate, use AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(18.0 \text{ in.})(\tfrac{1}{2} \text{ in.})$ $= 292 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{(36 \text{ ksi})(18.0 \text{ in.})(\tfrac{1}{2} \text{ in.})}{1.67}$ $= 194 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

For tensile rupture of the single plate, use AISC *Specification* Equation J4-2:

$$\begin{aligned} A_e &= t_p\left[L - n\left(d_h + 1/16\right)\right] \\ &= \left(1/2 \text{ in.}\right)\left[18.0 \text{ in.} - 6\left(0.875 \text{ in.}\right)\right] \\ &= 6.38 \text{ in.}^2 \end{aligned}$$

LRFD	ASD
$\phi R_n = \phi F_u A_e$ $= 0.75\left(58 \text{ ksi}\right)\left(6.38 \text{ in.}^2\right)$ $= 278 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{2.00}$ $= \dfrac{\left(58 \text{ ksi}\right)\left(6.38 \text{ in.}^2\right)}{2.00}$ $= 185 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

By inspection, block shear rupture will not control and therefore will not be checked.

Design of Stiffener Connections

Check single plates to strong-axis moment connection stiffeners

Since the eccentricity is resisted by equal and opposite horizontal forces at the beam-to-column and gusset-to-column connections, the horizontal force must be transferred between the connections through the single plate. In this case the single plate is interrupted by column stiffeners required for the moment connection to the strong-axis of the column. Therefore, the welds of the single plates to the stiffeners must be capable of transmitting this force.

Check the single plates for shear yielding

Determine the available shear strength of the single plates using AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi V_n = \phi 0.60 F_y A_{gv}$ $= 1.00\left(0.60\right)\left(36 \text{ ksi}\right)\left(7\tfrac{3}{8} \text{ in.}\right)\left(1/2 \text{ in.}\right)$ $= 79.7 \text{ kips} > 65.7 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{1.50}$ $= \dfrac{0.60\left(36 \text{ ksi}\right)\left(7\tfrac{3}{8} \text{ in.}\right)\left(1/2 \text{ in.}\right)}{1.50}$ $= 53.1 \text{ kips} > 43.8 \text{ kips}$ **o.k.**

Check the single plates for tensile yielding

Determine the available tensile yielding strength of the single plates from AISC *Specification* Section J4.1, Equation J4-1, as follows (note that M_{ub} and M_{ab} are the moments required to keep the plate from rotating when all the forces shown in Fig. A-5 are in place):

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90\left(36 \text{ ksi}\right)\left(7\tfrac{3}{8} \text{ in.}\right)\left(1/2 \text{ in.}\right)$ $= 119 \text{ kips}$	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{\left(36 \text{ ksi}\right)\left(7\tfrac{3}{8} \text{ in.}\right)\left(1/2 \text{ in.}\right)}{1.67}$ $= 53.1 \text{ kips}$

LRFD	ASD
$R_u = \dfrac{4M_{ub}}{L}$ $= \dfrac{4(40.8 \text{ kip-in.})}{(7\frac{3}{8} \text{ in.})}$ $= 22.1 \text{ kips} < 119 \text{ kips}$ **o.k.**	$R_a = \dfrac{4M_{ab}}{L}$ $= \dfrac{4(27.2 \text{ kip-in.})}{(7\frac{3}{8} \text{ in.})}$ $= 14.8 \text{ kips} < 79.5 \text{ kips}$ **o.k.**

Design weld of single plates to moment connection stiffeners

LRFD	ASD
Resultant load: $R = \sqrt{(24.6 \text{ kips})^2 + (65.7 \text{ kips})^2}$ $= 70.2 \text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{24.6 \text{ kips}}{65.7 \text{ kips}}\right)$ $= 20.5°$ $D = \dfrac{R}{2l(1.392 \text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{70.2 \text{ kips}}{2(7\frac{3}{8} \text{ in.})(1.392 \text{ kip/in.})(1.0 + 0.5\sin^{1.5} 20.5°)}$ $= 3.10$ sixteenths	Resultant load: $R = \sqrt{(16.4 \text{ kips})^2 + (43.8 \text{ kips})^2}$ $= 46.8 \text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{16.4 \text{ kips}}{43.8 \text{ kips}}\right)$ $= 20.5°$ $D = \dfrac{R}{2l(0.928 \text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{46.8 \text{ kips}}{2(7\frac{3}{8} \text{ in.})(0.928 \text{ kip/in.})(1.0 + 0.5\sin^{1.5} 20.5°)}$ $= 3.10$ sixteenths

Therefore, use a ¼-in. fillet weld of the single plates to the ¾-in. strong-axis moment connection stiffener, and use this same weld for the single plates-to-column connection for simplicity.

If a portion of the vertical beam reaction had not been redistributed to the gusset connection, the moment would have been 160 kip-in. (LRFD) or 106 kip-in. (ASD). The effect is demonstrated in the following.

Check tensile yielding on single plates

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(7\tfrac{3}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})$ $= 119\text{ kips}$ $R_u = \dfrac{4M_{ub}}{L_g}$ $= \dfrac{4(160\text{ kip-in.})}{7\tfrac{3}{8}\text{ in.}}$ $= 86.8\text{ kips} < 119\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{(36\text{ ksi})(7\tfrac{3}{8}\text{ in.})(\tfrac{1}{2}\text{ in.})}{1.67}$ $= 79.5\text{ kips}$ $R_a = \dfrac{4M_{ab}}{L_g}$ $= \dfrac{4(106\text{ kip-in.})}{7\tfrac{3}{8}\text{ in.}}$ $= 57.5\text{ kips} < 79.5\text{ kips}$ **o.k.**

Design weld of single plates to moment connection stiffeners

LRFD	ASD
Resultant load: $R_u = \sqrt{(96.6\text{ kips})^2 + (65.7\text{ kips})^2}$ $= 117\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{96.6\text{ kips}}{65.7\text{ kips}}\right)$ $= 55.8°$ $D = \dfrac{R_u}{2l(1.392\text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{117\text{ kips}}{2(7\tfrac{3}{8}\text{ in.})(1.392\text{ kip/in.})(1.0 + 0.5\sin^{1.5} 55.8°)}$ $= 4.14$ sixteenths	Resultant load: $R_a = \sqrt{(64.0\text{ kips})^2 + (43.8\text{ kips})^2}$ $= 77.6\text{ kips}$ Resultant load angle: $\theta = \tan^{-1}\left(\dfrac{R_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{64.0\text{ kips}}{43.8\text{ kips}}\right)$ $= 55.6°$ $D = \dfrac{R_a}{2l(0.928\text{ kip/in.})(1.0 + 0.5\sin^{1.5}\theta)}$ $= \dfrac{77.6\text{ kips}}{2(7\tfrac{3}{8}\text{ in.})(0.928\text{ kip/in.})(1.0 + 0.5\sin^{1.5} 55.6°)}$ $= 4.12$ sixteenths

In this case the ½-in.-thick plate and single pass weld are still okay even without the vertical beam end reaction distributed to minimize the moment at the critical section.

APPENDIX B
Use of the Directional Strength Increase for Fillet Welds

In this Design Guide the authors include the increased strength of the fillet welds due to loading transverse or oblique to the longitudinal axis of the weld. In many cases, the stresses the welds must resist are based on the resultant forces from shears, axial loads and moments. The authors base the calculations on the largest resultant normal stress in such cases, though obviously the moment will add to the axial stress on one side and subtract on the other. Below is an explanation of the consequences of this procedure. It is not always conservative, but the maximum overstress in the weld resulting from the assumption is limited to about 3% and is therefore neglected.

When a normal force and a moment act on a gusset edge, as shown in Figure B-1, it is convenient for analysis to convert the moment into a statically equivalent normal force and then to add this to the original normal force to arrive at an equivalent normal force:

$$N_e = T + \frac{4M}{L} \tag{B-1}$$

The steps necessary to produce N_e are shown in Figure B-1. If there is a shear V acting on the gusset edge, then the resultant force on the edge is:

$$R = \sqrt{N_e^{\,2} + V^2} \tag{B-2}$$

And the resultant force per unit length of the edge is:

$$r = \frac{R}{L} = \sqrt{\left(\frac{N_e}{L}\right)^2 + \left(\frac{V}{L}\right)^2} \tag{B-3}$$

Let D_{plus} equal the required weld where the axial force and moment add and D_{minus} equals the required weld where the axial force and moment subtract:

$$D_{plus} = \frac{\sqrt{\left(\dfrac{T}{L}+\dfrac{4M}{L^2}\right)^2+\left(\dfrac{V}{L}\right)^2}}{2(1.392)\left\{1+0.5\left[\dfrac{\left(\dfrac{T}{L}+\dfrac{4M}{L^2}\right)}{\sqrt{\left(\dfrac{T}{L}+\dfrac{4M}{L^2}\right)^2+\left(\dfrac{V}{L}\right)^2}}\right]^{1.5}\right\}} \tag{B-4a}$$

$$D_{minus} = \frac{\sqrt{\left(\dfrac{T}{L}-\dfrac{4M}{L^2}\right)^2+\left(\dfrac{V}{L}\right)^2}}{2(1.392)\left\{1+0.5\left[\dfrac{\left(\dfrac{T}{L}-\dfrac{4M}{L^2}\right)}{\sqrt{\left(\dfrac{T}{L}-\dfrac{4M}{L^2}\right)^2+\left(\dfrac{V}{L}\right)^2}}\right]^{1.5}\right\}} \tag{B-5a}$$

Let

$$x = \frac{T}{L} + \frac{4M}{L^2} \tag{B-6}$$

Introducing the parameters Z and Y, let

$$xZ = \frac{T}{L} - \frac{4M}{L^2}, \text{ where } 0 < Z < 1.0$$

and

$$xY = \frac{V}{L}, \text{ where } Y \text{ is any positive number}$$

Thus, D_{plus} and D_{minus} can be rewritten as:

$$D_{plus} = \frac{x}{2(1.392)}\left[\frac{\sqrt{1+Y^2}}{1+0.5\left(\frac{1}{\sqrt{1+Y^2}}\right)^{1.5}}\right] \quad \text{(B-4b)}$$

$$D_{minus} = \frac{x}{2(1.392)}\left[\frac{\sqrt{Z^2+Y^2}}{1+0.5\left(\frac{Z}{\sqrt{Z^2+Y^2}}\right)^{1.5}}\right] \quad \text{(B-5b)}$$

It can be shown that as Z approaches 1.0, D_{minus} approaches the value D_{plus}. Therefore D_{minus} is most different from D_{plus} when Z equals zero.

With $Z = 0$, D_{minus} can be rewritten as:

$$D_{minus} = \frac{x(Y)}{2(1.392)} \quad \text{(B-7)}$$

The ratio of D_{minus} to D_{plus} can be written as:

$$\frac{D_{minus}}{D_{plus}} = \frac{Y}{\sqrt{1+Y^2}}\left[1+0.5\left(\frac{1}{\sqrt{1+Y^2}}\right)^{1.5}\right] \quad \text{(B-8)}$$

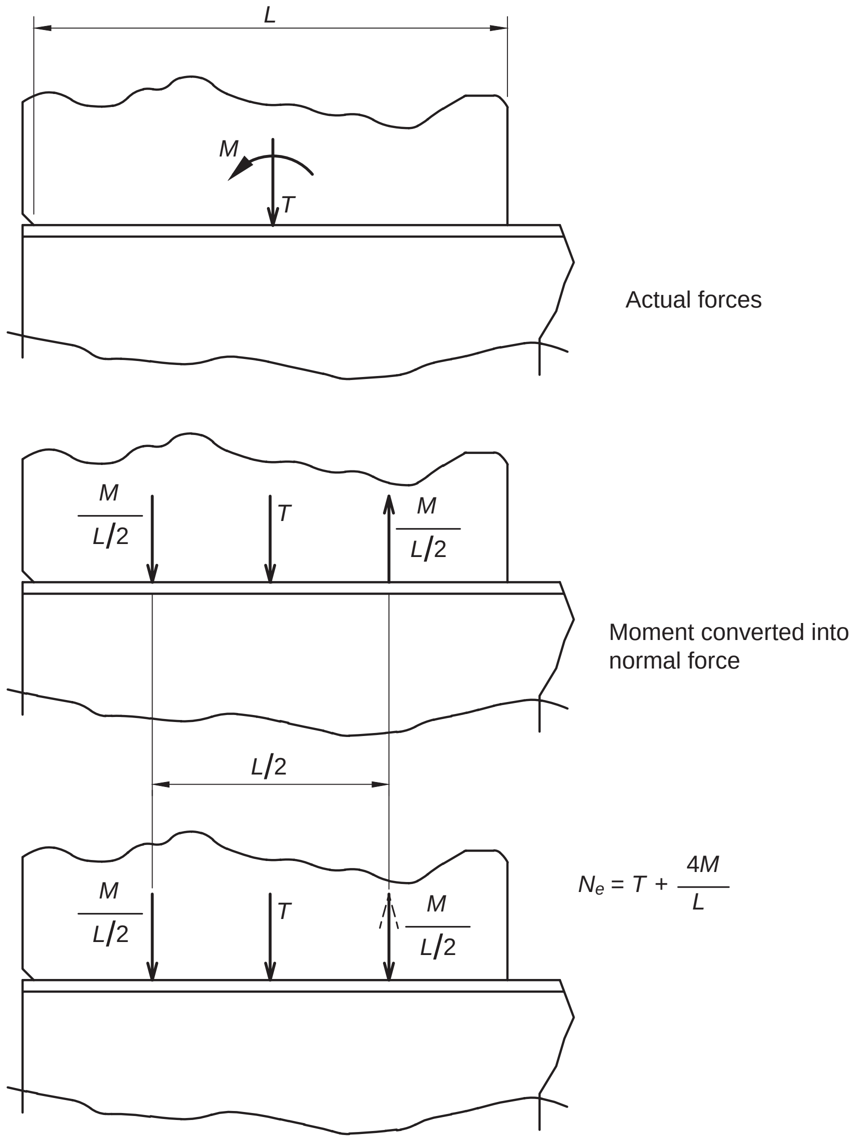


Fig. B-1. Development of equivalent axial force.

Taking the derivative of this ratio with respect to Y and setting it equal to 0 to find the maximum, results in $Y = 2.65$. Therefore, the maximum ratio between D_{minus} and D_{plus} is:

$$\frac{D_{minus}}{D_{plus}} = \frac{2.65}{\sqrt{1+(2.65)^2}}\left[1+0.5\left(\frac{1}{\sqrt{1+(2.65)^2}}\right)^{1.5}\right]$$
$$= 1.03$$

The maximum possible variance between D_{minus} and D_{plus} is 3.37%. Because it can be shown that the weld will never be more than 3.37% overstressed, it is reasonable to base the directional strength increase on the addition of the moment and the axial stresses. It should be noted that though this analysis was performed using LRFD, the results would be identical using ASD—only the constant 1.392 would change to 0.928.

APPENDIX C
Buckling of Gusset Plates

C.1. BUCKLING AS A STRENGTH LIMIT STATE—THE LINE OF ACTION METHOD

The approach to gusset buckling used in the AISC *Manual* and in this Design Guide recognizes that the critical buckling limit state is along the line of action of the brace. If buckling occurs here, the axial strength of the connection will be compromised. The method is called the "line of action" method (LOAM) in this Design Guide.

The method, as shown in Figure C-1, was originally developed by Thornton (1984). It uses the Whitmore Section to estimate the actual stress and an equivalent column of length: $l_b = l_1$ or $l_b = \dfrac{l_1 + l_2 + l_3}{3}$ to estimate buckling stress.

As originally proposed by Thornton, the column effective length factor, K, was chosen as 0.65, but research by Gross (1990) indicated that $K = 0.50$ is conservative and this value was adopted by the AISC *Manual* (AISC 1992, 1994, 2001 and 2005). The method has been called the "Thornton" Method (Cheng and Grondin, 1999) or the "pseudo column" Method (Chambers and Ernst, 2005). Dowswell (2006) has shown that when compared to the results of physical tests by many researchers, the line of action method (LOAM) with revised K factors almost always gives a conservative estimate for the buckling strength of the gusset plate. The revised K factors suggested by Dowswell are used in this Design Guide and are reproduced in Table C-1 with other pertinent information, for convenience. It should be noted that prior to the publication of the LOAM in 1984, there was no published method available for line of action buckling of gusset plates. Many practitioners simply stiffened the gusset to preclude buckling or ignored buckling (Zetlin et al., 1975).

C.2 BUCKLING AS A HIGH CYCLE FATIGUE LIMIT STATE—GUSSET PLATE EDGE BUCKLING

In the design of highway bridges, gusset free edge buckling or flexing can cause fatigue cracks. For this reason, the *LRFD Bridge Design Specifications* (AASHTO, 2012) includes a criterion for stiffening of gusset plate free edges in Section 6.14.2.8 that defines the maximum free edge length of the gusset plate without required stiffening as:

$$a_{max} = 2.06\sqrt{\frac{E}{F_y}}\left(t\right) \qquad \text{(C-1)}$$

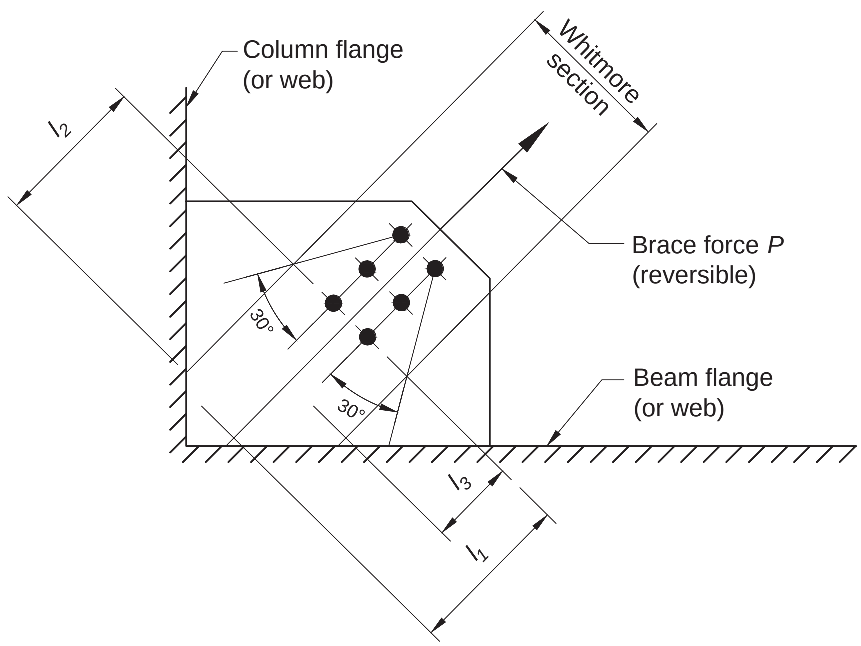


Fig. C-1. Line of action buckling method (LOAM).

Table C-1. Summary of Effective Length Factors		
Gusset Configuration [a]	**Effective Length Factor, K**	**Buckling Length, l** [b]
Compact Corner [c]	— [d]	— [d]
Noncompact Corner [c]	1.0	l_{avg}
Extended Corner	0.6	l_1
Single Brace	0.7	l_1
Chevron	0.65	l_1

[a] See Figure C-2 for gusset configurations.
[b] See Figure C-3 for buckling length.
[c] See Figure C-4 for determination of compactness.
[d] Yielding is the applicable limit state for compact corner gusset plates; therefore, the effective length factor and the buckling length are not applicable.

where

a = length of the gusset plate free edge, in.
t = thickness of gusset plate, in.
E = modulus of elasticity, psi
F_y = specified minimum yield stress, psi

Both of the variables, E and F_y, may be applied with any consistent set of units. This criterion has been in the bridge code since at least the 1944 Edition, and is also included in *Guide Specification of Seismic Design of Steel Bridges* (Caltrans, 2001).

The basis for Equation C-1 is elastic buckling of a plate with the boundary conditions shown in Figure C-5. This plate is imagined embedded in a gusset plate, for instance, as shown in Figure C-6. When the plate has an aspect ratio of a/b that is approximately equal to 3 and the plate elastic bucking stress is set equal to F_y, Equation C-1 results, as will now be demonstrated.

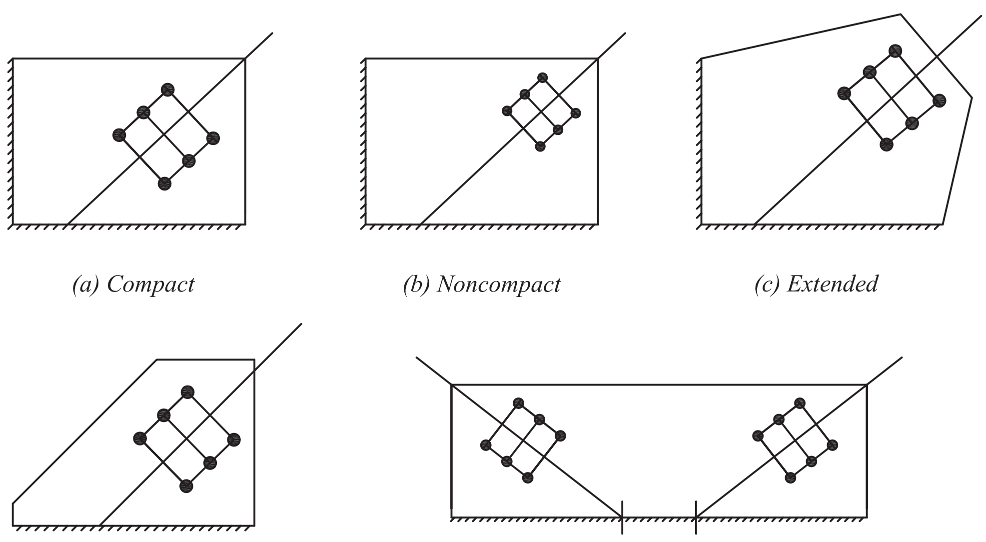
The elastic plate bucking equation can be written as:

$$\frac{a}{t} = \sqrt{\frac{k\pi^2 E\left(\dfrac{a}{b}\right)^2}{12\left(1-\upsilon^2\right)F_{cr}}} \tag{C-2}$$

where

k = plate buckling coefficient
≈ 0.5 for $a/b = 3$ (Salmon and Johnson, 1971)
υ = Poisson's ratio (= 0.3 for steel)
F_{cr} = plate buckling stress, psi

Substituting these values and $a/b = 3$ into Equation C-2 results in:



(a) Compact (b) Noncompact (c) Extended

(d) Single-brace (e) Chevron-brace

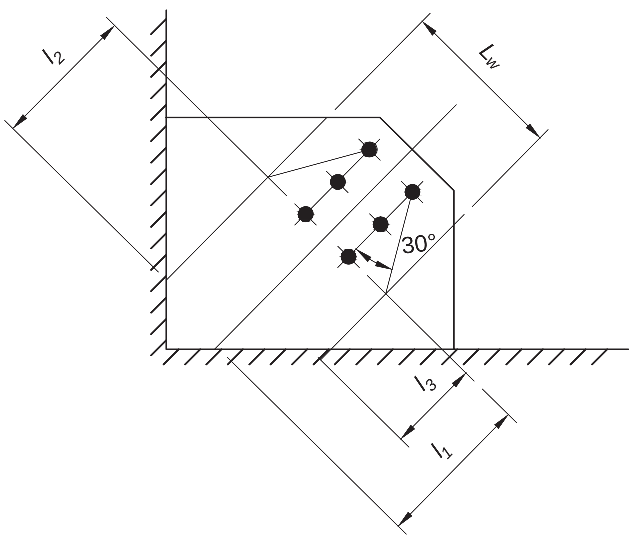
Fig. C-2. Gusset configurations (Dowswell, 2006).

$$\frac{a}{t} = \sqrt{\frac{0.5\pi^2 E (3.0)^2}{12(1-0.3^2)F_{cr}}}$$

$$= 2.02\sqrt{\frac{E}{F_{cr}}}$$

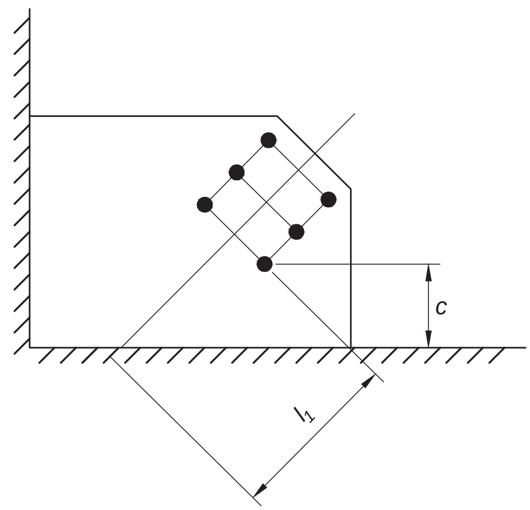
Setting $F_{cr} = F_y$ and recognizing that k and a/b are approximate (i.e., eliminating the precision implied by the factor 2.02), the expression is equivalent to that presented in Equation C-1.

The criterion adopted by AASHTO is generally very conservative. It assumes that the material yield stress will be achieved on the gusset plate free edge. In actual gussets, the maximum possible gusset edge stress based on an



$$l_{avg} = \frac{l_1 + l_2 + l_3}{3}$$

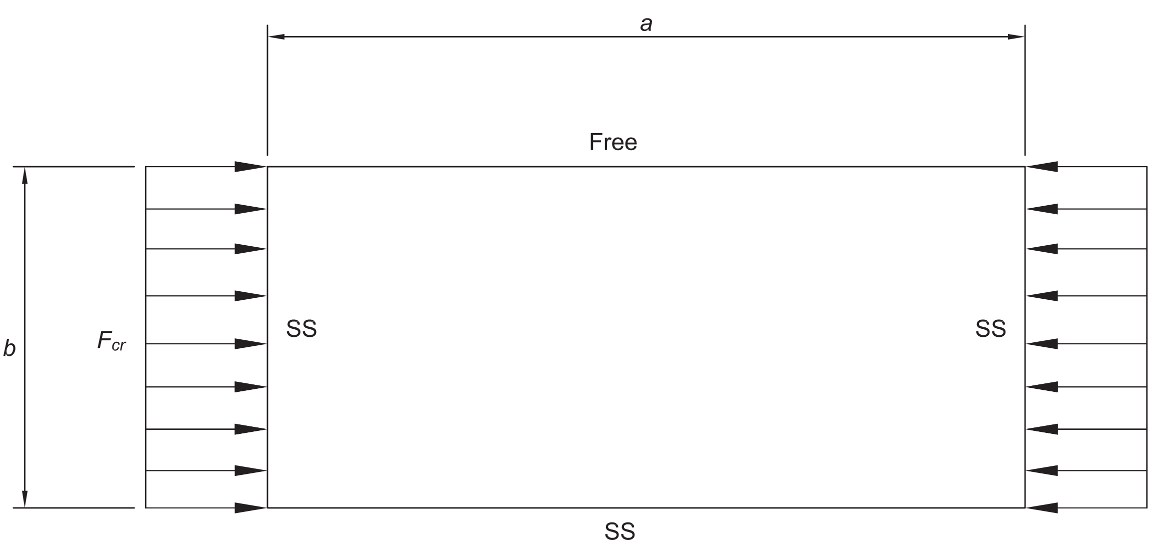
Fig. C-3. Buckling length (Dowswell, 2006).



If $t_g \geq t_\beta$, then compact
If $t_g < t_\beta$, then noncompact
where t_g = thickness of gusset

$$t_\beta = 1.5\sqrt{\frac{F_y c^3}{E l_1}}$$

Fig. C-4. Determination of compact/noncompact (Dowswell, 2006).



SS = Simple support

Fig. C-5. Basic gusset plate stability model.

admissible force system is $0.6F_y$ at working loads, for both ASD and LRFD. While Equation C-1 is generally conservative, as noted here, if the plate aspect ratio is less than 3, the equation can be unconservative. The authors believe that any reasonable approach to gusset plate edge buckling should be based on the statically admissible force system assumed for all of the other limit states that govern the design of a bracing connection. A subsequent section of this Appendix will present such a method.

C.3. BUCKLING AS A LOW-CYCLE FATIGUE LIMIT STATE—GUSSET PLATE FREE EDGE BUCKLING

In low-cycle fatigue, with excursions into the material inelastic region, it is perceived that gusset plate free edge flexing could cause premature fracture in the gusset and its connections. To prevent this flexing, Astaneh-Asl (1998), following the work of Brown (1988), suggests treating the gusset plate free edge as a column extending 1 in., or other linear unit, from the free edge and completely separated from the remainder of the gusset. This is equivalent to using the top 1 in. of the gusset plate shown in Figure C-5, and is explicitly shown in Figure C-7. The boundary conditions for this column, as shown in Figure C-7, are the fixed-sliding type, with a theoretical K of 1.0 and a recommended value of 1.2. Note that the top 1 in. of the gusset plate shown in Figure C-5, which is pinned-pinned, also has a theoretical K value of 1.0. Thus, the gusset model of Figure C-5 can be expected to yield similar results for the simple-support-to-simple-support case and the fixed-sliding case.

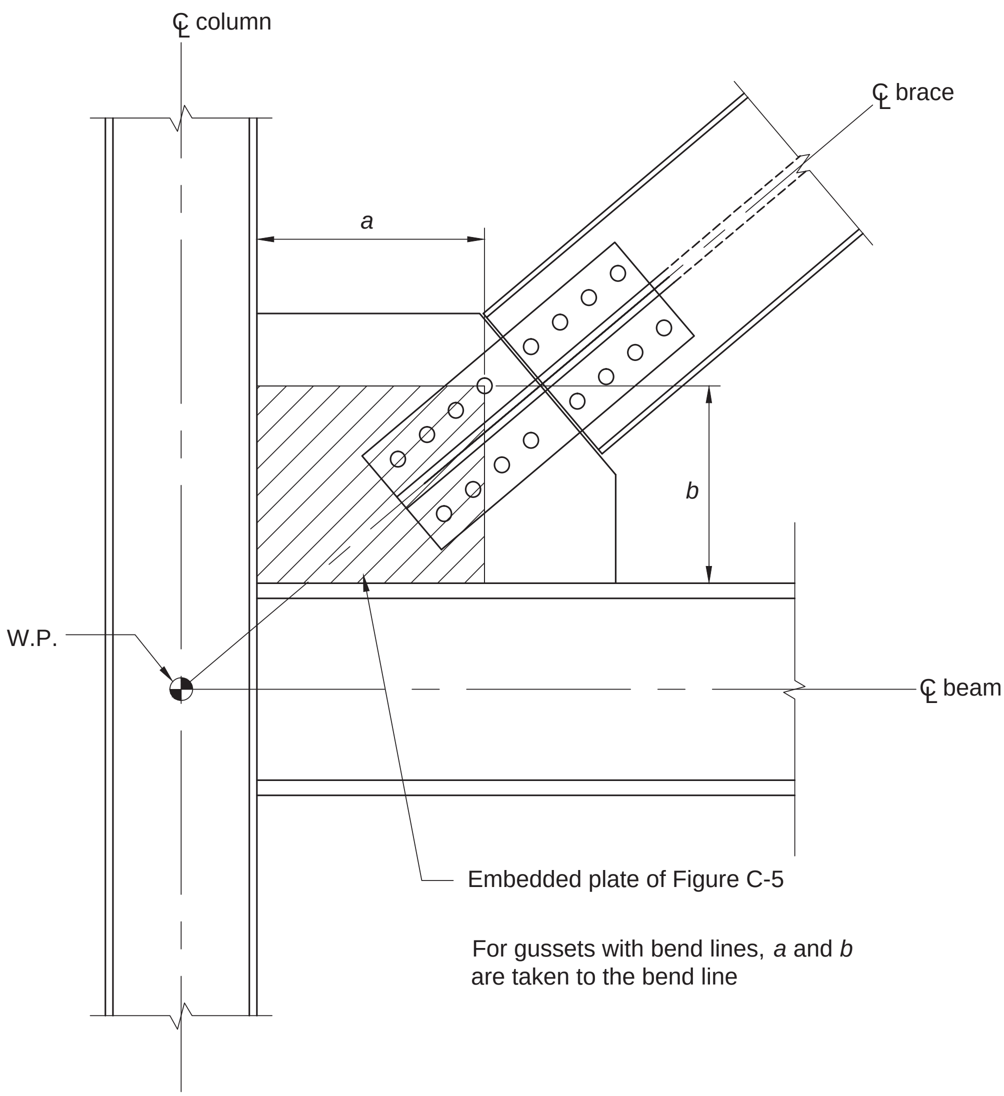


Fig. C-6a. Embedded plate for horizontal (free) edge gusset plate buckling.

The critical elastic buckling stress for the strip column of Figure C-7 is:

$$F_{cr} = \frac{\pi^2 E}{12\left(1-\upsilon^2\right)\left(\dfrac{Ka}{t}\right)^2} \tag{C-3}$$

where the term $(1–\upsilon^2)$ is included to model the increased stiffness of a plate over a simple "stand-alone" column, which will have an inelastic curvature. Solving Equation C-3 for a/t:

$$\frac{a}{t} = \sqrt{\frac{\pi^2 E}{12\left(1-\upsilon^2\right)K^2 F_{cr}}}$$

Setting $F_{cr} = F_y$ and $K = 1.2$:

$$\frac{a}{t} = 0.792\sqrt{\frac{E}{F_y}}$$

and rounding to eliminate the precision implied by the factor 0.792:

$$\left(\frac{a}{t}\right)_{max} = \frac{4}{5}\sqrt{\frac{E}{F_y}} \tag{C-4}$$

Astaneh-Asl (1998) further rounds Equation C-4 based on the results of one test to:

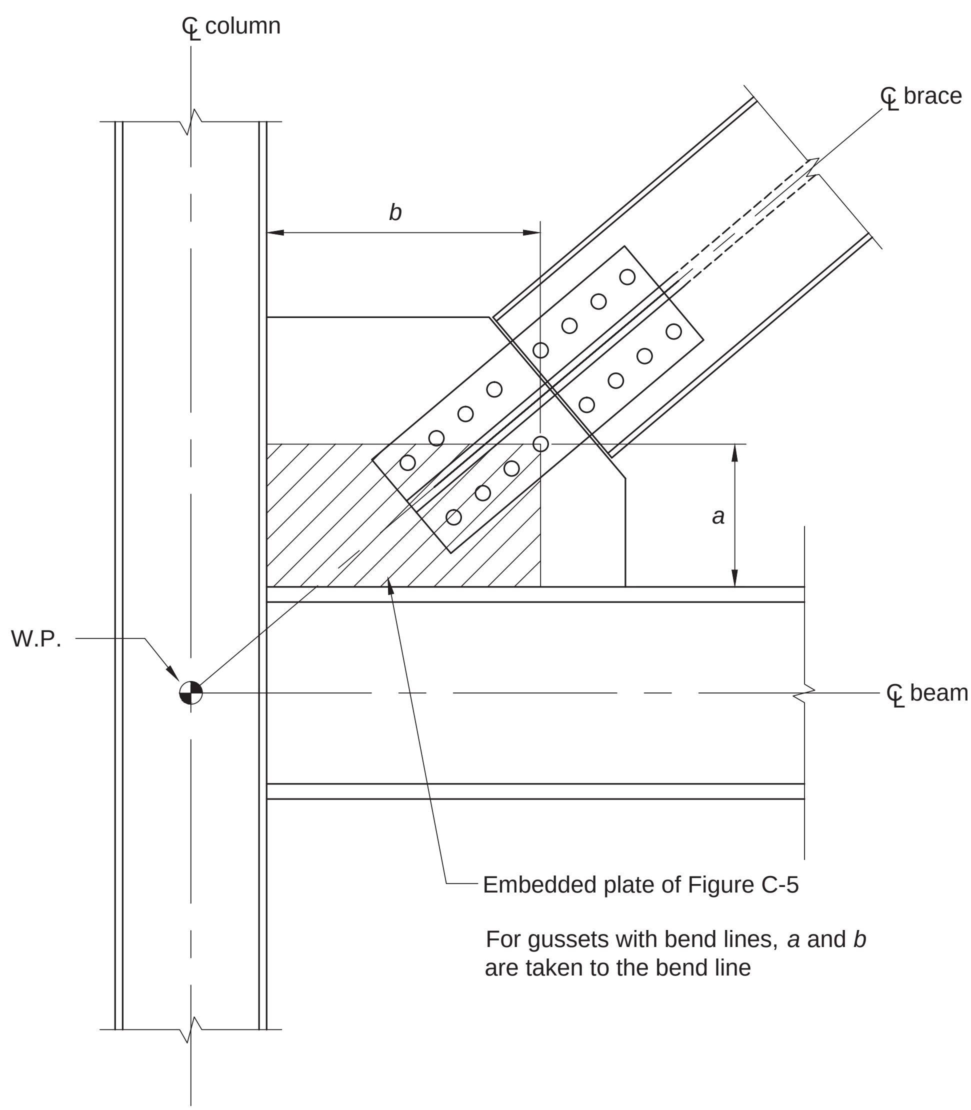


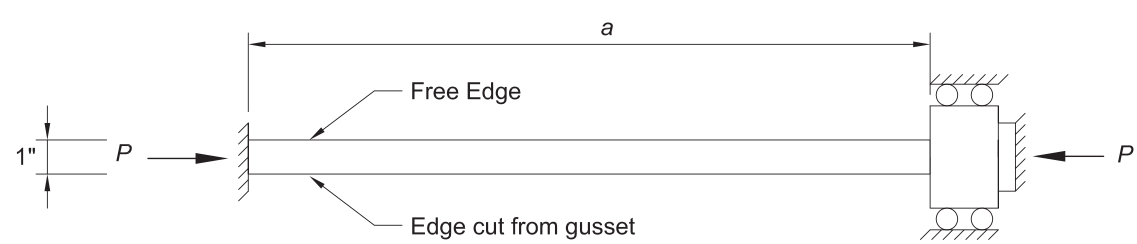
Fig. C-6b. Embedded plate for vertical (free) edge gusset plate buckling.

$$\left(\frac{a}{t}\right)_{max} = 0.75\sqrt{\frac{E}{F_y}} \quad \text{(C-5)}$$

It can be seen from the above derivation that Equation C-5 is extremely conservative. It is very likely that a gusset plate edge will not buckle even if Equation C-5 is not satisfied. Not withstanding this observation, Equation C-5 was in wide use in high-seismic regions and for that reason was adopted for use in the 1st Edition AISC *Seismic Design Manual* (AISC, 2006). It has been dropped from explicit use in the 2nd Edition AISC *Seismic Design Manual* (AISC, 2012). The reason for this is explained in the AISC *Seismic Provisions* Commentary Section F2.6c (AISC, 2010b), which states:

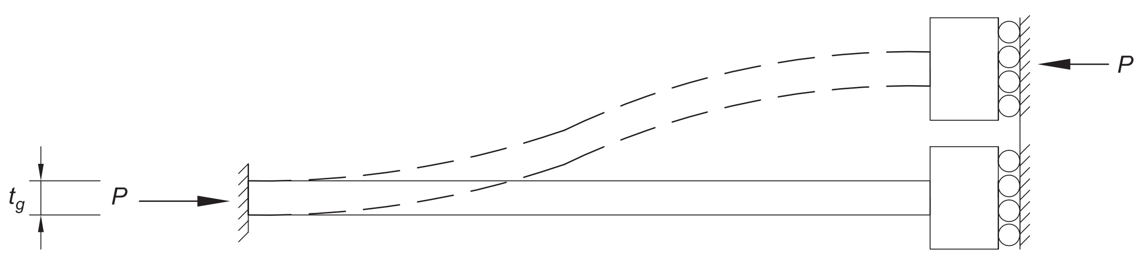
> Certain references suggest limiting the free edge length of gusset plates, including SCBF brace-to-beam connection design examples in the *Seismic Design Manual*, (AISC, 2006), and other references (Astaneh-Asl et al., 2006; ICC, 2006). However, the committee has reviewed the testing cited and has concluded that such edge stiffeners do not offer any advantages in gusset plate behavior. There is therefore no limitation on edge dimensions in these provisions.

It should be noted that the use of a gusset plate free edge buckling criterion does not eliminate the need for a line of action method (LOAM) check. The stiffening of gusset plate edges will not necessarily increase the ultimate compressive strength of a gusset plate (Rabinovitch, 1993). If gusset plate free edge stiffeners are determined by the designer to be necessary, it may be more cost effective to use a thicker gusset plate to eliminate the stiffeners. Finally, if gusset plate edge stiffeners are used, they should not be attached to the beam or column. If they are, they will completely change the force distribution assumed by the UFM (or any of the other methods discussed in this Design Guide) with an increased likelihood of premature local fractures. These local fractures could propagate and cause complete connection failure.



(a) In plane of gusset

(Top 1 in. of gusset from free edge)



(b) Normal to plane of gusset

(Basis for AISC *Seismic Design Manual* Free-Edge Buckling)

Fig. C-7. Fixed-sliding buckling model.

C.4. AN APPROACH TO GUSSET PLATE FREE EDGE BUCKLING USING STATICALLY ADMISSIBLE FORCES (THE ADMISSIBLE FORCE MAINTENANCE METHOD)

The Commentary to the *Seismic Provisions* (AISC, 2010b) indicates that edge stiffeners do not offer any advantages in gusset plate behavior. However, unusual conditions may be encountered where the engineer may wish to determine the free-edge buckling strength of a gusset. While free-edge buckling seldom, if ever, controls the gusset ultimate compressive strength as determined by LOAM, edge buckling could compromise the statically admissible force distribution determined for the system. For instance, the connection shown in Figures 6-5a and 6-5b shows a horizontal force of 214 kips (LRFD) and 142 kips (ASD) in tension, but the brace load is reversible, so the vertical gusset plate edge will see a compressive force of 214 kips (LRFD) and 142 kips (ASD), which gives rise to a compressive stress, f_a = (214 kips)/[(1.00 in.)(28.0 in.)] = 7.64 ksi for LRFD and f_a = (142 kips)/(1.00 in.)(28.0 in.) = 5.07 ksi for ASD. For this stress to be achieved, the horizontal (top) edge of the gusset plate must have a compressive strength exceeding 7.64 ksi (LRFD) and 5.07 ksi (ASD). Similarly, the horizontal edge at the beam sees an equivalent axial force of:

LRFD	ASD
$T_{equiv} = 218\text{ kips} + \dfrac{(1{,}740\text{ kip-in.})(4)}{35\frac{1}{4}\text{ in.}}$ $= 415\text{ kips}$	$T_{equiv} = 145\text{ kips} + \dfrac{(1{,}160\text{ kip-in.})(4)}{35\frac{1}{4}\text{ in.}}$ $= 277\text{ kips}$

The vertical (inclined) edge must not buckle at a load less than 415 kips (LRFD) and 277 kips (ASD) in order to sustain the postulated admissible internal force system.

The method to be used in this Design Guide to check gusset plate edges for buckling is based on the plate model of Figure C-5. The critical buckling stress for this model is given by Muir and Thornton (2004) as:

$$F_{cr} = QF_y \tag{C-6}$$

where

$Q = 1$ for $\lambda \le 0.7$ (yield controls)

$= 1.34 - 0.486\lambda$ for $0.7 \le \lambda \le 1.41$ (inelastic buckling controls)

$= \dfrac{1.3}{\lambda^2}$ for $\lambda > 1.41$ (elastic buckling controls)

and

$$\lambda = \frac{(b/t)\sqrt{F_y}}{5\sqrt{475 + \dfrac{1{,}120}{(a/b)^2}}} \tag{C-7}$$

The parameter λ is the effective slenderness ratio of the plate. The formulation of Equation C-6 reduces to AISC *Specification* Section E7.1(c) for single angles when a/b goes to ∞. Note that Equation C-6 is the same as that used for certain double-coped beams in the AISC *Manual*. It will be referred to here as the admissible force maintenance method (AFMM). Applying the AFMM theory to the design of the connection in Figure 6-1, for the vertical edge to carry 214 kips (LRFD) or 142 kips (ASD), as calculated previously f_a = 7.64 ksi (LRFD) or f_a = 5.07 ksi (ASD).

Approximate length of horizontal free edge to the bend line is a = 16.0 in.

Depth of the gusset at the column, b = 28.0 in.:

$$\frac{a}{b} = \frac{16.0\text{ in.}}{28.0\text{ in.}} = 0.571$$

$$\frac{b}{t} = \frac{28.0\text{ in.}}{1.00\text{ in.}} = 28.0$$

$$\lambda = \frac{28.0\sqrt{50\text{ ksi}}}{5\sqrt{475+\dfrac{1{,}120}{\left(0.571\right)^2}}} \tag{C-7}$$

$$= 0.633$$

$$Q = 1.00$$

LRFD	ASD
$\phi F_{cr} = 0.90\left(1.00\right)\left(50\text{ ksi}\right)$ $= 45.0\text{ ksi} > 7.64\text{ ksi}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = \dfrac{\left(1.00\right)\left(50\text{ ksi}\right)}{1.67}$ $= 29.9\text{ ksi} > 5.07\text{ ksi}$ **o.k.**

For the horizontal beam-to-gusset plate edge to carry 415 kips (LRFD) and 277 kips (ASD):

LRFD	ASD
$f_u = \dfrac{415\text{ kips}}{\left(3/4\text{ in.}\right)\left(35\tfrac{1}{4}\text{ in.}\right)}$ $= 15.7\text{ ksi}$	$f_a = \dfrac{277\text{ kips}}{\left(3/4\text{ in.}\right)\left(35\tfrac{1}{4}\text{ in.}\right)}$ $= 10.5\text{ ksi}$

$a = 12$ in. (distance from beam flange to lowest edge of HSS brace—this is the "c" dimension of Figure 6-2)

$b = 35\tfrac{1}{4}$ in.

$$\frac{a}{b} = \frac{12.0\text{ in.}}{35\tfrac{1}{4}\text{ in.}}$$

$$= 0.340$$

$$\frac{b}{t} = \frac{35\tfrac{1}{4}\text{ in.}}{1.00\text{ in.}}$$

$$= 35.25$$

$$\lambda = \frac{35.25\sqrt{50\text{ ksi}}}{5\sqrt{475+\dfrac{1{,}120}{\left(0.34\right)^2}}}$$

$$= 0.494$$

$$Q = 1.00$$

LRFD	ASD
$\phi F_{cr} = 0.90\left(1.00\right)\left(50\text{ ksi}\right)$ $= 45.0\text{ ksi} > 15.7\text{ ksi}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = \dfrac{\left(1.00\right)\left(50\text{ ksi}\right)}{1.67}$ $= 29.9\text{ ksi} > 10.5\text{ ksi}$ **o.k.**

Therefore, the gusset is sufficiently thick to prevent edge buckling under the given admissible force distribution and that distribution can thus be maintained.

The AFMM is presented here to provide a viable and reasonable method for considering gusset buckling. As the example here shows, it did not control the design. Usually the LOAM method for compression and the block shear tearout limit state for tension will control the gusset thickness. This is the case for every corner gusset example presented in this Design Guide. In fact, the authors, with their considerable experience in the design of these connections, have never found a corner connection case where the AFMM controlled the gusset thickness. For this reason, it has not been included in the design examples of corner connections presented in this Design Guide. However, for the case of gussets in chevron bracing connections, if both braces are in compression or even in tension, the long horizontal edge may require a stiffener by the AFMM. This is demonstrated in Example 5.9.

C.5. APPLICATION OF THE FREE EDGE APPROACH TO EXAMPLE 6.1a

This section will apply the free edge approach of Astaneh-Asl et al. (2006) and Equation C-6 to Example 6.1a.

The length of the top sloping free edge is 16 in. to the bend line if measured to the face of the column and thus:

$$\begin{aligned}\frac{a}{t} &= \frac{16\text{ in.}}{\tfrac{3}{4}\text{ in.}}\\ &= 21.3\end{aligned}$$

$$0.75\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}} = 18.1$$

$$\frac{a}{t} = 21.3 > 18.1$$

To size the stiffener, Dowswell (2009) can be used. From Dowswell, the required stiffener moment of inertia is:

$$I_c = 0.92\left(\frac{a}{b}\right)t^4\sqrt{\left(\frac{b}{t}\right)^2 - 144} \tag{C-8}$$

where the terms are as previously defined. Equation C-7 is valid for aspect ratios, a/b, from ½ to 2.

For the top edge of the gusset of Example 6.1a (Figures 6-1 and 6-5):

$$\frac{a}{b} = 0.571$$

$$\frac{b}{t} = 28.0$$

$$t = 1.00\text{ in.}$$

$$\begin{aligned}I_c &= 0.92(0.571)(1.00\text{ in.})^4\sqrt{(28.0)^2 - 144}\\ &= 13.3\text{ in.}^4\end{aligned}$$

Try a stiffener ¾ in. × 6.00 in.:

$$\begin{aligned}I_c &= \frac{1}{12}(\tfrac{3}{4}\text{ in.})(6.00\text{ in.})^3\\ &= 13.5\text{ in.}^4 > 13.3\text{ in.}^4 \quad \textbf{o.k.}\end{aligned}$$

The stiffener length of 14 in. is chosen to keep it clear of the column flange. The stiffener is attached to the gusset plate with 3⁄16-in. fillet welds, the minimum required size based on AISC *Specification* Table J2.4.

APPENDIX D
Transfer Forces

The AISC *Code of Standard Practice for Steel Buildings and Bridges* (AISC, 2010a), hereafter referred to as the AISC *Code of Standard Practice*, states that, when the fabricator is required to design connection details, the structural design drawings shall provide, among other things, "data concerning the loads, including shears moments, axial forces and transfer forces, that are to be resisted by the individual members and their connections, sufficient to allow the fabricator to select or complete connection details…"

This statement is most remarkable for what it does not require. It does not require the fabricator to be provided with all of the loads, or even loads that satisfy equilibrium. It merely requires that *sufficient* information be provided. It should also be noted that the term transfer force is not defined in the glossary for either the AISC *Code of Standard Practice* or the AISC *Specification*. In fact a survey of steel design handbooks will demonstrate that the term is neither ubiquitous nor well-understood. It is therefore useful to begin any discussion of transfer forces with a definition. A possible definition for a transfer force is a force that must be transferred from one element to another through connection elements and whose magnitude and direction cannot be ascertained from the maximum member end forces.

An example is useful to demonstrate the concept. Assume there is bracing as shown in Figure D-1. It would be logical to assume that there is some transfer force or transfer of lateral load from the braced frame of the left-hand side of the column to the braced frame on the right-hand side of the column. In order to determine the magnitude and direction of the transfer force, the forces in the member framing to the joint must be examined. These forces are indicated in Table D-1. The transfer force, *TF*, can be defined using the *x* direction components of the forces as:

$$TF = 1_x + 2_x + 6_x \text{ or } TF = 3_x + 7_x$$

Performing these calculations results in the transfer forces shown in Table D-1. The maximum member end forces are presented in Table D-2. As can be seen the correct transfer forces could not be determined from the maximum member end forces alone, since there is no way to ensure that equilibrium has been satisfied. An assumption could be made that all of the members on one side of the column act in the same direction. This has been done to obtain the transfer forces shown in Table D-2. This results in excessive transfer forces—on the order of 8 times those actually required. Trying to design for these forces would be expensive and would in many cases require reinforcement of the members in order to transfer the force. It is also interesting to note that the load combination that produces the largest transfer force contains the maximum member end force for only one of the five members of interest.

D.1. THE EFFECT OF CONNECTION CONFIGURATION ON THE TRANSFER FORCE

The connection configuration can often have a significant impact on the calculation of the transfer force. For example, if the configuration shown in Figure D-1 was changed so that the members were connected using near side and far side plates as shown in Figure D-2, there is a change in the transfer force information required to design the connection. In the configuration shown in Figure D-1, the connection will deliver the vertical component of the brace load to the column. Therefore, there is no need to supply a vertical transfer force. However, it is unclear how much of the horizontal component must be transferred through the joint, and how

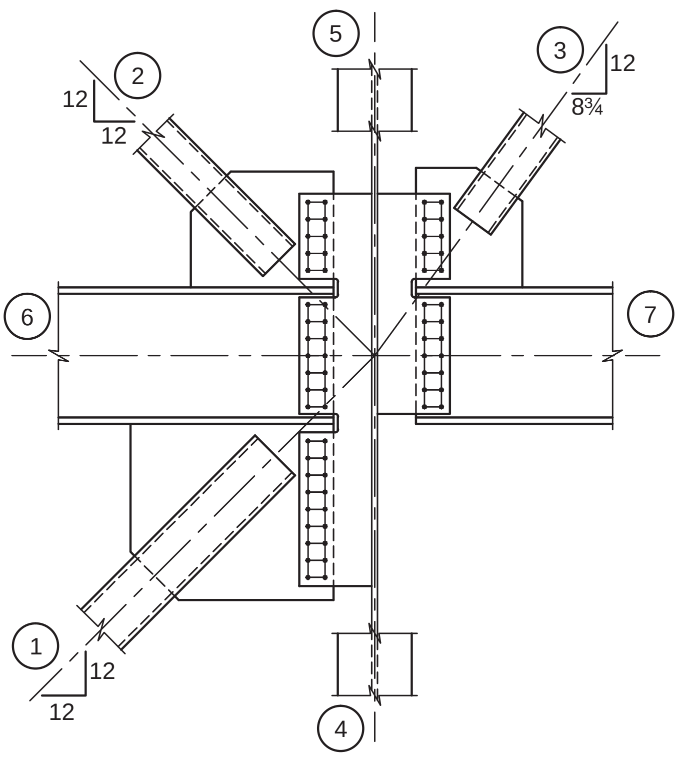


Fig. D-1. Configuration used to calculate transfer forces.

Table D-1. Member End Forces and Transfer Forces for Connection Shown in Figure D-1								
Load Combination	Member ID							TF
	Axial 1	Axial 2	Axial 3	Axial 4	Axial 5	Axial 6	Axial 7	TF
	kips	kips	kips	kips	kips	kips	kips	kips
1	–509	217	–198	–125	–479	120	30.2	–86.7
2	366	–386	113	–867	–426	24.9	–55.6	10.9
3	323	–436	87.6	–1160	–697	68.2	–63.4	–11.6
4	–466	267	–173	170	–209	76.8	38	–64.1
5	–185	–219	–50.3	–1130	–1070	194	–62	–91.6
6	41.6	50.3	34.8	145	167	–49	36.5	16.0

+ = Tension
– = Compression

much stays within the left-hand bay. Either statically consistent loadings or a horizontal transfer force must be provided to ensure the connection is properly designed.

In the configuration shown in Figure D-2, the situation is reversed. Since all of the horizontal load can be transferred directly through the near side and far side plates, the horizontal transfer is only a secondary concern. However, the magnitude of the vertical load that must be delivered to the column is unclear. Again, either statically consistent loadings or a vertical transfer force must be provided to ensure the connection is properly designed.

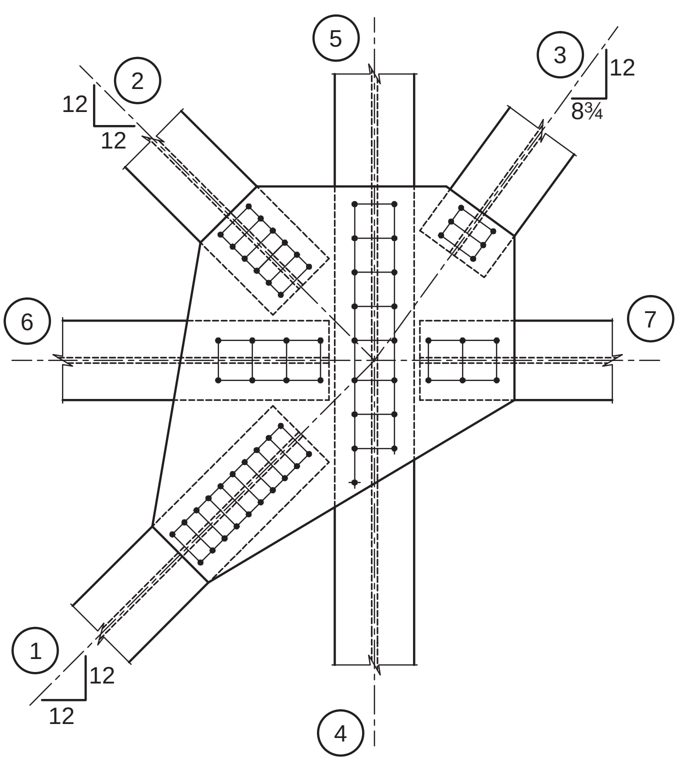


Fig. D-2. Alternate connection configuration.

Assuming the members carry only axial loads, the transfer force, *TF*, can be defined as:

$$TF = 1_y + 2_y + 3_y$$

D.2. PRESENTATION OF TRANSFER FORCES IN DESIGN DOCUMENTS

There are two possible methods that can be used to satisfy the AISC *Code of Standard Practice* requirement that sufficient load information be provided to complete the connection design. The first method involves providing a matrix of statically consistent load combinations that can be used to determine the required transfer force. An example of this type of presentation based on Table D-1 is shown in Figure D-3.

The procedure for generating such a matrix can be described as follows:

1. Define the assumed connection configuration.
2. Determine the load combinations that produce the maximum tension and maximum compression end forces for each member framing into a joint. Additionally, determine the load combinations that produce the maximum tension and maximum compression transfer forces.
3. For each load combination found in Step 1, give the axial forces in all members associated with that load combination so that joint equilibrium is provided for each. The maximum number of load cases will be equal to 2(n+1), where n is the number of members framing into the joint.

Possible advantages of this approach are as follows:

- Joint equilibrium is always provided for each load case, making the load paths very clear.

Table D-2. Maximum Member End Forces and Apparent Transfer Forces for Connection Shown in Figure D-1						
	Member ID					
	Axial 1	**Axial 2**	**Axial 3**	**Axial 6**	**Axial 7**	**TF**
	kips	**kips**	**kips**	**kips**	**kips**	**kips**
Maximum Compression	509	436	198	49	63.4	717
Maximum Tension	366	267	113	194	36.5	620

- The most economical connection can be provided because all forces are given.
- The generation of the required load data can be automated with a post-processor program.

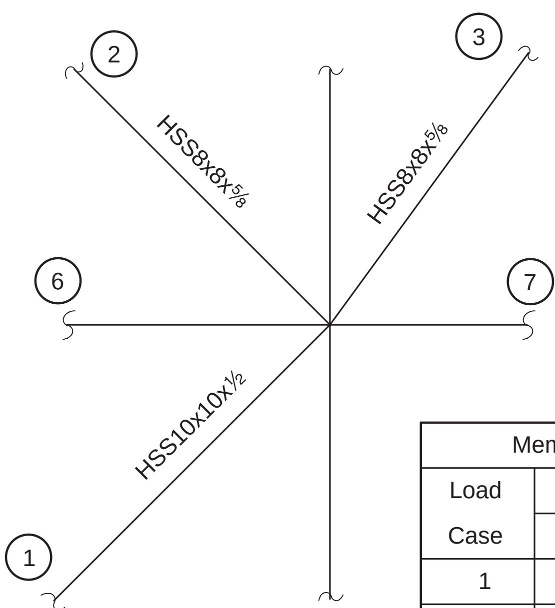
Possible disadvantages of this approach are as follows:

- A large volume of data must be conveyed to the fabricator.
- The fabricator must perform the calculations necessary to extract the transfer force. This is a transfer of labor from the engineer of record to the fabricator.
- As the generation of the connection data will undoubtedly be automated, there is a potential that less checking of computer output will be performed.

The second method of providing sufficient load information to complete the connection design is the approach most commonly used. It involves providing simply the maximum tension and compression axial forces and a required transfer force. An example of this type of presentation based on Table D-1 is shown in Figures D-4(a) and (b).

Possible advantages of this approach are as follows:

- The volume of load data is greatly reduced.



Member Axial End Forces and Transfer Forces						
Load Case	Member ID					TF
	1	2	3	6	7	
1	-509	217	-198	120	30.2	-86.7
2	366	-386	113	24.9	-55.6	10.9
3	323	-436	87.6	68.2	-63.4	-11.6
4	-466	267	-173	76.8	38	-64.1
5	-185	-219	-50.3	194	-62	-91.6
+ = Tension - = Compression						

Fig. D-3. Presentation of load information in tabular form.

- The process of generating the transfer force may encourage more checking of the computer output.
- The fabricator is not burdened with additional labor that has historically been performed by the engineer of record.

Possible disadvantages of this approach are as follows:

- It would be more difficult to automate this approach.
- The fabricator has less flexibility to design more economical, alternative connections.
- Without statically consistent loadings, the fabricator has a greatly diminished understanding of the engineer of record's intent and requirements.

It should be noted that, regardless of the presentation, the transfer forces must be calculated. Also the possibility exists, though unlikely, that the three-step process described previously may not capture the most critical condition for some of the internal connection plate stresses. It is difficult to conceive of a case where this could occur using the configuration illustrated in Figure D-1, but may be possible using the configurations illustrated in Figures D-2 and D-5.

D.3. ADDITIONAL CONSIDERATIONS

In the absence of loadings that satisfy equilibrium, the fabricator is required to make assumptions about the direction of the given loads. The most reasonable assumption when faced with a bracing connection where braces frame both above and below the beam is that while one brace is in tension, the other is in compression. This would usually be consistent with the lateral loading of a braced frame. However, this assumption is not always correct. For example, in Table D-1 it can be seen that for most of the load combinations members 1 and 2 have opposite signs, conforming to the stated assumption. However, load combination 5 puts both braces into compression; while load combination 6 puts both braces into tension.

It can also be problematic to check gusset plates when the given loads do not satisfy equilibrium. As an example, Figure D-5 shows a chevron brace, which in addition to resisting lateral loads, also supports a column. Given only the maximum member forces, as shown, it is impossible with any certainty to derive the correct design forces, and the design of the gusset plate must be based on potentially erroneous assumptions.

It might be concluded for instance that the column is unlikely to undergo significant uplift. The connection designer may therefore assume a force distribution as shown in Figure D-6. The free-body diagram shown satisfies equilibrium, with the force in the right-hand brace reduced to a value of 207 kips, but this is only an assumption made in an attempt to satisfy equilibrium. Although it is unlikely that the column will experience significant uplift, this does not preclude the possibility entirely. Any uplift, even with a magnitude considerably less than 500 kips, will increase the magnitude of the horizontal shear force.

Another approach might be to assume that the maximum brace forces shown act simultaneously. In this case, to satisfy equilibrium, the column force must be increased to 707 kips. This is shown in Figure D-7. Though the vertical force is maximized, the horizontal shear force is nonexistent. Since, in Figure D-7 a force is increased to satisfy equilibrium, while in Figure D-6 a force is decreased, it might be assumed that Figure D-7 represents the most critical case. However, this logic does not always hold true.

Still another possibility exists. The column may not experience any uplift, but may unload. This is illustrated in Figure D-8. Because the horizontal shear applied in Figure D-8 is

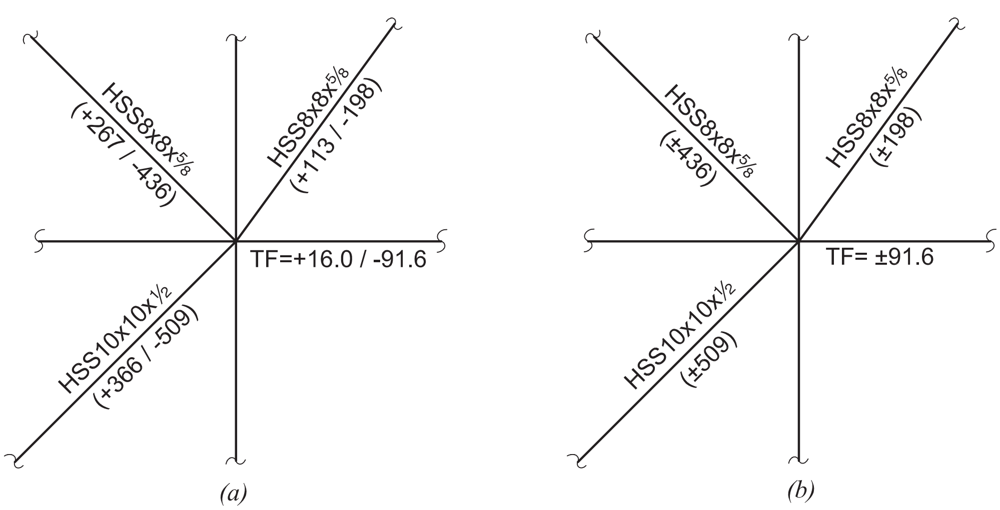


Fig. D-4. Presentation of load information using only the maximum member forces.

equal to the vertical transverse load applied in Figure D-7, the required plate thickness and weld size will be greater for the forces shown in Figure D-8. However, this is not necessarily the critical case either, for while the force shown in Figure D-7 will cause significant compression at the bottom edge of the gusset and the potential for buckling, no such concern exists for the forces shown in Figure D-8. This illustrates the intractable situation the connection designer is faced with when provided forces that do not satisfy equilibrium.

D.4. EFFECTS OF MODELING ASSUMPTIONS ON TRANSFER FORCES

The analysis and design of a concentrically braced frame may at first seem to be a trivial task. Since the structure is determinate, the calculation of forces within the bracing system is not complicated. However, the means of delivering the forces to the bracing system can have a significant impact on the connection design.

For example, once the lateral force is known it can be delivered to the system in various ways. In Figure D-9 the loads are delivered to the bracing system in varying ways, which will affect the design of both the connections and the members. The lateral force at structure (a) would require a transfer force at the left-hand side of the frame. The lateral force at structure (b) would require a transfer force at the right-hand side of the frame, while structure (c) with the load presumably delivered through the diaphragm directly to the braced frame would not require a transfer force at all.

If during the analysis the load is assumed to be delivered at a joint (or node), then it will not show up as a member force. In such cases care must be taken to ensure that the transfer is

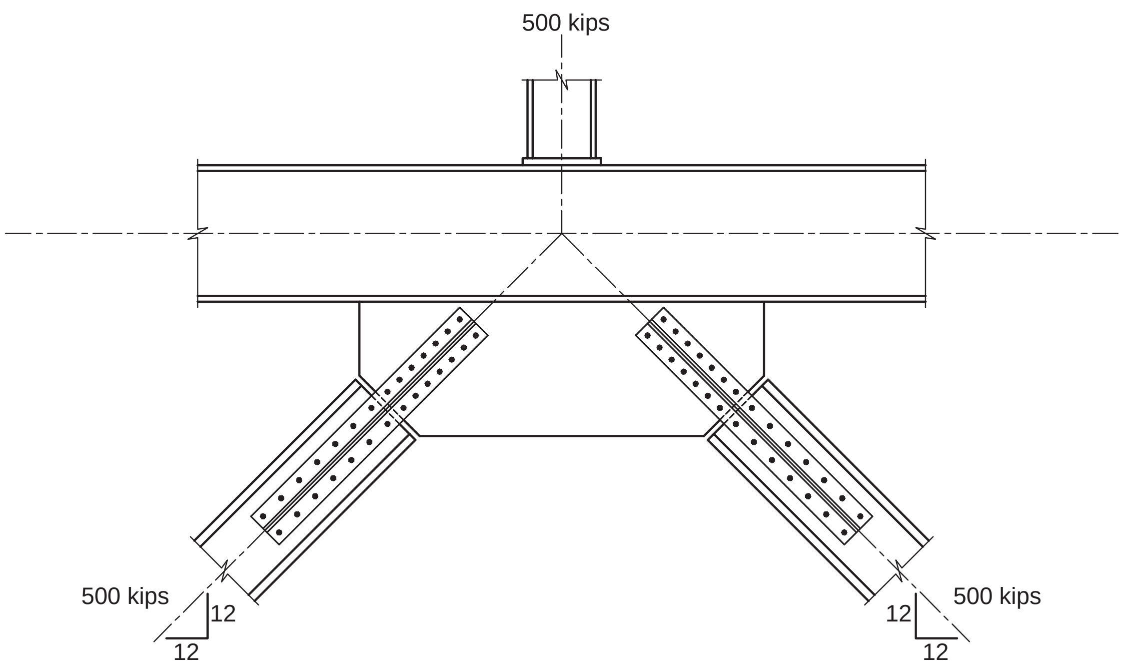


Fig. D-5. Bracing subjected to gravity and lateral loads.

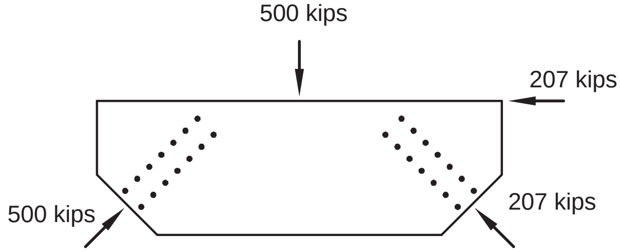


Fig. D-6. A possible free body diagram for the gusset plate shown in Figure D-5.

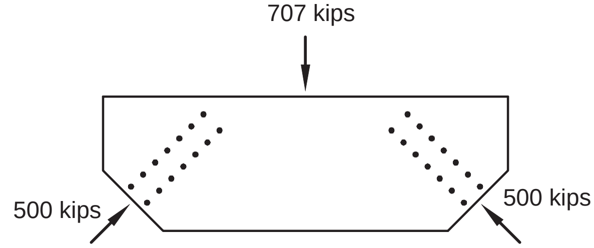


Fig. D-7. A possible free body diagram for the gusset plate shown in Figure D-5.

Table D-3. Member End Forces and Transfer Forces for Connection Shown in Figure D-2								
Load Combination	Member ID							TF
	Axial 1	Axial 2	Axial 3	Axial 4	Axial 5	Axial 6	Axial 7	
	kips	kips	kips	kips	kips	kips	kips	kips
1	–509	217	–198	–125	–479	120	30.2	354
2	366	–386	113	–867	–426	24.9	–55.6	–441
3	323	–436	87.6	–1160	–697	68.2	–63.4	–466
4	–466	267	–173	170	–209	76.8	38	379
5	–185	–219	–50.3	–1130	–1070	194	–62	–64.6
6	41.6	50.3	–34.8	145	167	–49	36.5	–21.9

+ = Tension
– = Compression

not neglected. A nodal load column could be added to Table D-3, but this too presents problems in terms of connection design. Applying the load to an infinitesimally small point in the model may seem a perfectly reasonable assumption during analysis, but there are few practical ways to transfer the load directly to the joint without first delivering it to the other members, which may or may not be capable of resisting the additional load. Therefore, providing a theoretical load transfer directly to the node again places the connection designer in the unenviable position of making assumptions about the proper mode of load transfer.

Practical problems can also occur in the case of Figure D-9(c). There is an assumed transfer of force between the diaphragm and the beam. However, the presence of bracing connections at the top flange of the beam could limit the number of shear studs that can be placed in this location. The situation is illustrated in Figure D-10.

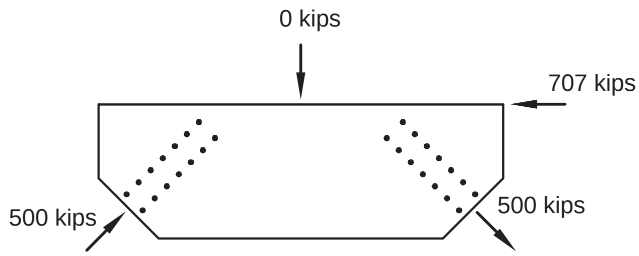


Fig. D-8. A possible free body diagram for the gusset plate shown in Figure D-5.

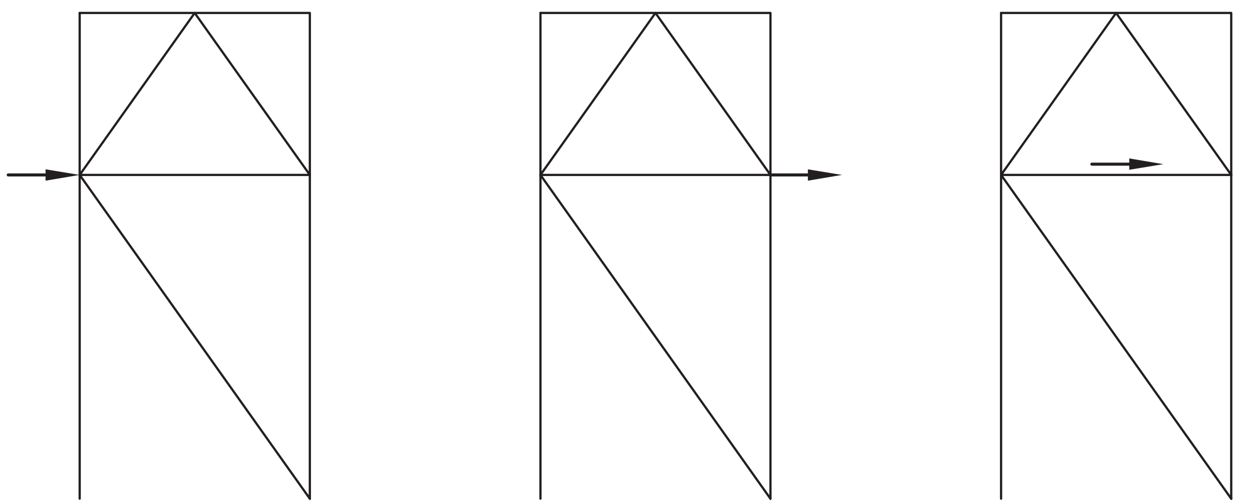


Fig. D-9. Design assumptions for delivering lateral force to the braced frame.

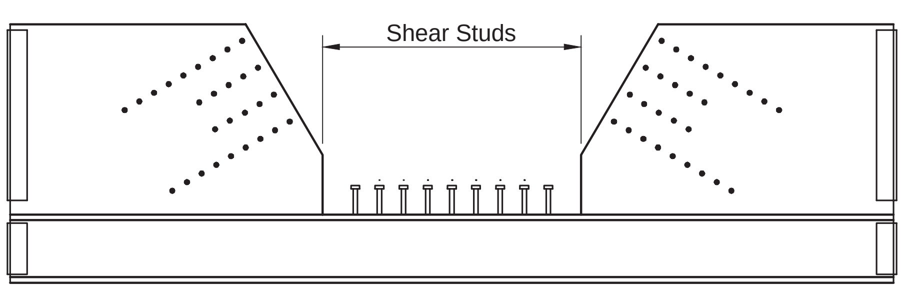


Fig. D-10. Number of shear studs limited by brace connections.

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